EE 1291

Electrical Engg (PART A)

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Chapter 2

Single phase circuits

Lecture 3

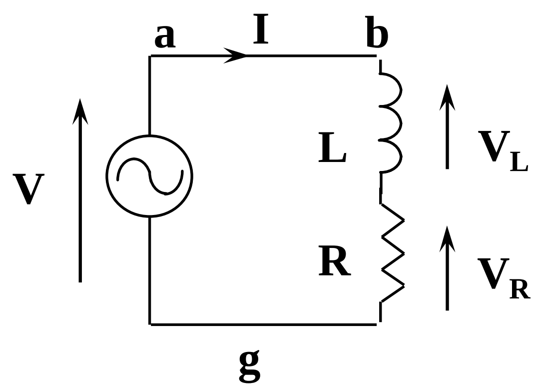
Single-phase Circuits

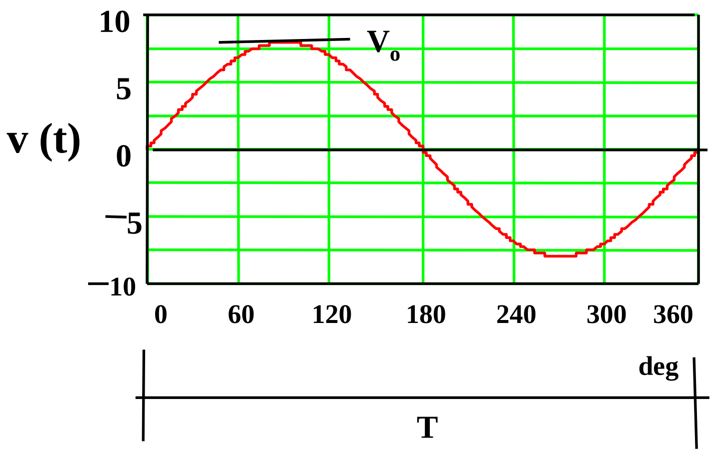
2.1 Basic definition

Single Phase Circuit

Review

- **<u>Single phase circuit components</u>:**
- Voltage or current sources
- Impedances (resistance, inductance, and capacitance)
- The components are connected in series or in parallel.
- The figure shows a simple circuit where a voltage source (generator) supplies a load (resistance and inductance in series).





Single Phase Circuit

Review

- **The voltage source produces a sinusoidal voltage wave**

$$v(t) = \sqrt{2}\ V_{rms}\ \cos(\omega t + \delta)$$

where: V_{rms} is the rms value of the voltage (volts)

ω is the angular frequency (rad/sec)

$$\omega = 2\pi f = \frac{2\pi}{T} \quad \text{rad/sec} \qquad\qquad f = \frac{1}{T} \quad \text{Hz}$$

f is the frequency (60 Hz in USA, 50 Hz in Europe).

T is the time period (seconds).

Single Phase Circuit

Review

The rms value is calculated by:

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v(t)^2\ dt}$$

The peak value (max value) of the voltage is:

$$V_0 = \sqrt{2}\ V_{rms}$$

Single Phase Circuit

Review

- **The current is also sinusoidal**

$$i\,(\mathbf{t}) = \sqrt{2}\, I_{rms}\ \sin(\omega t - \phi)$$

where: I_{rms} is the rms value of the current.

ϕ is the phase-shift between current and voltage.

- **The rms current is calculated by the Ohm's Law:**

where: Z is the impedance

$$I_{rms} = \frac{V_{rms}}{Z}$$

Single Phase Circuit

Review

Complex Notation

- **Voltage phasor:**

$$\mathbf{V} = |V|\, e^{j\delta} \qquad \text{or}$$

$$\mathbf{V} = |V| \angle \delta = |V| \cos\delta + \mathbf{j}|V| \sin\delta$$

where : V is the rms value, and δ is the phase angle

Note: The supply voltage phase angle is often selected as the reference with $\delta = 0$

Single Phase Circuit

Review

Complex Notation

- **Current phasor**

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{|V|\, e^{j\delta}}{|Z|\, e^{j\phi}} = \left|\frac{V}{Z}\right| e^{j(\delta-\phi)} = \left|\frac{V}{Z}\right| \left[\mathbf{cos}(\delta-\phi) + \mathbf{j}\,\mathbf{sin}(\delta-\phi)\right]$$

