Complex amplitude technique

Step 1. Associate the actual voltage sources (or current sources) waveform with the complex amplitudes

Actual waveform		**Complex amplitude**
$v(t) = V_M \sin(\omega t)$	==>	$\mathbf{V_M}$
$i(t) = I_M \sin(\omega t)$	==>	$\mathbf{I_M}$
$v(t) = V_M \sin(\omega t+\phi)$	==>	$\mathbf{V_M} \angle \phi$; (equivalent to the Euler's form $V_M e^{j\phi}$)
$i(t) = I_M \sin(\omega t+\phi)$	==>	$\mathbf{I_M} \angle \phi$; (equivalent to the Euler's form $I_M e^{j\phi}$)

Complex amplitude technique

Step 1. Associate the actual voltage sources (or current sources) waveform with the complex amplitudes

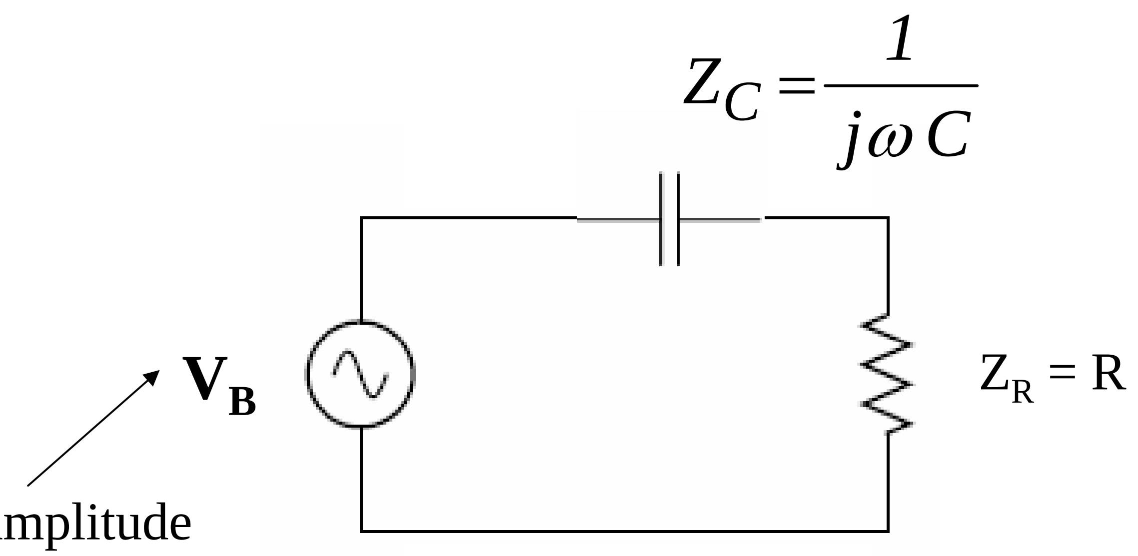
Actual waveform ⇒ **Complex amplitude**

Examples

$i(t) = 2.3 \times \sin(360 * t)$ A	==>	**2.3 A**
$v(t) = 6 \times \sin(5E7 * t + 1.57)$ V	==>	$\mathbf{6} \times e^{j\,1.57}$ **V**
	or	$\mathbf{6}\ \angle 1.57$ **V**

Complex amplitude technique

Step 2. Assign impedances (or admittances) to all the circuit components



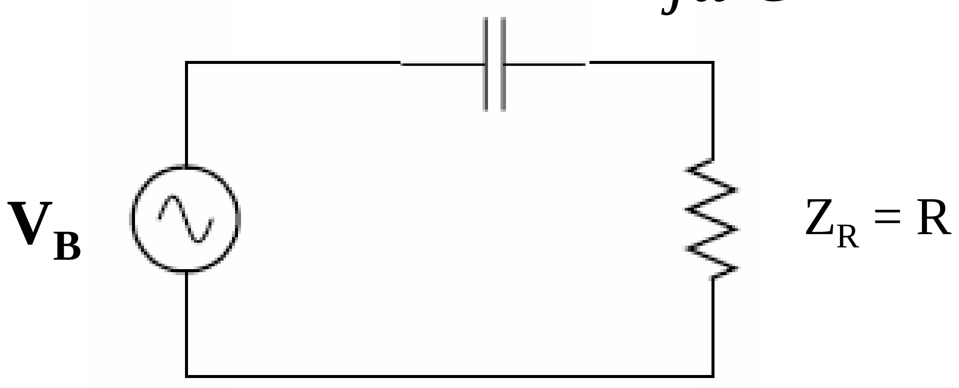
Complex amplitude
of the source voltage

Complex amplitude technique

Step 3. Find the complex current – complex voltage relations in the circuit using the "AC Ohm's law", KVL and KCL.

Any method known for DC circuit analysis (series, parallel, mesh analysis, etc.) can be used.

$$Z_C = \frac{1}{j\omega C}$$



In this series circuit, the complex current amplitude,

$$\mathbf{I_M} = \frac{\mathbf{V_B}}{Z_C + Z_R}$$

Complex amplitude technique

Step 4. Convert that found complex current and/or voltage amplitudes into the polar (Euler's) form

$$\mathbf{I_M} = I_{MR} + j\, I_{MIm} \quad \Rightarrow \quad I_M\, e^{j\varphi} = I_M \angle \varphi$$

Real and Imaginary components of the complex current (or voltage) amplitudes found in the step 3.

At this step, I_{MR} and I_{MIm} are the numbers.

Complex amplitude technique

Step 5. Convert the complex current and/or voltage amplitudes into the actual waveform

The found complex current amplitude has the form (I_M and φ are real numbers):

$$I_M e^{j\varphi}$$

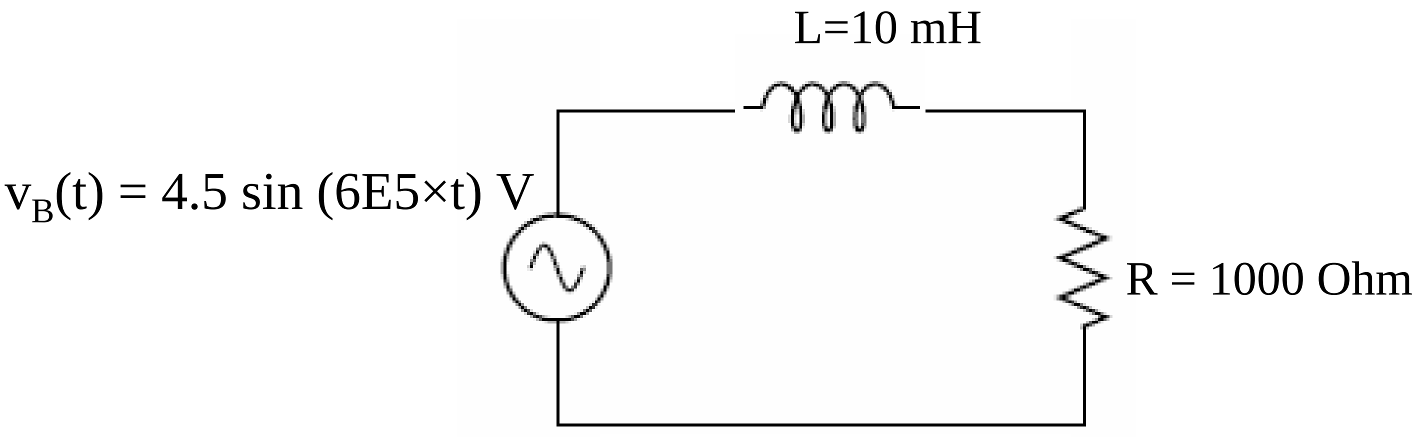
I_M is the modulus of the found complex amplitude;
ϕ is the angle of the found complex amplitude

The actual current waveform has the same amplitude I_M and the same phase angle φ:

$$i(t) = I_M \sin(\omega t + \varphi)$$

The current amplitude = I_M
The current phase angle = φ.

