

EE1291

Electrical Engineering

Chapter 2

Three phase circuits and Unit system

Lecture 3

3.1 Three phase quantities

Three Phase Circuit

Balanced three phase system

- Three generators produce voltages
 - same magnitude
 - 120 deg phase shift

Balanced load

$$\begin{aligned}i_a(t) &= I_M \cos(\omega t - \theta) \\i_b(t) &= I_M \cos(\omega t - \theta - 120^\circ) \\i_c(t) &= I_M \cos(\omega t - \theta - 240^\circ)\end{aligned}$$

$$\begin{aligned}v_{an}(t) &= V_M \cos(\omega t) \\v_{bn}(t) &= V_M \cos(\omega t - 120^\circ) \\v_{cn}(t) &= V_M \cos(\omega t - 240^\circ) = V_M \cos(\omega t + 120^\circ)\end{aligned}$$

$$i_a(t) + i_b(t) + i_c(t) = 0$$

Three Phase Circuit

Balanced system

$$v_{an}(t) + v_{bn}(t) + v_{cn}(t) = 0 \quad i_a(t) + i_b(t) + i_c(t) = 0$$

Power

The instantaneous power is constant

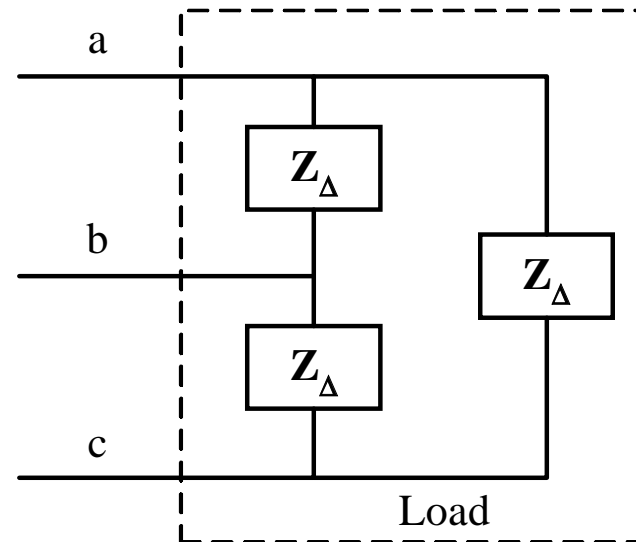
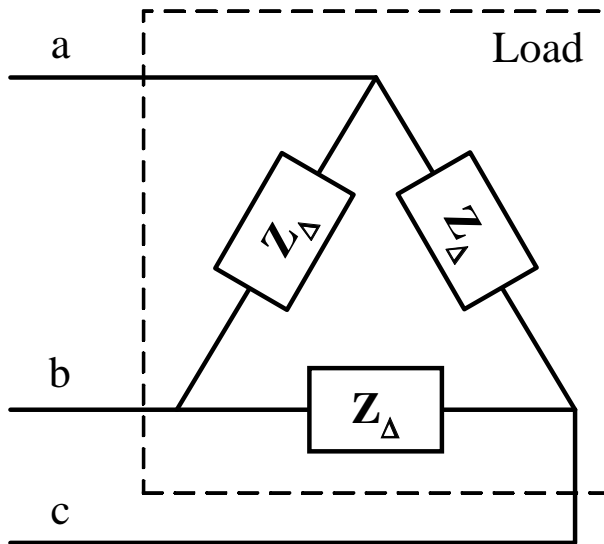
$$p(t) = p_a(t) + p_b(t) + p_c(t) = 3 \frac{V_M I_M}{2} \cos(\theta) = 3 V_{rms} I_{rms} \cos(\theta)$$

