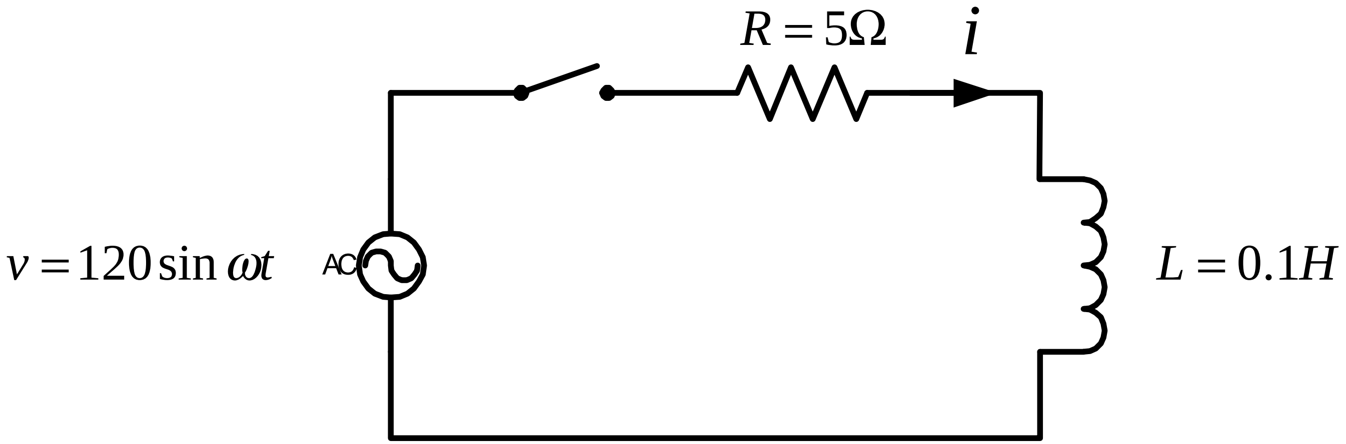
R-L and R-C Circuits

R-L Circuit

Considering a R-L circuit:



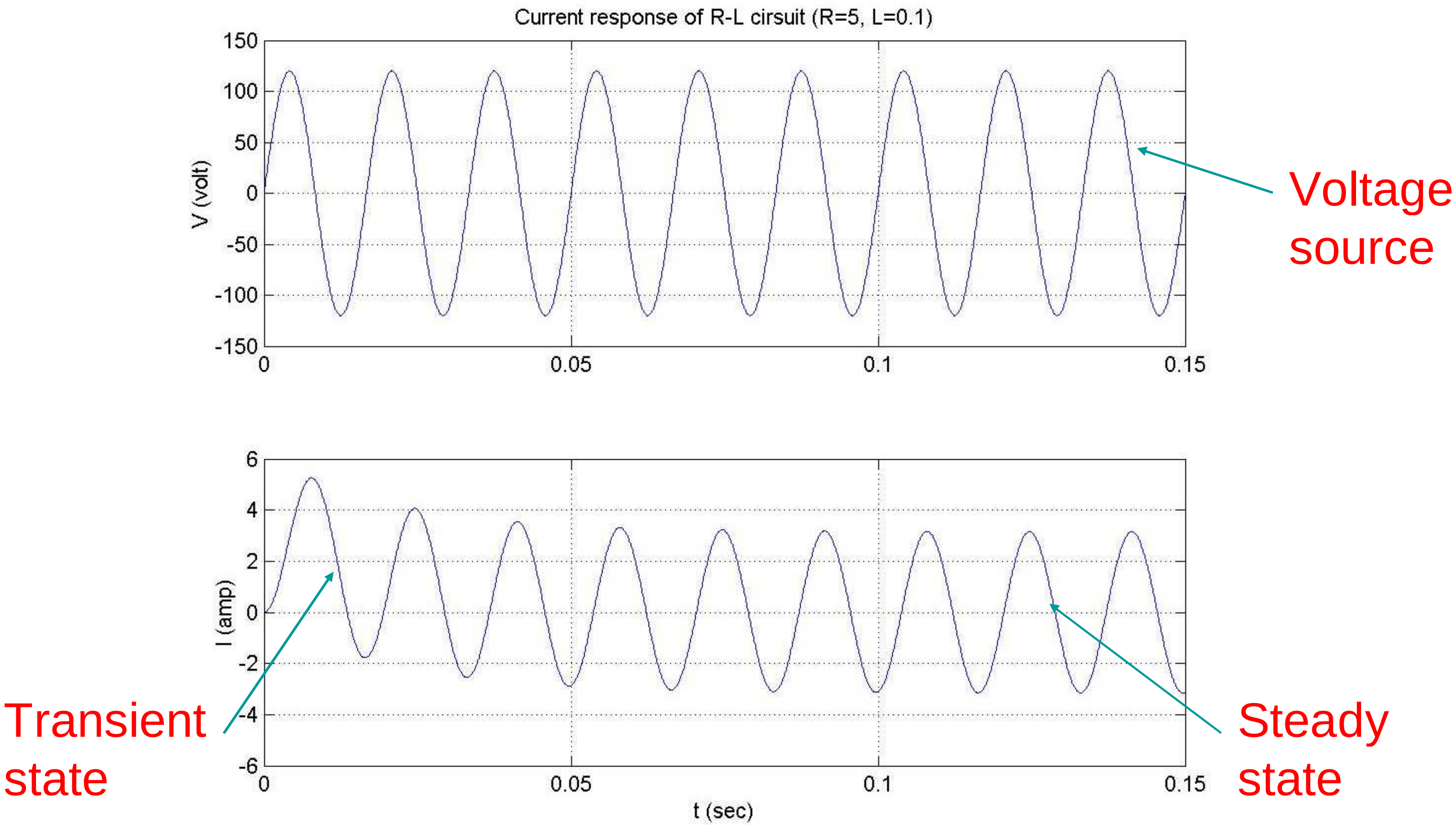
If at t=0, the switch is closed. What will be the response inductor current i?

The system differential equation is:

$$v = Ri + L\frac{di}{dt}$$

R-L Circuit

After some time (transient state), the current reaches steady state as shown in the figure.



R-L Circuit

At steady state, v and i can be represented by phasor V and I, where

$$V = V_{rms}\angle\theta_V \qquad V_{rms} = 120\Big/\sqrt{2}$$

$$I = I_{rms}\angle\theta_I \qquad \theta_I = \omega t + \theta_{I,0}$$

θ_V is the phase angle of V.

θ_I is the phase angle of I. It is determined by θ_V and impedance angle θ ($\theta_I = \theta_V - \theta$)

or

$$I = I_{rms}e^{j(\omega t+\theta_{I,0})} \Rightarrow \frac{dI}{dt} = j\omega I_{rms}e^{j(\omega t+\theta_{I,0})} = j\omega I$$