Single Phase Circuit

Review

- The sinusoidal current, voltage is expressed in complex form:

$$V_o = I_{rms} \cdot e^{j \cdot \delta} = V_{rms} \cdot (\cos(\delta) + j \cdot \sin(\delta))$$

-

$$I_o = I_{rms} \cdot e^{j \cdot \Phi} = I_{rms} \cdot (\cos(\Phi) + j \cdot \sin(\Phi))$$

2.2 Impedances

Single Phase Circuit

Review

- **The Ohm law states:** $V_o = I_o \cdot Z_o$

- $$Z_o = \frac{V_o}{I_o} = \frac{V_{rms}}{I_{rms}} \cdot e^{j(\delta - \Phi)} = \frac{V_{rms}}{I_{rms}} \cdot e^{j(\theta)}$$

$$Z_o = |Z| \cdot (\cos(\theta) + 1j \cdot \sin(\theta))$$

Zo is the impedance

Single Phase Circuit

Review

- The impedance has resistive and reactive part,
- The reactance depend on the frequency:

$$Z_o = R_o + j\,X_o(\omega)$$

- The absolute value and phase angle is:

$$|Z_o| = \sqrt{R_o{}^2 + X_o(\omega)^2} \qquad \Phi = \operatorname{atan}\left(\frac{X_o}{R_o}\right).$$

Single Phase Circuit

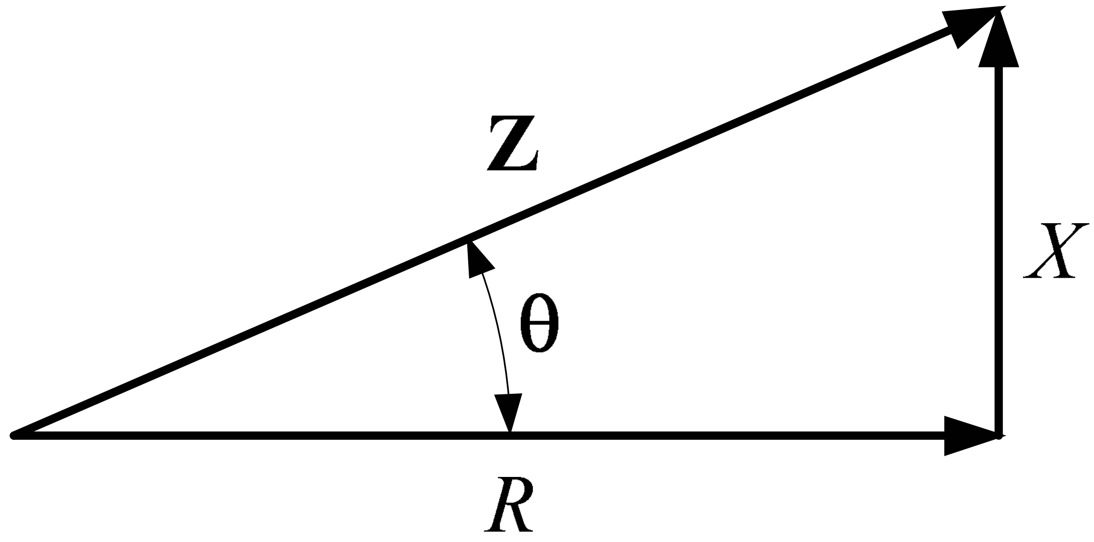
Review

Fig 2.02 Impedance Triangle

$$\mathbf{Z} = |Z| e^{j\phi}$$

$$\mathbf{Z} = \sqrt{R^2 + X^2}$$

$$\phi = a\tan\left(\frac{X}{R}\right)$$



Single Phase Circuit

Review

- **The impedances (in Ohms) are :**
  - **a) Resistance (R)**
  - **b) Inductive reactance** · $X_L = j \cdot \omega \cdot L_{ind}$
  - **c) Capacitive reactance** $X_c = \dfrac{1}{j \cdot \omega \cdot C_{cap}}$