Three phase power

$$\mathbf{S}_T = \mathbf{S}_A + \mathbf{S}_B + \mathbf{S}_C = 3 \mathbf{S}_1$$

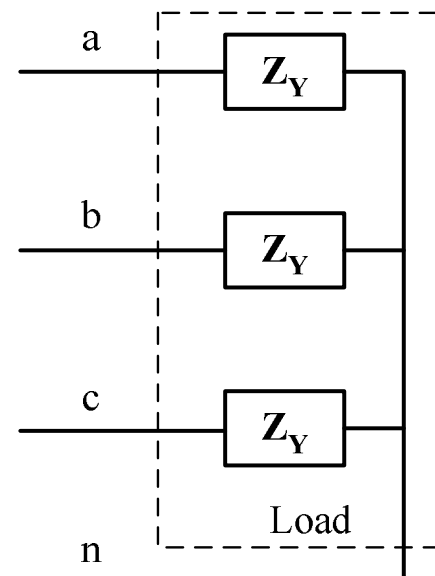
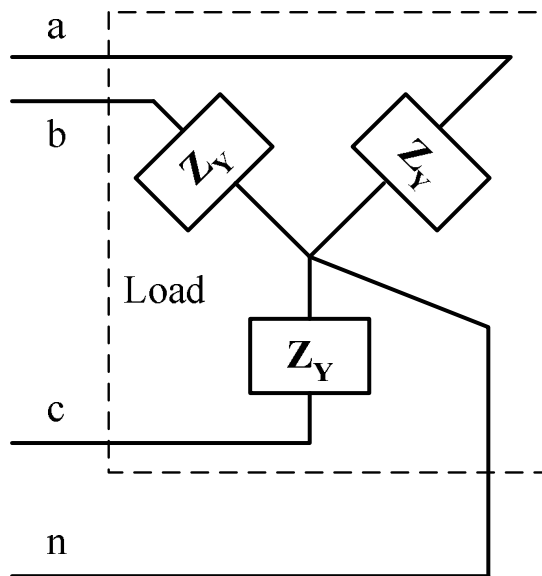
Three Phase Circuit