The differential equation becomes

$$V = (R + j\omega L)I$$

R-L Circuit

Let $X_L = \omega L$ Inductor Reactance

$$V = (R + jX_L)I$$

Let

$$Z = R + jX_L = |Z|\angle\theta \qquad \theta = \tan^{-1}(\frac{X_L}{R}) > 0$$

We have $\theta_V - \theta_I = \theta$

Generally we choose V as reference, then θ_V =0

$$V = |V|\angle 0 \qquad I = {}^{V}\!/_{Z} = |I|\angle -\theta$$

I lags V

R-L Circuit

The power supplied by the power source is

$$S = VI^* = |V||I|\cos\theta + j|V||I|\sin\theta$$

$$= P + jQ$$

Lagging – positive reactive power

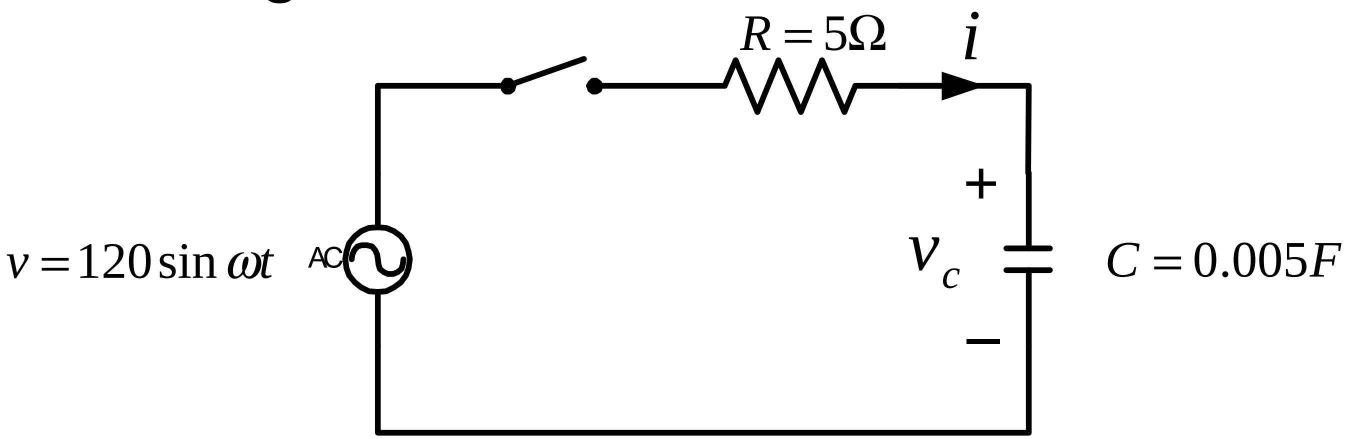
Or in other form,

$$S = VI^* = (R + jX_L)I \cdot I^*$$

$$= |I|^2 R + j|I|^2 X_L = P + jQ$$

R-C Circuit

Considering a R-C circuit:



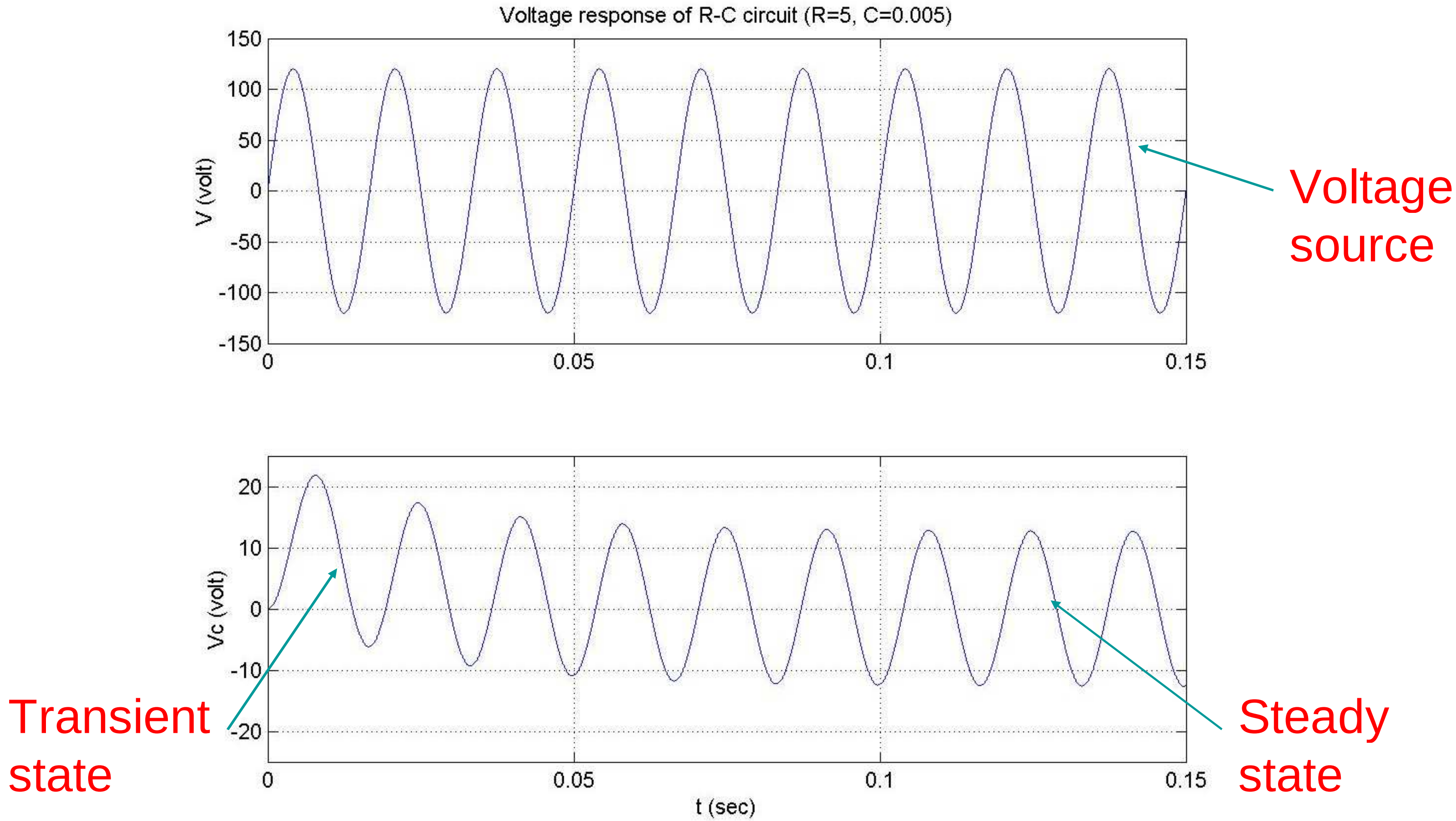
If at t=0, the switch is closed. What will be the response capacitor voltage v_c?

The system differential equation is:

$$v = RC\frac{dv_c}{dt} + v_c$$

R-C Circuit

After some time (transient state), the voltage reaches steady state as shown in the figure.



R-C Circuit

At steady state, v and v_c can be represented by phasor V and V_C, where

$$V = V_{rms} \angle \theta_V \qquad V_{rms} = 120 / \sqrt{2}$$

$$V_C = V_{C,rms} \angle \theta_{Vc}$$

$$\frac{dV_C}{dt} = j\omega V_C \qquad I = C\frac{dV_C}{dt} = j\omega C V_C$$

or

$$V_C = I\left(\frac{1}{j\omega C}\right) = I\left(-j\frac{1}{\omega C}\right)$$

R-C Circuit

so $$V=(R-j\frac{1}{\omega C})I$$

Let $$X_C={1}/{\omega C}$$ Capacitor Reactance

$$V=(R-jX_C)I$$

Let $Z=R-jX_C=|Z|\angle-\theta \quad \theta=\tan^{-1}(\frac{X_C}{R})$

We have $\theta_V-\theta_I=-\theta$

Generally we choose V as reference, then θ_V=0

$$V=|V|\angle 0 \qquad I={V}/{Z}=|I|\angle\theta$$ I leads V

R-C Circuit

The power supplied by the power source is

$$S = VI^* = |V||I|\cos\theta - j|V||I|\sin\theta$$

$$= P - jQ$$

Leading – negative reactive power

Or in other form,

$$S = VI^* = (R - jX_C)I \cdot I^*$$

$$= |I|^2 R - j|I|^2 X_C = P - jQ$$