Single Phase Circuit

Review

Series connection

$$Z_e = \sum_{k=1}^{N} Z_k$$

Single Phase Circuit

Review

Series connection Example: (resistance, capacitor, and inductance connected in series)

$$\mathbf{Z}_{\mathrm{e}} = \mathbf{Z}_{\mathrm{R}} + \mathbf{Z}_{\mathrm{L}} + \mathbf{Z}_{\mathrm{C}} = R + j\omega L_{ind} + \frac{1}{j\omega C_{cap}}$$

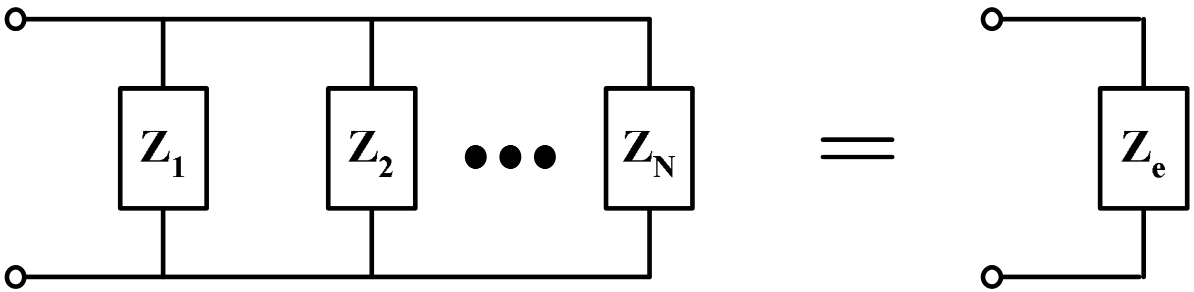
$$= |\mathbf{Z}| e^{j\theta} = |\mathbf{Z}| \left[\cos(\theta) + j\sin(\theta)\right]$$

$$|\mathbf{Z}| = \sqrt{R^2 + \left(\omega L_{ind} - \frac{1}{\omega C_{cap}}\right)^2} \qquad \theta = \arctan\left(\frac{\omega L_{ind} - \frac{1}{\omega C_{cap}}}{R}\right)$$

Single Phase Circuit

Review

Fig 2.3 Parallel connection



$$\mathbf{Z}_e = \frac{1}{\sum_{k=1}^{N} \frac{1}{\mathbf{Z}_k}}$$

Single Phase Circuit

Review

Parallel connection

Impedance phasor: (resistance, capacitor, and inductance connected in parallel)

$$\mathbf{Z} = \frac{1}{\mathbf{Y}} = \frac{1}{\frac{1}{R} + \frac{1}{j\omega L} + \frac{1}{\frac{1}{j\omega C}}} = \frac{1}{\frac{1}{R} + \frac{1}{j\omega L} + j\omega C}$$

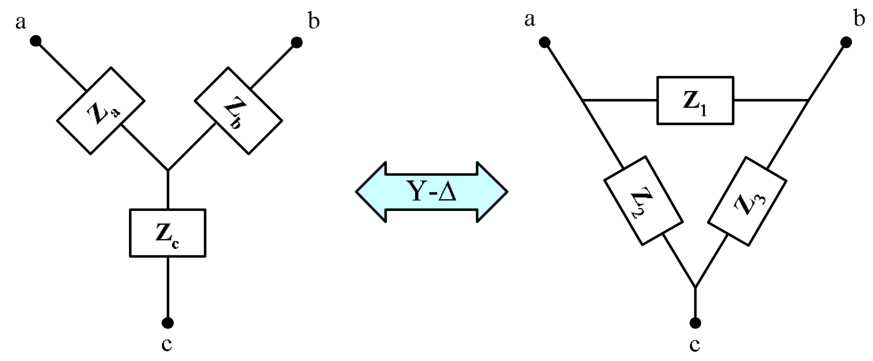
Two impedances connected in parallel

$$\mathbf{Z} = \frac{1}{\frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_1}} = \frac{\mathbf{Z}_1\,\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}$$