Delta connection of load



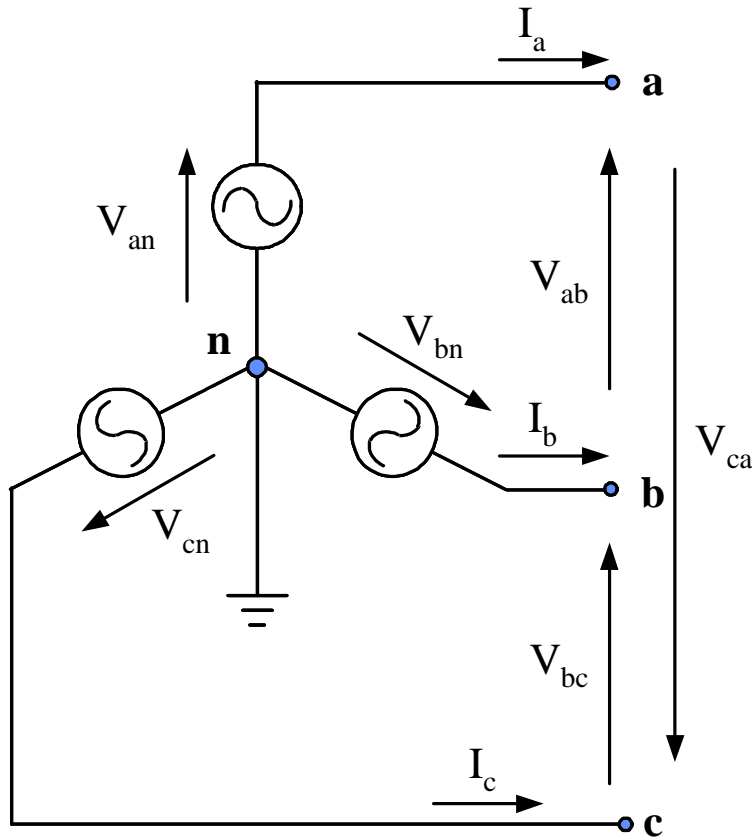
Three Phase Circuit

Wye connection of Load



3.2 WYE connected generators

Three Phase Circuit



Line-to-neutral voltages

$$\mathbf{V}_{an} = V_P \angle \delta$$

$$\mathbf{V}_{bn} = V_P \angle \delta - 120^\circ$$

$$\mathbf{V}_{cn} = V_P \angle \delta - 240^\circ$$

Balanced system

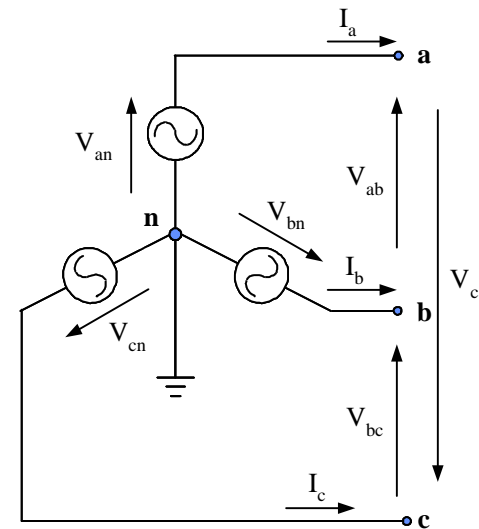
$$\mathbf{V}_{an} + \mathbf{V}_{bn} + \mathbf{V}_{cn} = 0$$

Three Phase Circuit

$$\begin{aligned}V_{ab} &= V_{an} - V_{bn} \\ &= V_{an} - V_{an} \angle -120^\circ \\ &= V_{an} (1 - e^{-j120^\circ}) \\ &= V_{an} \sqrt{3} \angle 30^\circ\end{aligned}$$

The two other can be calculated similarly

- The three line-to-line voltages, from the $n \rightarrow a \rightarrow b \rightarrow n$ loop



Three Phase Circuit

The line to line voltages

$$V_{ab} := V_{an} - V_{bn} \quad \bigg| \quad V_{bc} := V_{bn} - V_{cn} \quad \bigg|$$

$$V_{ca} := V_{cn} - V_{an} \quad \bigg|$$

The equations can be modified

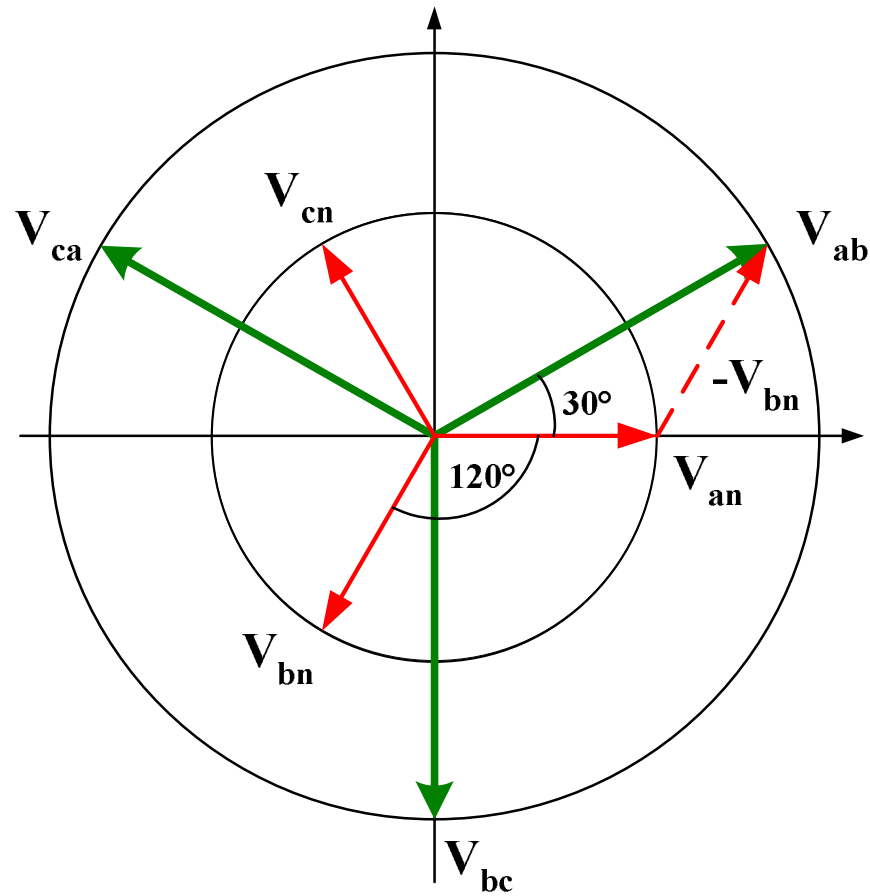
$$V_{ab} = V_{an} \sqrt{3} \angle 30^\circ$$