Example 1: Series L-R circuit



1. Complex amplitude of the voltage source, $\mathbf{V_B}$= 4.5 V

2. Find the impedances. The angular frequency $\omega = 6E5\ s^{-1}$

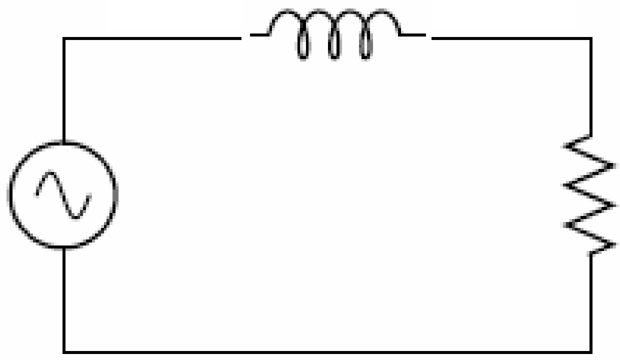
$Z_L = j\omega L = j * 6E5 * 10E{-3} = j\ 6E3$ Ohm;

$Z_R = R = 1E3$ Ohm

3. This is a series circuit. Total impedance:

$Z = Z_L + Z_R = 1E3 + j * 6E3$ Ohm

$v_B(t) = 4.5 \sin(6E5 \times t)$ V



4. Complex current amplitude using "the AC Ohm's law",

$I = \mathbf{V_B} / Z = \mathbf{V_B} / (R + j\omega L)$

I =4.5 /(1E3 + j *6E3) A …=1.22E-4-j7.3E-4

5. Convert complex current amplitude into a polar form

4.5 /(1E3 + j *6E3) => Polar

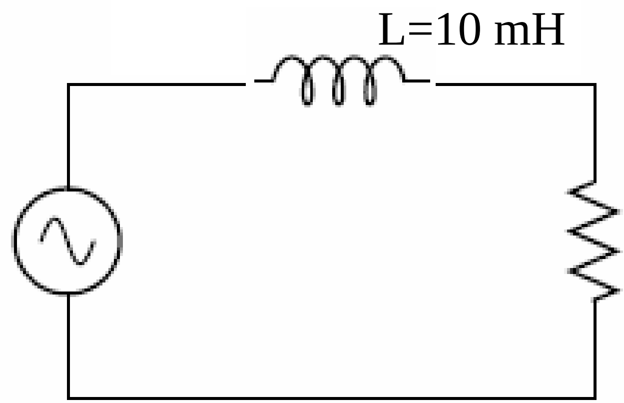
The complex current amplitude: **I =7.4E-4 ∠ -1.41 A;**

6. Convert complex amplitude into the actual current waveform

$i(t) = 7.4E\text{-}4 \sin(6E5 \times t - 1.41)$ A

The current amplitude = 7.4E-4 A; the current phase angle =-1.41 *rad*

$v_B(t) = 4.5 \sin(6E5 \times t)$ V



R = 1000 Ohm

7. Find the voltage drop across the resistor R

The current, $i(t) = 7.4E\text{-}4 \sin(6E5 \times t - 1.41)$ A

The voltage across the resistor,

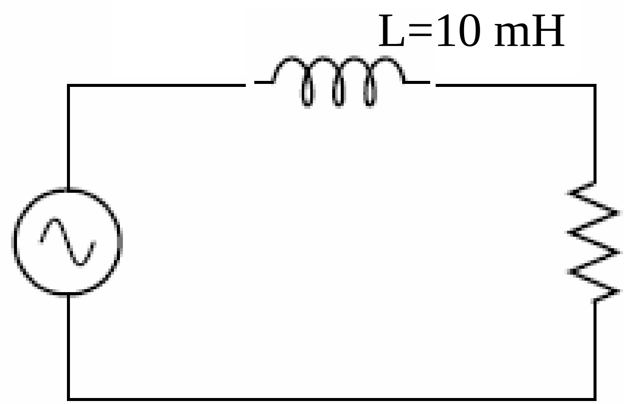
$v_R(t) = R \times i(t) = 1000 \times 7.4E\text{-}4 \sin(6E5 \times t - 1.41)$ V =

$= 0.74 \sin(6E5 \times t - 1.41)$ V

The resistor voltage amplitude = 0.74 V.

The resistor voltage phase angle = -1.41 *rad*

$v_B(t) = 4.5 \sin(6E5 \times t)$ V



R = 1000 Ohm

8. Find the voltage drop across the inductor L

The complex current amplitude:

$$\mathbf{I = 7.4E\text{-}4 \angle -1.41\ A}$$

Complex impedance of the inductor:

$$\mathbf{Z_L} = j\ 6E3 \text{ Ohm}$$

Complex voltage across the inductor:

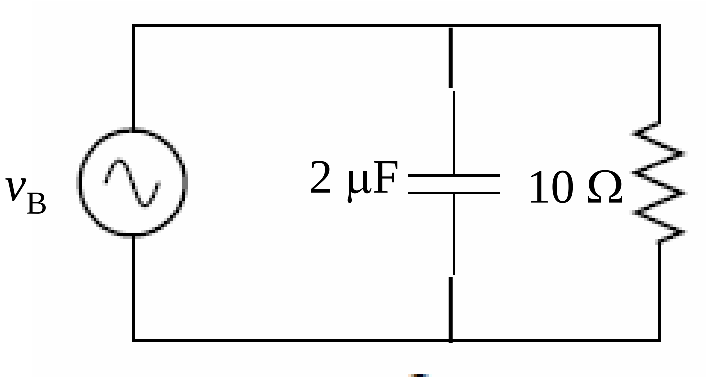
$$\mathbf{V_L} = \mathbf{Z_L} \times \mathbf{I = j6E3} \times \mathbf{(7.4E\text{-}4 \angle -1.41) \Rightarrow Polar \Rightarrow 4.44\ e^{j0.161}\ V = 4.44 \angle 0.161\ V}$$

Actual voltage waveform: $v(t) = 4.44 \sin(6E5 \times t + 0.161)$ V

The inductor voltage amplitude = 4.44 V.

The inductor voltage phase angle = 0.161 (*rad*)

Example 2: Parallel R-C circuit



Given the AC voltage source with the amplitude $V_M = 9.3$ V and the frequency f = 5 kHz,

find the total current in the circuit

1. $\omega = 2\pi f = 6.28 * 5E3 = 3.14E4\ s^{-1} = 3.14 \times 10^4\ s^{-1}$

2. Complex amplitude of the voltage source $\mathbf{V_M = 9.3\ V}$**;**

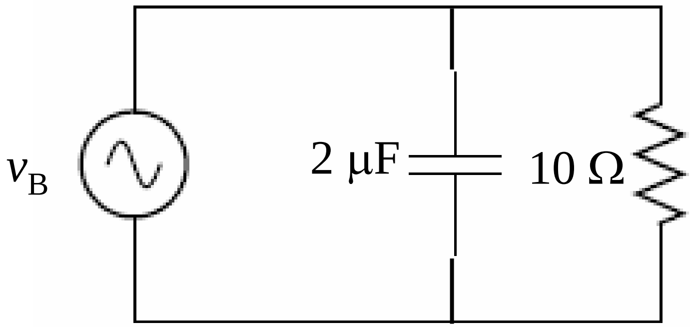
3. Capacitor impedance: $Z_C = 1/(j\omega C) = -j15.9$ Ohm.
Resistor impedance: $Z_R = R = 10$ Ohm.