Single Phase Circuit

Review

Fig 2.10 Delta-wye transformation



$$\mathbf{Z}_a = \frac{\mathbf{Z}_1 \mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3}$$

$$\mathbf{Z}_b = \frac{\mathbf{Z}_1 \mathbf{Z}_3}{\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3}$$

$$\mathbf{Z}_c = \frac{\mathbf{Z}_2 \mathbf{Z}_3}{\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3}$$

Reverse

$$\mathbf{Z}_1 = \frac{\mathbf{Z}_a \mathbf{Z}_b + \mathbf{Z}_b \mathbf{Z}_c + \mathbf{Z}_c \mathbf{Z}_a}{\mathbf{Z}_c}$$

$$\mathbf{Z}_2 = \frac{\mathbf{Z}_a \mathbf{Z}_b + \mathbf{Z}_b \mathbf{Z}_c + \mathbf{Z}_c \mathbf{Z}_a}{\mathbf{Z}_b}$$

$$\mathbf{Z}_3 = \frac{\mathbf{Z}_a \mathbf{Z}_b + \mathbf{Z}_b \mathbf{Z}_c + \mathbf{Z}_c \mathbf{Z}_a}{\mathbf{Z}_a}$$

Same values $\mathbf{Z}_\Delta = 3\,\mathbf{Z}_\mathbf{Y}$

Single Phase Circuit

Review

Example: Fig 2.4: Calculate the impedance of a compensated line

$R_{mi} := 0.32 \frac{\Omega}{mi}$ $\qquad X_{mi} := 0.75 \frac{\Omega}{mi}$ $\qquad d_{line} := 3mi$

$C_{comp} := 1572 \mu F$

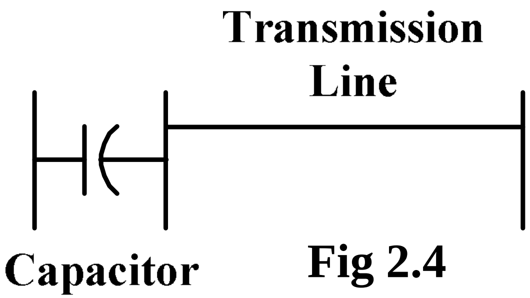


Fig 2.4

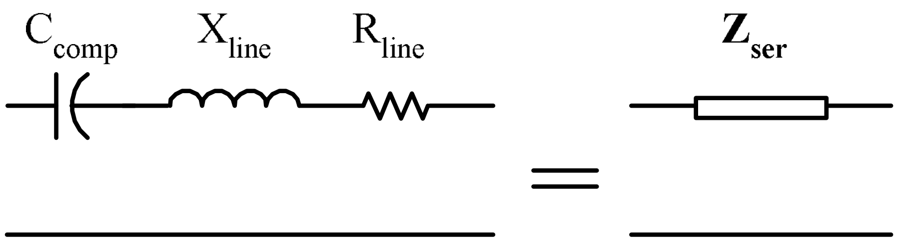


Fig 2.5

Single Phase Circuit

Review

Example: Fig 2.4: Calculate the impedance of a compensated line

The three impedances are connected in series

$R_{mi} := 0.32\,\frac{\Omega}{mi}$ $X_{mi} := 0.75\,\frac{\Omega}{mi}$ $d_{line} := 3mi$

$C_{comp} := 1572\mu F$ $f := 60Hz$ $\omega := 2\cdot\pi\cdot f$ $\omega = 376.991\ Hz$

$$Z_{line} := R_{mi}\cdot d_{line} + 1j\cdot X_{mi}\cdot d_{line} + \frac{1}{1j\omega\cdot C_{mi}\cdot d_{line}}$$

$$Z_{line} = 0.96 + 1.688i\,\Omega$$

Single Phase Circuit

Review

Example: Fig 2.4: Calculate the impedance of a motor and capacitor conncted in parallel