$$V_{bc} = V_{bn} \sqrt{3} \angle 30^\circ = V_{an} \sqrt{3} \angle -90^\circ$$

$$V_{ca} = V_{cn} \sqrt{3} \angle 30^\circ = V_{an} \sqrt{3} \angle -210^\circ$$

Three Phase Circuit

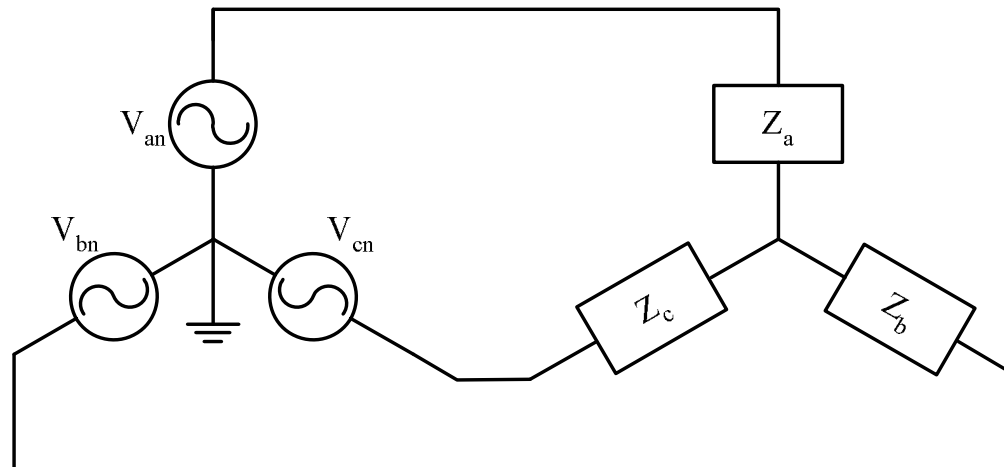
Vector diagram



3.3 WYE Connected Loads

Three Phase Circuit

Three wire system



Not part of the exam. Not used.

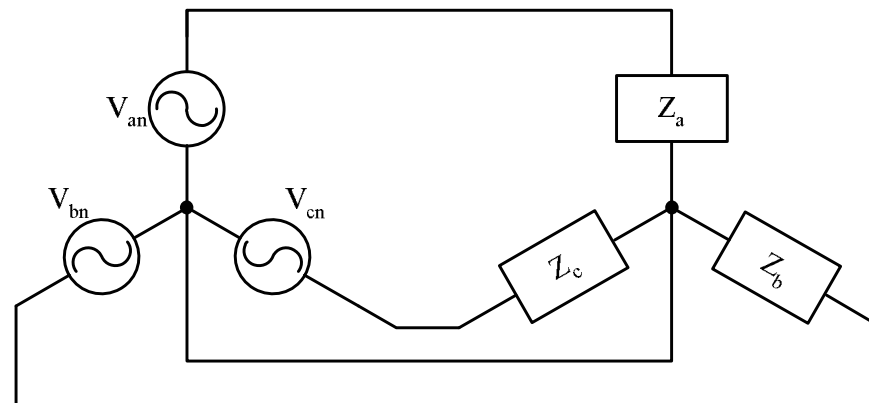
Three Phase Circuit

Four wire system

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_a} \quad \mathbf{I}_b = \frac{\mathbf{V}_{bn}}{\mathbf{Z}_b} \quad \mathbf{I}_c = \frac{\mathbf{V}_{cn}}{\mathbf{Z}_c}$$

$$\mathbf{S}_T = \mathbf{S}_a + \mathbf{S}_b + \mathbf{S}_c = \mathbf{V}_{an} \mathbf{I}_a^* + \mathbf{V}_{bn} \mathbf{I}_b^* + \mathbf{V}_{cn} \mathbf{I}_c^*$$