4. This is a parallel circuit. Find total impedance:

a) $1/Z = 1/Z_C + 1/Z_R = j0.063 + 0.1$.

b) $Z = 1/(j0.063 + 0.1) = 7.16 - j4.51$

Example 2: Parallel R-C circuit



Given the AC voltage source with the amplitude V_M = 9.3 V and the frequency f = 5 kHz,

find the total current in the circuit

4. The total impedance:

Z = 1/(j0.063 + 0.1) = 7.16 – j4.51

5. The total current complex amplitude:

I = $\mathbf{V_M/Z}$ = 9.3/(7.16 – j4.51) = 0.93+j0.586 A

6. Convert current complex amplitude into polar form:

0.93+j5.86 => Polar => 1.1 $e^{j0.562}$ = 1.1 ∠ 0.562

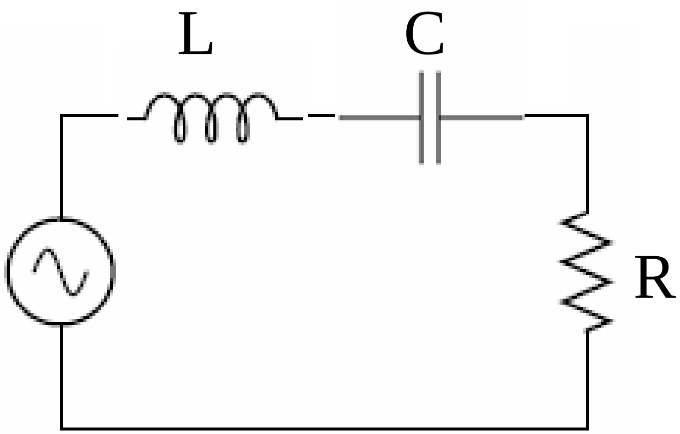
7. Total current amplitude = 1.1 A.

The phase angle = 0.562 rad

The frequency = 5 kHz

Example 3: Series L-C-R circuit

$v_B(t) = V_M \sin(\omega \times t)$



1. Find the impedances.

$Z_L = j\omega L$

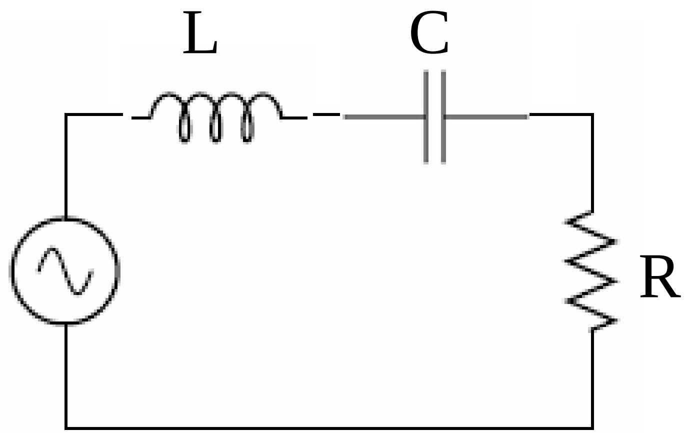
$Z_C = 1/ j\omega C$

$Z_R = R$

3. This is a series circuit. Total impedance:

$Z = Z_L + Z_C + Z_R = j\omega L + 1/ j\omega C + R$

Resonance in series L-C-R circuit



$v_B(t) = V_M \sin(\omega \times t)$

3. (cont) Total impedance:

$Z = Z_L + Z_C + Z_R = j\omega L + 1/\, j\omega C + R$

Note that

$$\frac{1}{j} = \frac{1}{j}\left(\frac{j}{j}\right) = \frac{j}{j^2} = -j$$

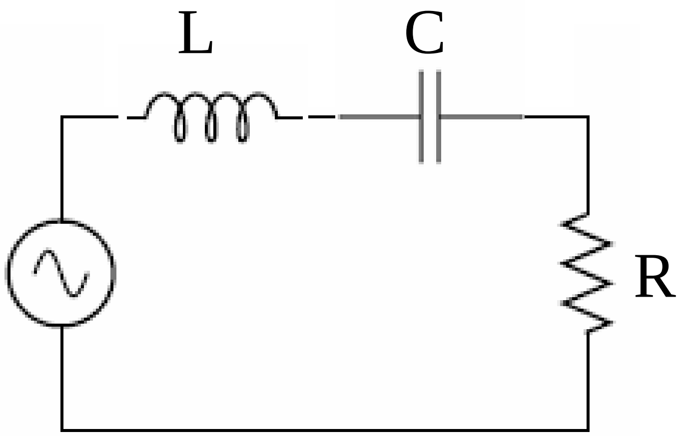
$Z = R + j(\omega L - 1/\omega C)$

Under certain conditions,

the imaginary part of this impedance = 0 (!)

Resonance in series L – C - R circuit

$v_B(t) = V_M \sin(\omega \times t)$



$Z = R + j(\omega L - 1/\omega C)$

Given L and C values, find the frequency f_0 at which

$Im(Z) = (\omega_0 L - 1/\omega_0 C) = 0$

$$\omega_0 L - \frac{1}{\omega_0 C} = \frac{\omega_0 \cdot L \cdot \omega_0 \cdot C - 1}{\omega_0 C} = \frac{\omega_0^2 \cdot L \cdot C - 1}{\omega_0 C} = 0$$

For Im(Z) = 0, $\omega_0^2 \cdot L \cdot C - 1 = 0$, or $\omega_0^2 \cdot L \cdot C = 1$,

$$\text{or } \omega_0 = \frac{1}{\sqrt{LC}} \quad \text{and } f_0 = \frac{1}{2\pi\sqrt{LC}}$$

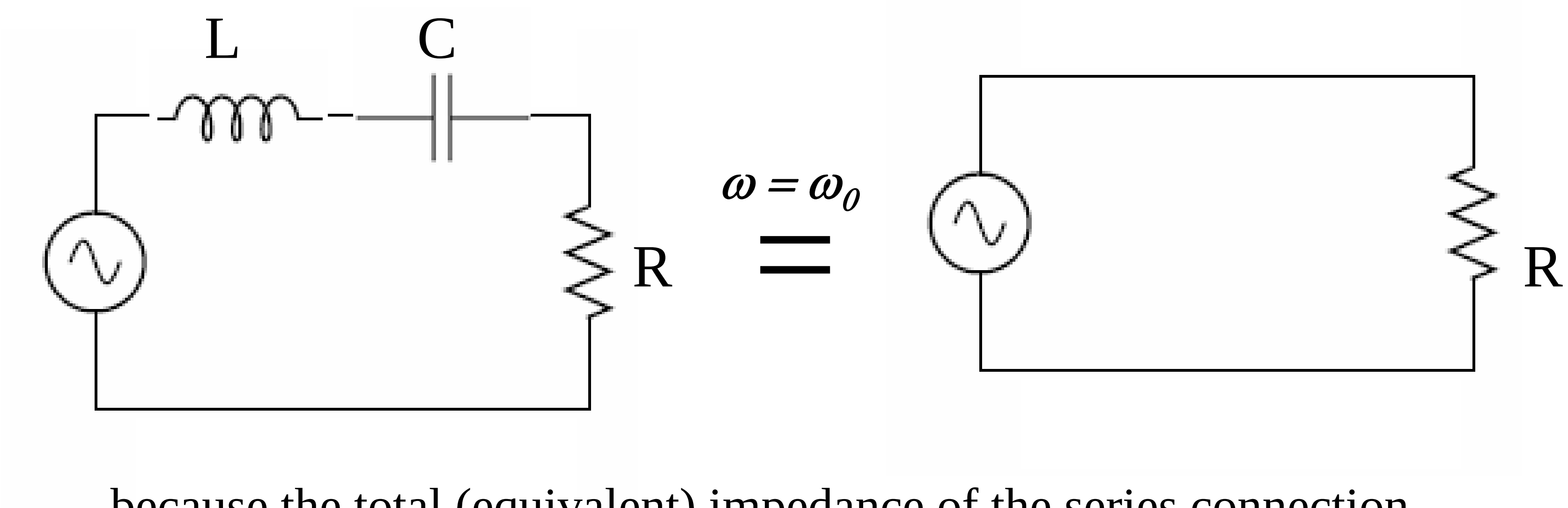
Resonance in series L-C-R circuit

For any L-C circuit

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad \text{and} \quad f_0 = \frac{1}{2\pi\sqrt{LC}}$$

are called the resonance angular frequency (ω_0) and the resonance frequency f_0 correspondingly.

