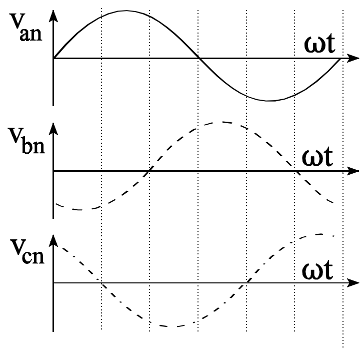
A2: Three-phase Power Circuits

Wildi Text Chapter 8, All Sections, All Questions and Problems

Poly-phase power circuits are used for two main reasons; to make more efficient use of the conductors in a transmission line; and to enable the use of poly-phase rotating machines.



A three phase generator is a device that produces three ac voltages with the same frequency and rms value and whose relative phases are one third of an electrical cycle. These voltages may be written as:

$$v_{an} = \sqrt{2}\ V\ \sin\ \omega t \qquad V$$

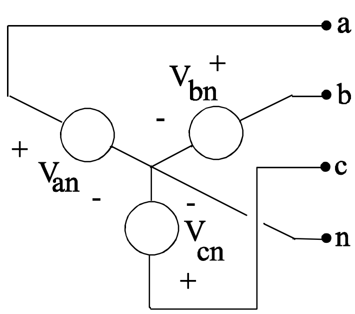
$$v_{bn} = \sqrt{2}\ V\ \sin\ (\omega t - \frac{2\pi}{3}) \qquad V$$

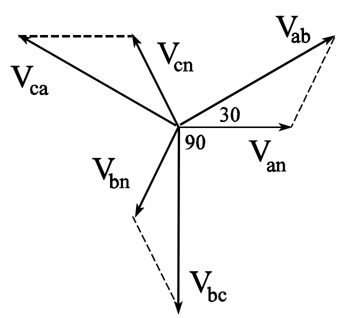
$$v_{cn} = \sqrt{2}\ V\ \sin\ (\omega t + \frac{2\pi}{3}) \qquad V$$

$$V_{an}\ \angle 0\ ,\ V_{bn}\ \angle -120\ ,\ V_{cn}\ \angle +120$$

There are three possible connections for three-phase devices: 4-wire **Y**, 3-wire **Y**, (3-wire) **Δ**.

4 Wire Wye Connection (Y)





Two sets of voltages LL, LN

$V_{ab} = V_{an} - V_{bn}$

$V_{bc} = V_{bn} - V_{cn}$

$V_{ca} = V_{cn} - V_{an}$

In general - convert between LL & LN

$$V_{LL} = \sqrt{3}\ \angle 30\ V_{LN}$$

If a "balanced" load is connected ($Z_a = Z_b = Z_c = |Z|\ \underline{/\theta}$) then the currents that flow are also balanced (i.e. symmetrical in magnitude and phase)

$$i_a = \sqrt{2}\ I\ \sin\ (\omega t - \theta) \qquad A$$

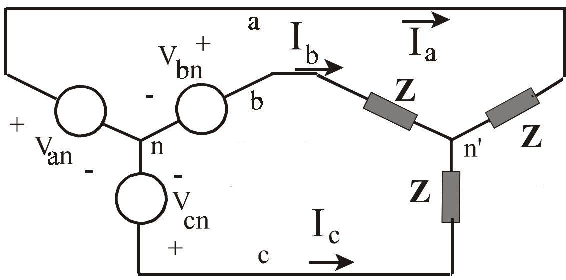
$$i_b = \sqrt{2}\ I\ \sin\ (\omega t - \theta - \frac{2\pi}{3}) \qquad A$$

$$i_c = \sqrt{2}\ I\ \sin\ (\omega t - \theta + \frac{2\pi}{3}) \qquad A$$

$$\text{note that } i_n = i_a + i_b + i_c = 0 \qquad A$$

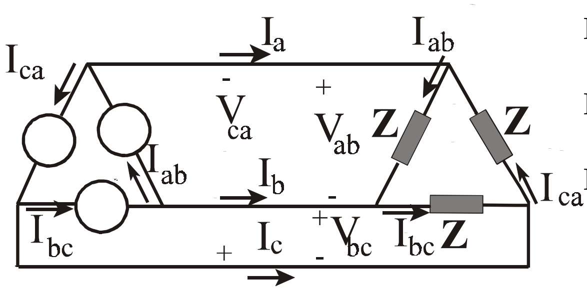
If the load is not balanced, a neutral current will exist.

3 Wire Wye Connection (Y)



In a 3-wire Wye configuration the neutral wire is omitted. In this case - an unbalanced load will cause the load voltages to become unbalanced and $v_{nn'} \neq 0$. If however the load is balanced then $v_{nn'} = 0$ and the presence of a neutral wire is immaterial. Note that $i_a + i_b + i_c$ is constrained to be zero.

Delta Connection (Δ)



Two sets of currents: line, delta (L, Δ)

$I_a = I_{ab} - I_{ca}$

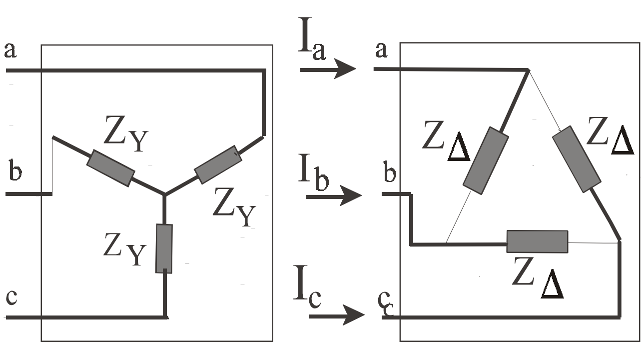
$I_b = I_{bc} - I_{ab}$

$I_c = I_{ca} - I_{bc}$

to convert between L & Δ

$$I_{\mathbf{L}} = \sqrt{3}\ \angle -30\ I_{\Delta}$$

Wye-Delta Transformations (Y to Δ and Δ to Y)



$$Z_{\Delta} = 3\ Z_{\mathbf{Y}}$$

$$Z_{\mathbf{Y}} = \frac{1}{3}\ Z_{\Delta}$$

Power Calculations

$$P = \sqrt{3}\ V_{\mathbf{LL}}\ I_{\mathbf{L}}\ \text{PF}\ (\text{Y or } \Delta) = 3\ V_{\mathbf{LN}}\ I_{\mathbf{L}}\ \text{PF}\ (\text{Y}) = 3\ V_{\mathbf{LL}}\ I_{\Delta}\ \text{PF}\ (\Delta),\ \text{where PF} = \cos\theta$$

$$Q = \sqrt{3}\ V_{\mathbf{LL}}\ I_{\mathbf{L}}\ \sin\theta\ (\text{Y or } \Delta) = 3\ V_{\mathbf{LN}}\ I_{\mathbf{L}}\ \sin\theta\ (\text{Y}) = 3\ V_{\mathbf{LL}}\ I_{\Delta}\ \sin\theta\ (\Delta)$$

$$S = \sqrt{3}\ V_{\mathbf{LL}}\ I_{\mathbf{L}}\ (\text{Y or } \Delta) = 3\ V_{\mathbf{LN}}\ I_{\mathbf{L}}\ (\text{Y}) = 3\ V_{\mathbf{LL}}\ I_{\Delta}\ (\Delta)$$

To solve for current or voltage, use a Y-conn. per-phase circuit and $V_{LN} = I_L \times Z_Y$