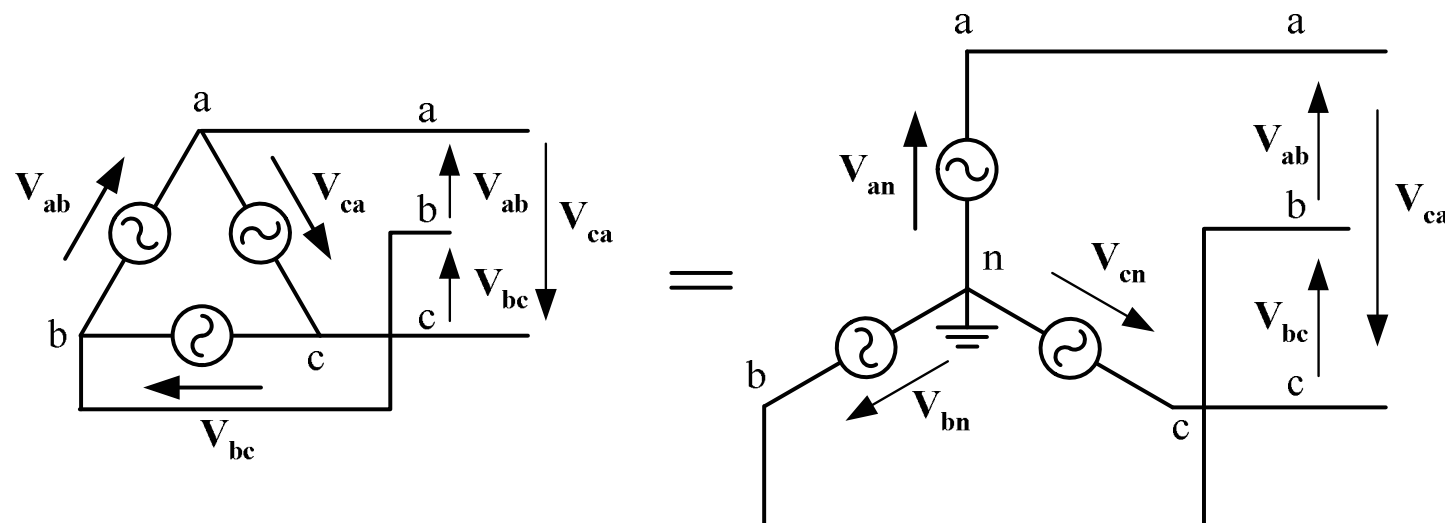
$$\mathbf{I}_0 = \mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c \neq 0$$