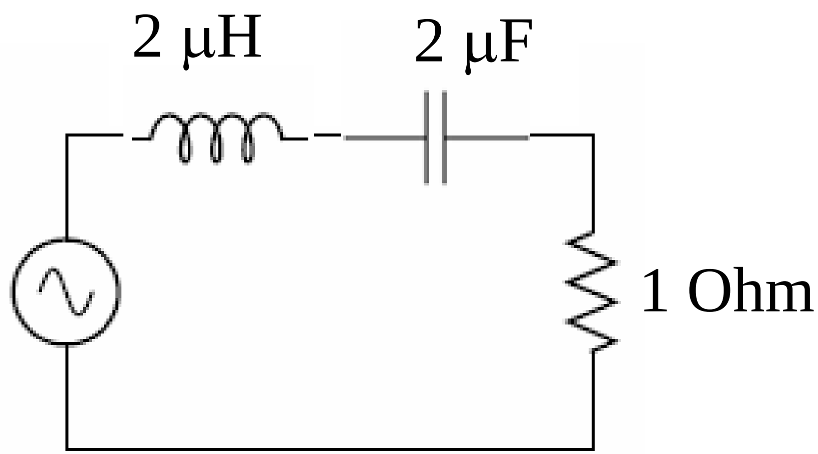
When the signal frequency f is chosen such that $f = f_0$,



because the total (equivalent) impedance of the series connection of L and C is zero: $Z_L + Z_C = 0$

Example 4: Resonance in series L-C-R circuit

Find the resonance frequency f_0 for the following circuit

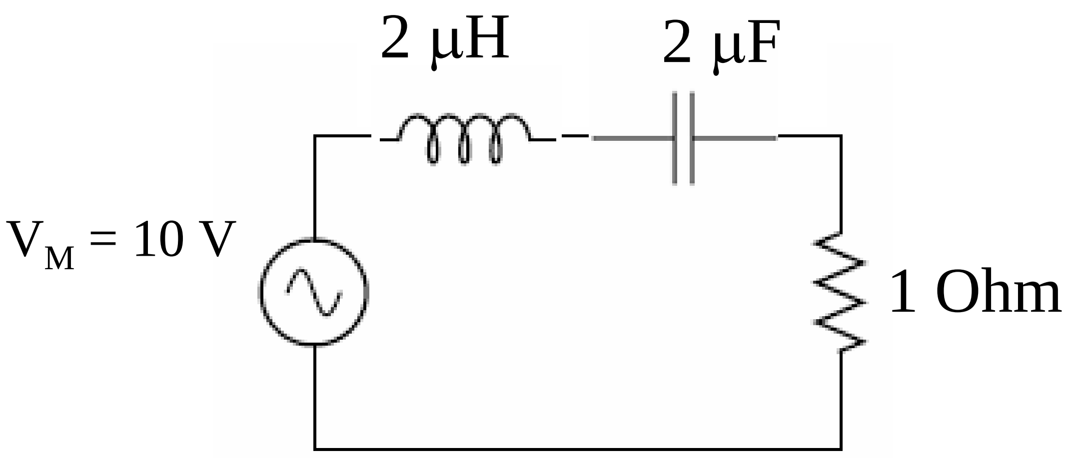


$$\omega_0 = \frac{1}{\sqrt{LC}}; \quad f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\cdot 3.14\cdot\sqrt{2\cdot 10^{-6}\cdot 2\cdot 10^{-6}}} = 7.96\cdot 10^4\ Hz$$

Example 4: Resonance in series L-C-R circuit

For the following circuit, find the current amplitude dependence on the signal frequency f.



Total impedanc

$$Z = Z_L + Z_C + Z_R = j\omega L + 1/\,j\omega C + R = R + j*(\omega L - 1/\omega C)$$

Complex amplitude of the current:

$$I = V_M /Z = V_M/ [R + j*(\omega L - 1/\omega C)]$$

here $\omega = 2\pi f$

Example 4: Resonance in series L-C-R circuit

MATLAB script to solve the problem

```
clear all
%circuit description
L=2e-6;
C=2e-6;
R=1;
VM=10;
% variable frequency
f=1e3:1e3:1e6;
%Circuit impedance
omega=2*pi*f;
Z=R+(omega*L-1./omega/C)*i;
I=VM./Z;
IM=abs(I);
figure(1)
plot(f,IM)
xlabel ('frequency, Hz')
ylabel ('Current amplitude, A')
```

Example 4: Resonance in series L-C-R circuit

MATLAB script explanation

```
clear all
%circuit description
L=2e-6;
C=2e-6;
R=1;
VM=10;
% variable frequency
f=1e3:1e3:1e6;
%Circuit impedance
omega=2*pi*f;
Z=R+(omega*L-1./omega/C)*i;
I=VM./Z;
IM=abs(I);
figure(1)
plot(f,IM)
xlabel ('frequency, Hz')
ylabel ('Current amplitude, A')
```

Definition of the circuit components

Example 4: Resonance in series L-C-R circuit

MATLAB script explanation

```
clear all
%circuit description
L=2e-6;
C=2e-6;
R=1;
VM=10;
% variable frequency
f=1e3:1e3:1e6;
%Circuit impedance
omega=2*pi*f;
Z=R+(omega*L-1./omega/C)*i;
I=VM./Z;
IM=abs(I);
figure(1)
plot(f,IM)
xlabel ('frequency, Hz')
ylabel ('Current amplitude, A')
```

Definition of the variable frequency

Example 4: Resonance in series L-C-R circuit

MATLAB script explanation

```
clear all
%circuit description
L=2e-6;
C=2e-6;
R=1;
VM=10;
% variable frequency
f=1e3:1e3:1e6;
%Circuit impedance
omega=2*pi*f;
Z=R+(omega*L-1./omega/C)*i;
I=VM./Z;
IM=abs(I);
figure(1)
plot(f,IM)
xlabel ('frequency, Hz')
ylabel ('Current amplitude, A')
```

Calculation of the complex current and current amplitude

Example 4: Resonance in series L-C-R circuit

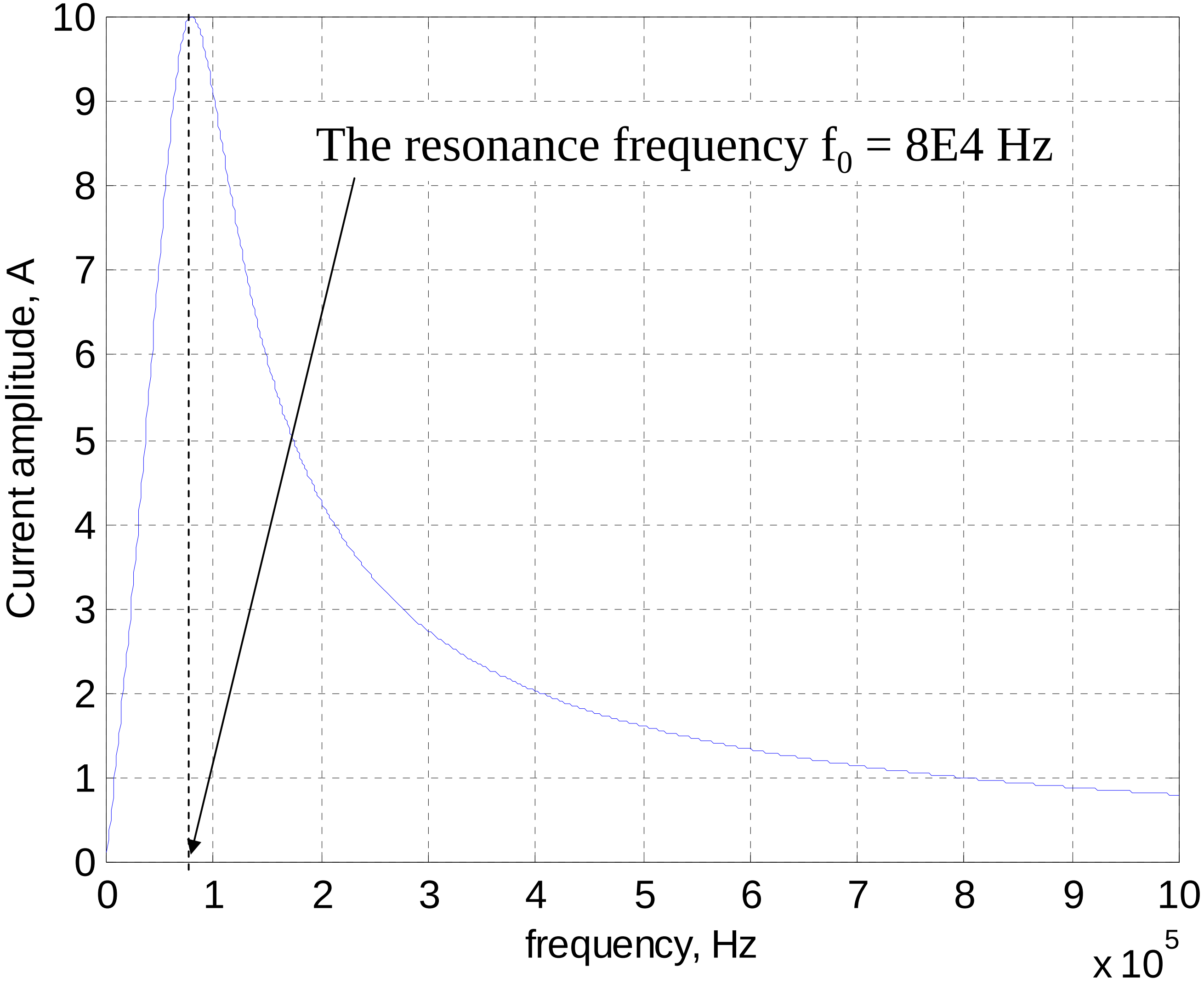
MATLAB script explanation

```
clear all
%circuit description
L=2e-6;
C=2e-6;
R=1;
VM=10;
% variable frequency
f=1e3:1e3:1e6;
%Circuit impedance
omega=2*pi*f;
Z=R+(omega*L-1./omega/C)*i;
I=VM./Z;
IM=abs(I);
figure(1)
plot(f,IM)
xlabel ('frequency, Hz')
ylabel ('Current amplitude, A')
```