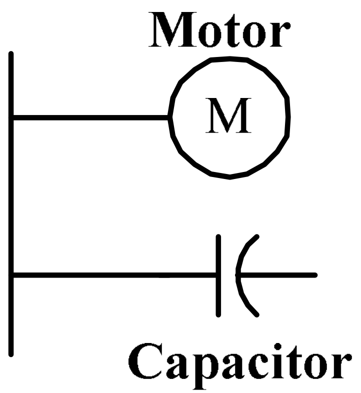


Fig 2.6

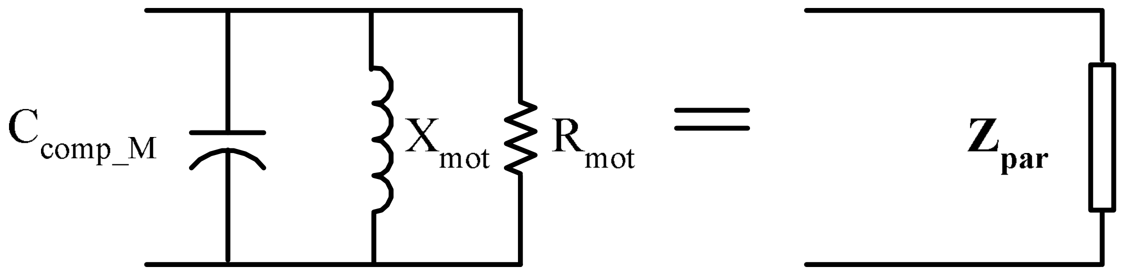


Fig. 2.7

$X_{mot} := 23\Omega$ $\quad R_{mot} := 20\Omega$ $\quad C_{comp\_M} := 500\mu F$

Single Phase Circuit

Review

Example: Fig 2.4: Calculate the impedance of a motor and capacitor connected in parallel

$X_{mot} := 23\Omega$ $\quad R_{mot} := 20\Omega$ $\quad C_{comp\_M} := 500\mu F$

$f := 60Hz \quad \omega := 2\cdot\pi\cdot f \quad \omega = 376.991Hz$

$$X_{mot\_C} := \frac{1}{1j\cdot\omega\cdot C_{mot\_C}} \qquad X_{mot\_C} = -5.305i\ \Omega$$

$$Z_{mot} := \frac{1}{\frac{1}{1jX_{mot}} + \frac{1}{R_{mot}} + \frac{1}{X_{mot\_C}}} \qquad Z_{mot} = 2.125 - 6.163i\ \Omega.$$

Single Phase Circuit

Review

2.3 Power

Single Phase Circuit

Review

Instantaneous power is the product of the instantaneous voltage and current

$$p(t) = v(t)\, i(t) = \sqrt{2}\ V_{rms}\ \cos(\omega t + \delta)\ \sqrt{2}\ I_{rms}\ \cos(\omega t + \phi)$$

Using the trigonometric identity

$$\cos(\alpha)\ \cos(\beta) = \tfrac{1}{2}\cos(\alpha - \beta) + \tfrac{1}{2}\cos(\alpha + \beta)$$

The instantaneous power is:

$$p(t) = V_{rms}\ I_{rms}\ \left[\cos(\delta - \phi) + \cos(2\omega t + \delta + \phi)\right]$$

Single Phase Circuit

Review

The average or active power is the integral

$$P = \frac{1}{T}\int_0^T p(t)\, dt = V_{rms}\, I_{rms}\, \cos(\delta - \phi)$$

Apparent power $\mathbf{S = V_{rms} * I_{rms}}$

Complex power:

$$\mathbf{S} = \mathbf{V_{rms}}\, \mathbf{I^*_{rms}} = V_{rms}\, e^{j\delta}\, I_{rms}\, e^{-j\phi}$$
$$= V_{rms}\, I_{rms}\, \cos(\delta - \phi) + jV_{rms}\, I_{rms}\, \sin(\delta - \phi)$$
$$= P + jQ$$

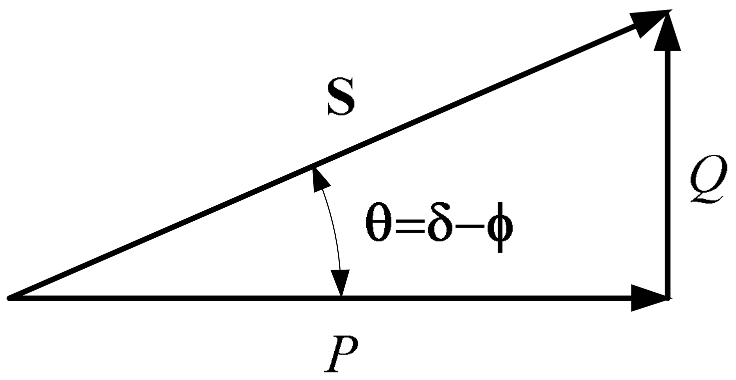
Single Phase Circuit

Review

Power Factor , Should be more than 0.8

$$pf = \frac{P}{V_{rms} I_{rms}} = \frac{P}{|\mathbf{S}|} = \cos(\delta - \phi)$$