Plotting the results

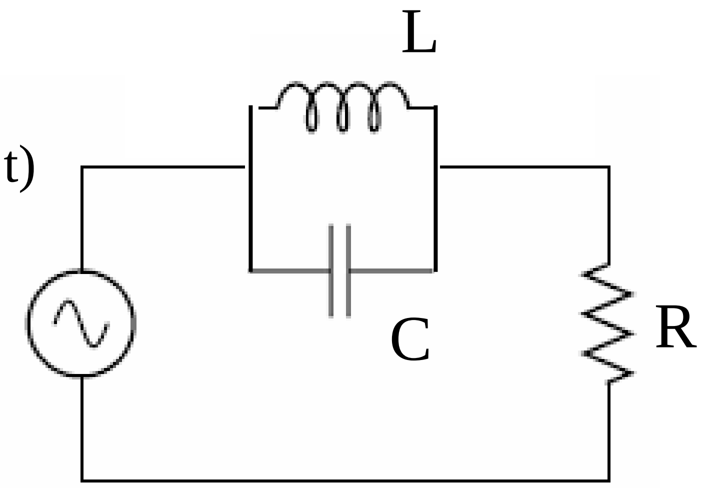
Example 4: Resonance in series L-C-R circuit

MATLAB results



Resonance in parallel L – C circuit

$v_B(t) = V_M \sin(\omega \times t)$



An impedance of parallel connection of L and C:

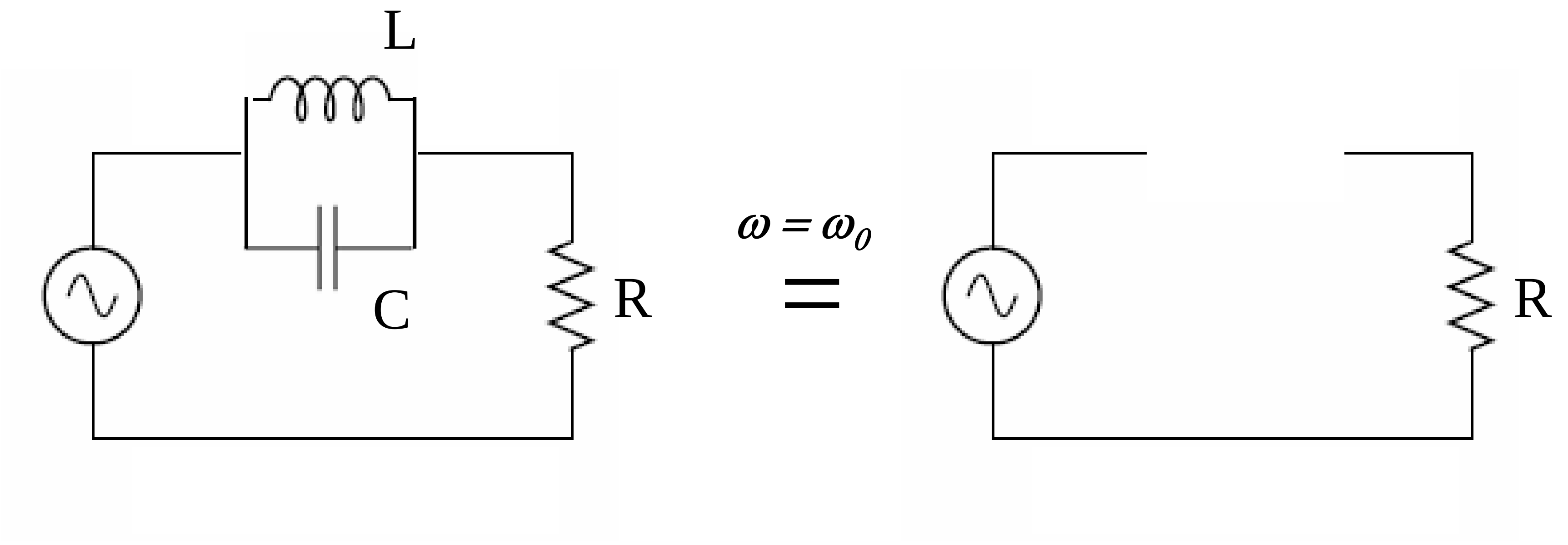
$$1/Z_L = j\left(\omega C - \frac{1}{\omega L}\right) = \frac{\omega^2 \cdot L \cdot C - 1}{\omega L}$$

$$Z_L = \frac{\omega L}{\omega^2 \cdot L \cdot C - 1}$$

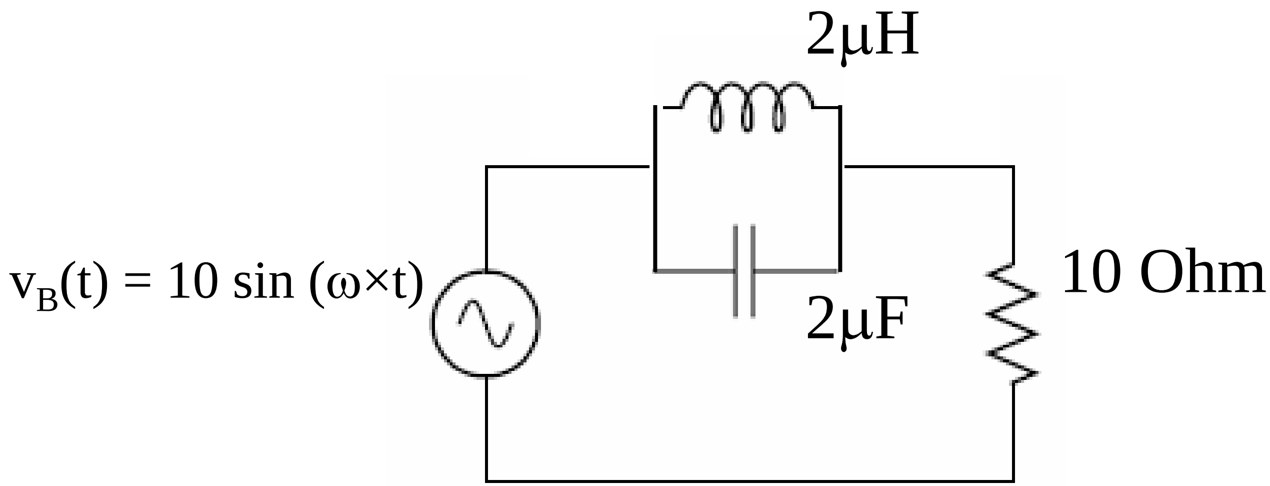
When $\omega = \omega_0$, where $\omega_0 = \frac{1}{\sqrt{LC}}$, $\quad \text{modulus}(Z_L) \Rightarrow \infty$

Resonance in parallel L – C circuit

At ω = $\omega_0 = \dfrac{1}{\sqrt{LC}}$



Example 5: Find the resonance frequency and current – frequency dependence for the parallel L – C circuit



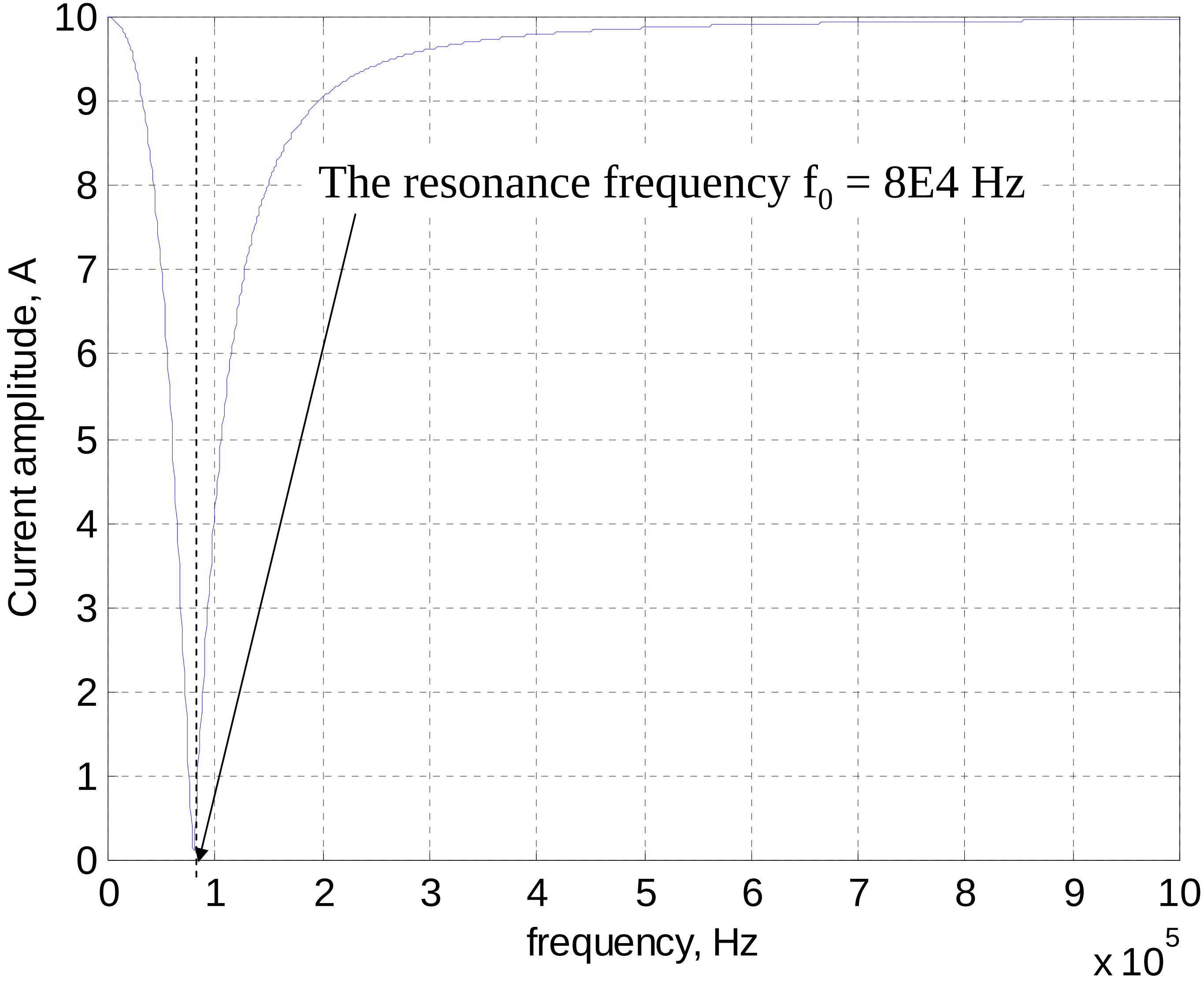
$$\omega_0 = \frac{1}{\sqrt{LC}}; \quad f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2 \cdot 3.14 \cdot \sqrt{2 \cdot 10^{-6} \cdot 2 \cdot 10^{-6}}} = 7.96 \cdot 10^4 \ Hz$$