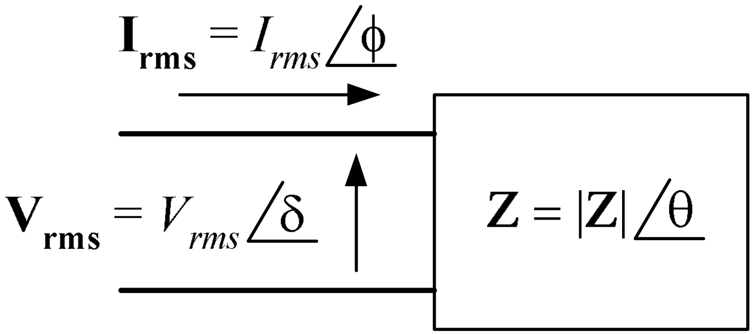
Fig 2.11 Power triangle



Single Phase Circuit

Review

General impedance load:



Power on a resistive load

$$P = I_{rms}^2 R = \frac{V_{rms}^2}{R}$$

Single Phase Circuit

Review

2.4 AC Circuit

Single Phase Circuit

Review

The circuit has voltage source and impedances

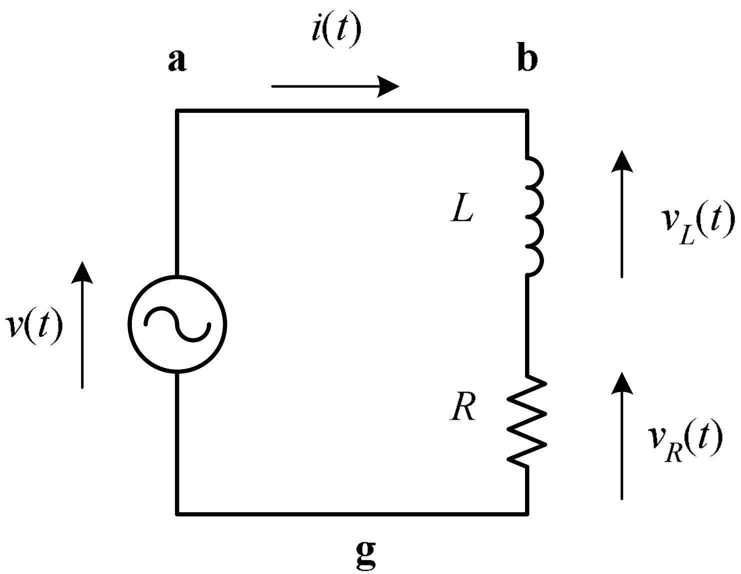


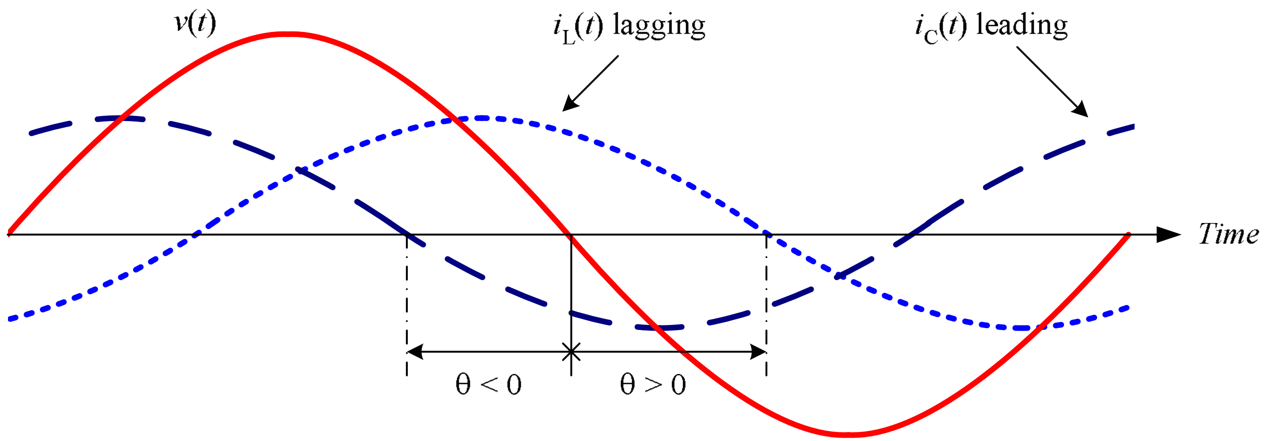
Fig. 2.13 Example single phase circuit

- **Generator current and voltage are in the same direction**
- **Load: current and voltage are in opposite direction**

Single Phase Circuit

Review

Lagging and leading current



Single Phase Circuit

Review

- The current is calculated by the Ohm Law

$$\mathbf{I_{rms}} = \frac{\mathbf{V_{rms}}}{\mathbf{Z}}$$

$$|\mathbf{Z}| = \sqrt{R^2 + X_{ind}{}^2}$$

Single Phase Circuit

Transmission line supplies a load

- The load voltage must be +/- 5% of the generator voltage
- Voltage regulation is calculated