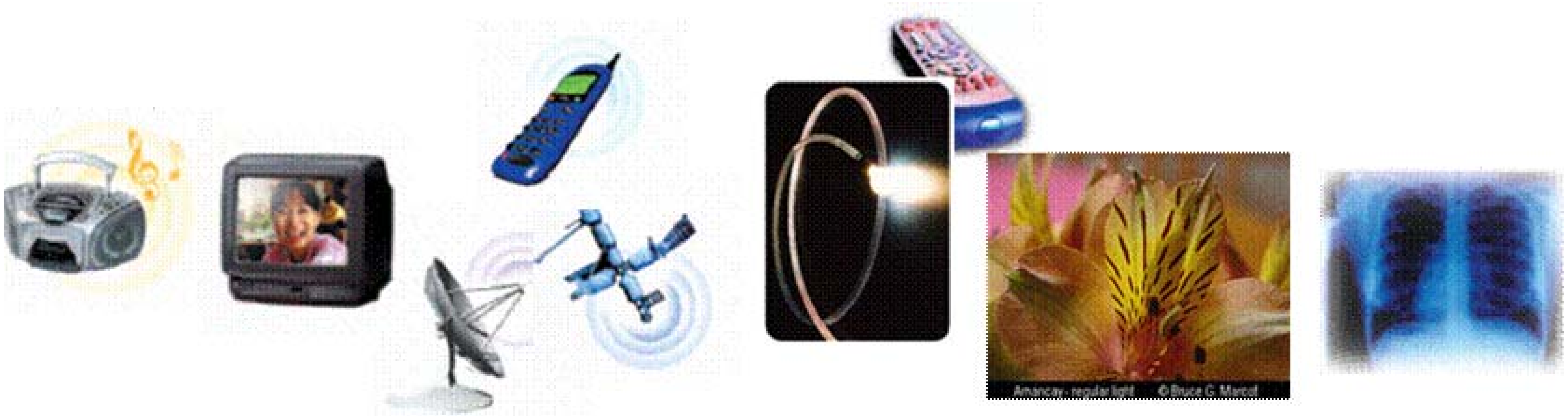
MATLAB script

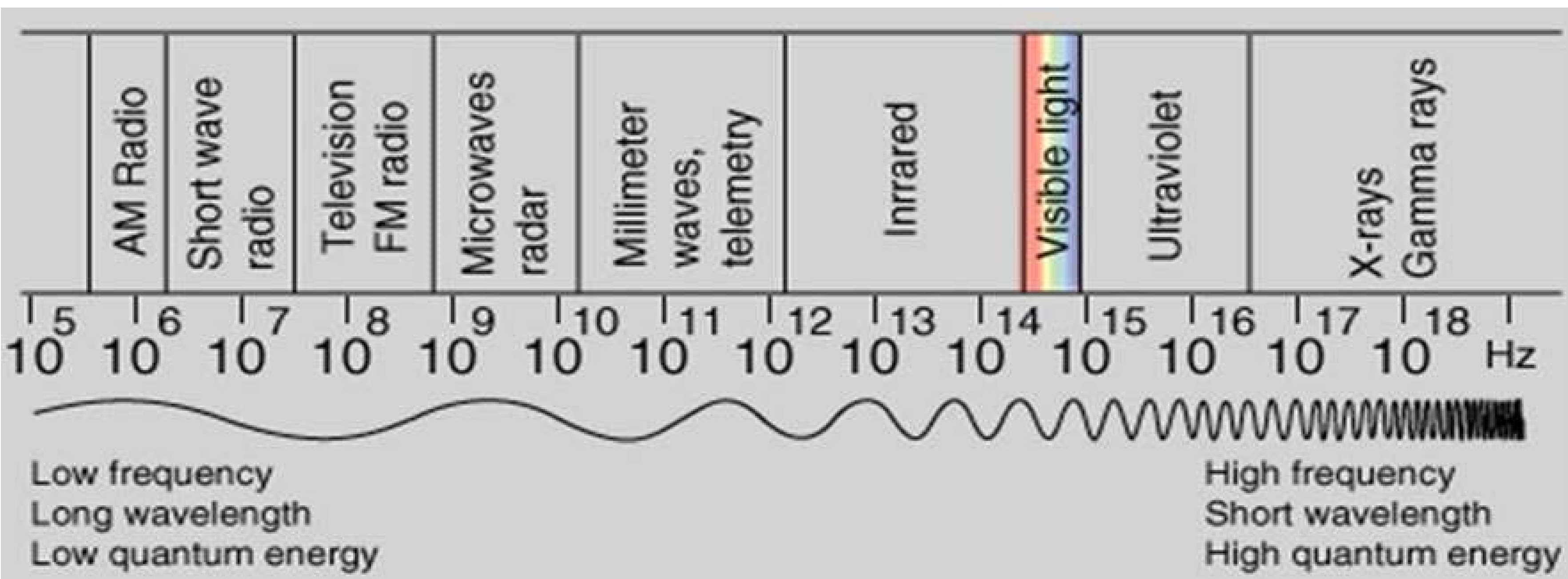
```
clear all
%circuit description
L=2e-6;
C=2e-6;
R=1;
VM=10;
% variable frequency
f=1e3:1e3:1e6;
%Circuit impedance
omega=2*pi*f;
YLC=i*(omega*C-1./omega/L);
ZLC=1./YLC;
Z=R+ZLC;
I=VM./Z;
IM=abs(I);
figure(1)
plot(f,IM)
xlabel('frequency, Hz')
ylabel('Current amplitude, A')
```

MATLAB results

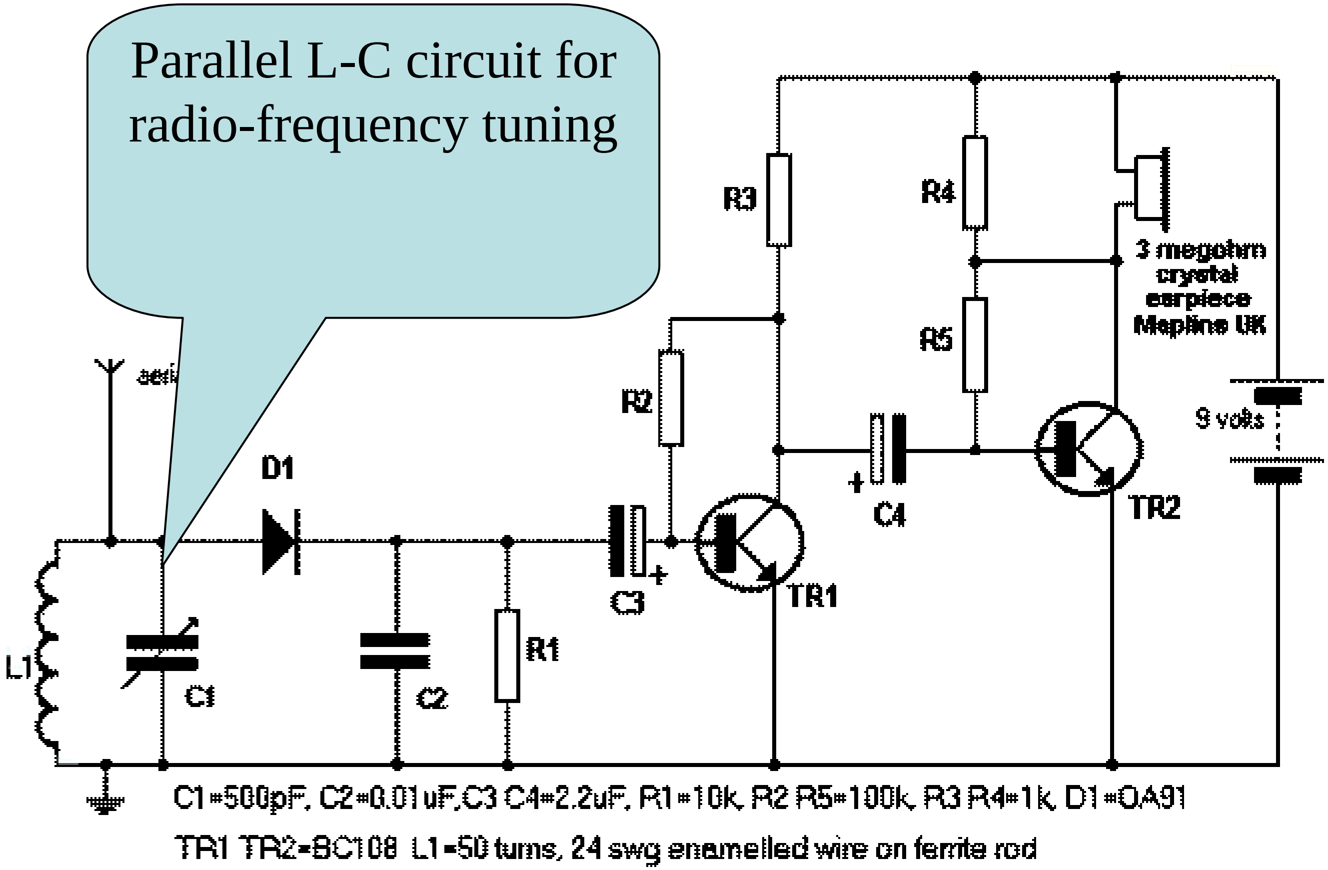


Frequency tuning in wireless communications

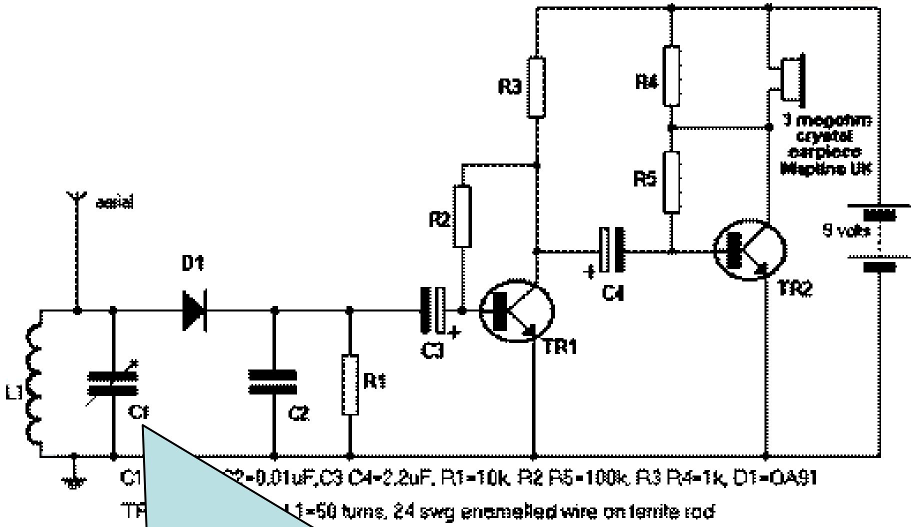




Transistor radio-receiver (1960)



Mechanical variable capacitor



Vacuum-tube TV receiver (1932)