- The loss must be minimized
- The efficiency of transmission is calculated

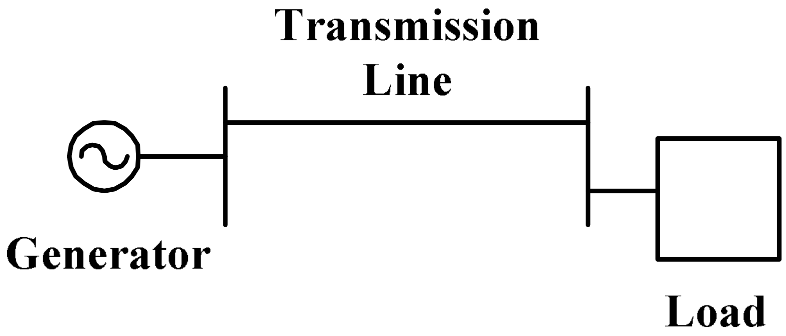


Fig 2.19 Load served by a line

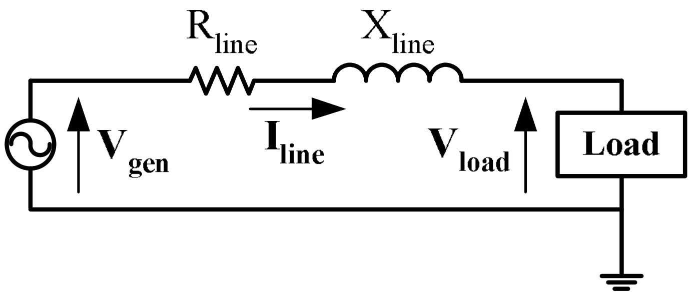


Fig 2.20 Equivalent circuit

Single Phase Circuit

Review

- The load current is:

$$I_{load} = \frac{V_{load}}{Z_{load}} = I_{line}$$

- The source voltage using loop equation is:

$$V_{gen} := V_{load} + I_{line}\left(R_{line} + j \cdot X_{line}\right)$$

Single Phase Circuit

Voltage regulation

$$Voltage\ Regulation = \frac{\left|\mathbf{V}_{\mathbf{no-load}}\right| - \left|\mathbf{V}_{\mathbf{load}}\right|}{\left|\mathbf{V}_{\mathbf{load}}\right|} \times 100\%$$

$$Voltage\ Regulation = \frac{\left|\mathbf{V}_{\mathbf{gen}}\right| - \left|\mathbf{V}_{\mathbf{load}}\right|}{\left|\mathbf{V}_{\mathbf{load}}\right|} \times 100\%$$

Efficiency

$$efficiency = \frac{P_{net}}{P_{gen}} = \frac{P_{gen} - P_{loss}}{P_{gen}}$$