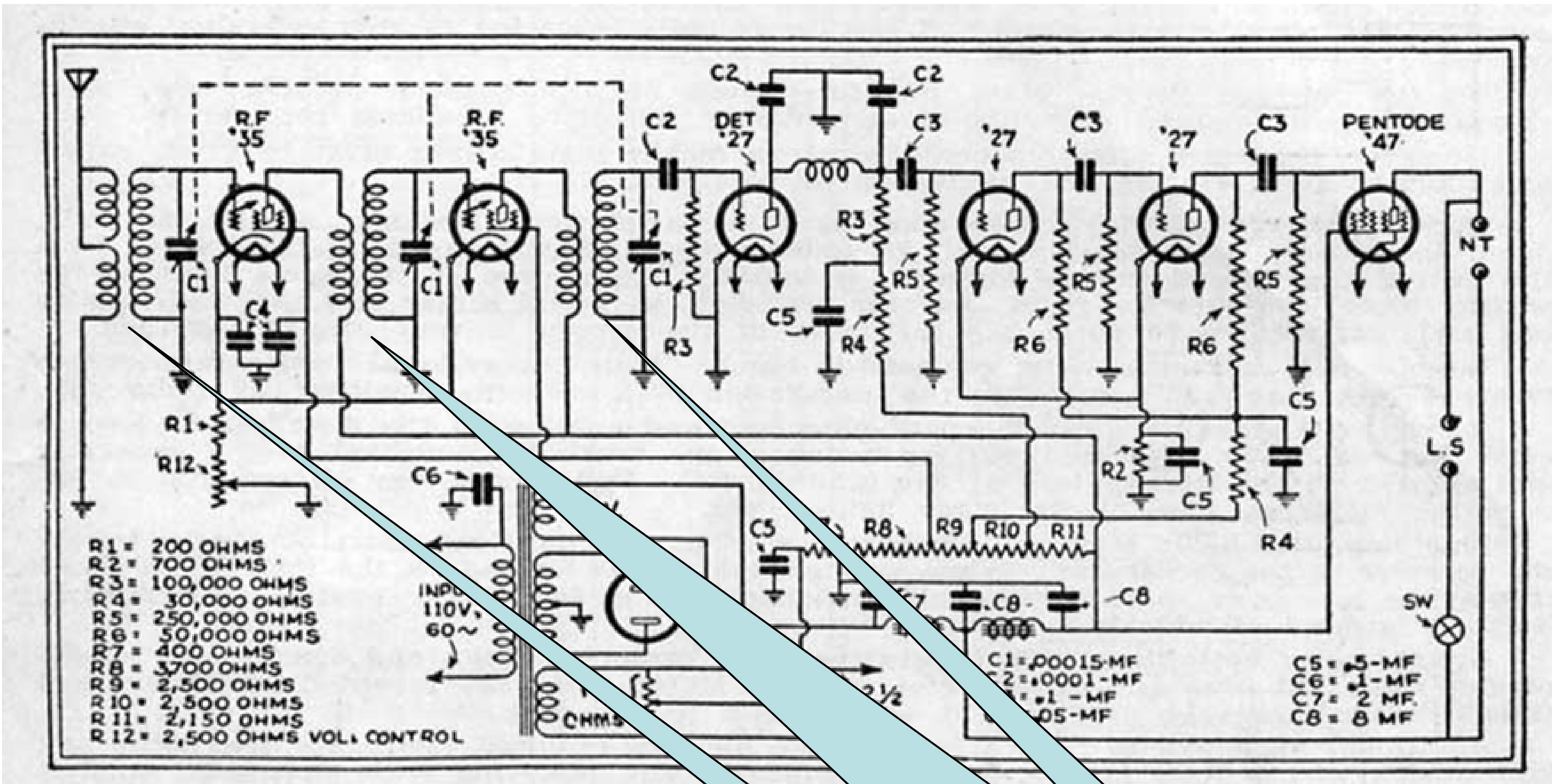


Fig. 57. A Direct-Coupled Television Re... watt resistors.

Parallel L-C circuits for TV tuning

Integrated TV receiver (2000)

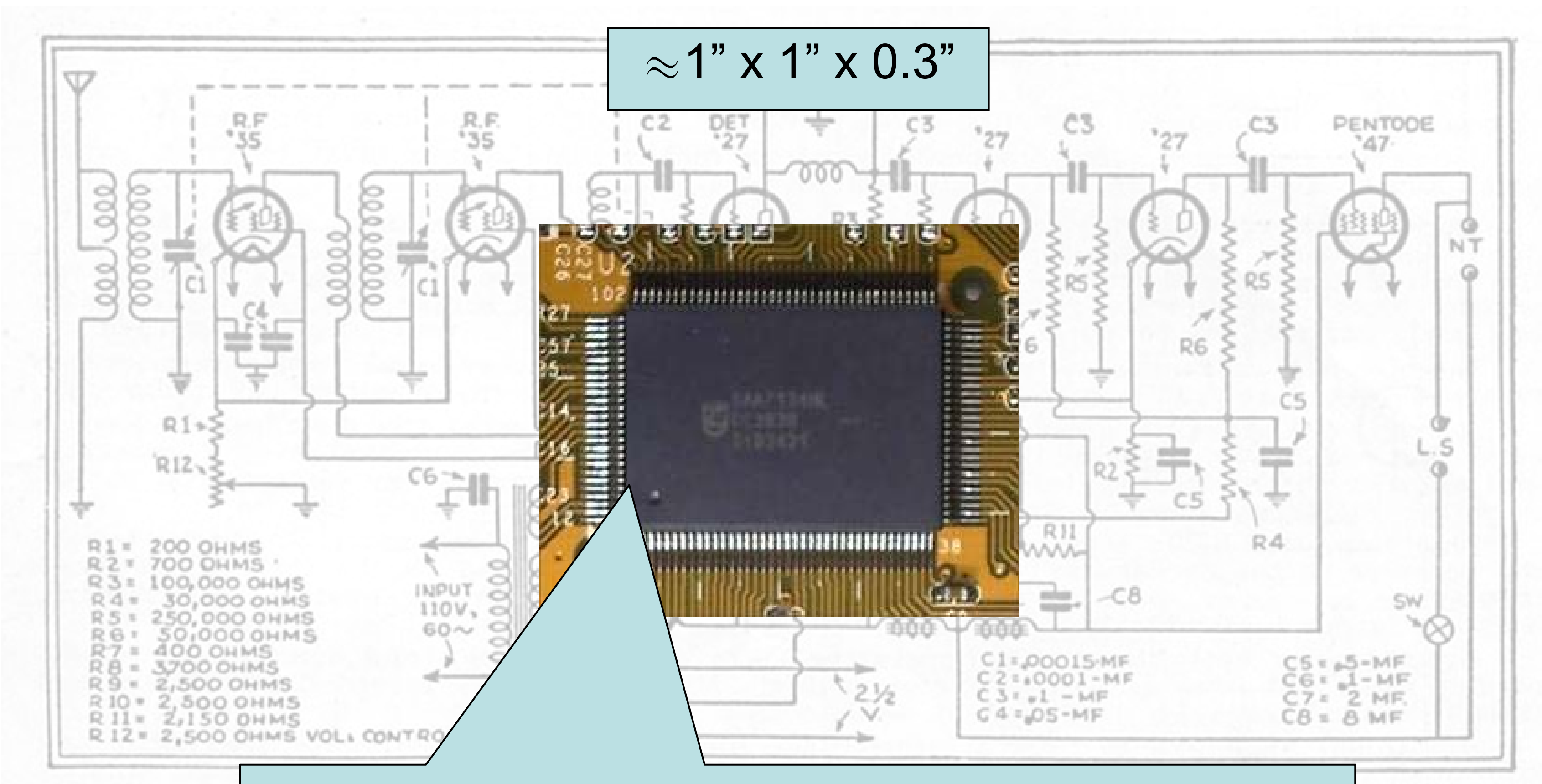


Fig. 57. A

resistors.

≈1" x 1" x 0.3"

All-channel TV tuning functions on-chip
Electronic tuning
100 smaller than discrete-component TV tuner