3.4 Delta Connected Loads

Three Phase Circuit

Delta connected generators



Can be converted to equivalent wye

$$\mathbf{V}_{an} = \frac{\mathbf{V}_{ab}}{\sqrt{3}} e^{-j30^\circ}$$

$$\mathbf{V}_{bn} = \frac{\mathbf{V}_{bc}}{\sqrt{3}} e^{-j30^\circ}$$

$$\mathbf{V}_{cn} = \frac{\mathbf{V}_{ca}}{\sqrt{3}} e^{-j30^\circ}$$

Three Phase Circuit

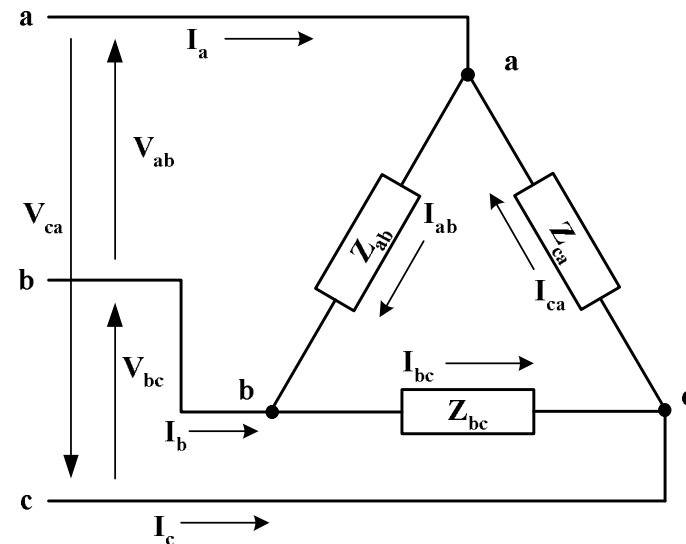
Delta connected load

$$\mathbf{I}_{ab} = \frac{\mathbf{V}_{ab}}{\mathbf{Z}_{ab}} \quad \mathbf{I}_{bc} = \frac{\mathbf{V}_{bc}}{\mathbf{Z}_{bc}} \quad \mathbf{I}_{ca} = \frac{\mathbf{V}_{ca}}{\mathbf{Z}_{ca}}$$

$$\mathbf{I}_a = \mathbf{I}_{ab} - \mathbf{I}_{ca}$$

$$\mathbf{I}_b = \mathbf{I}_{bc} - \mathbf{I}_{ab}$$

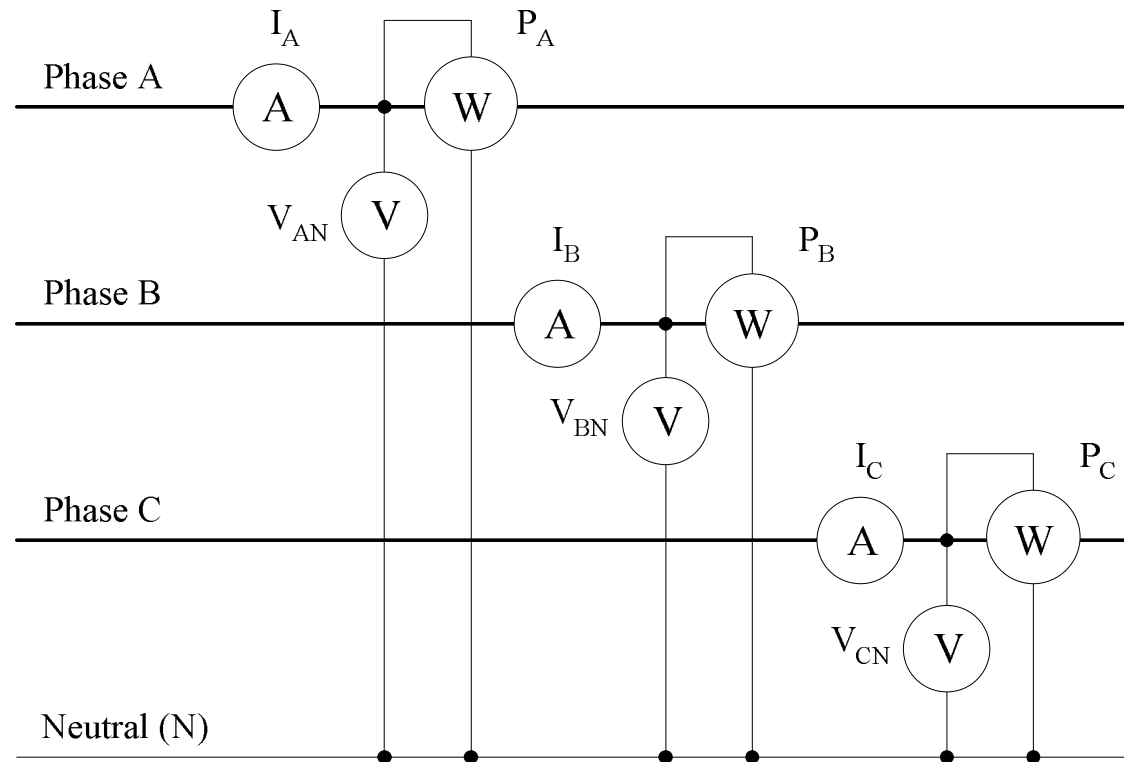
$$\mathbf{I}_c = \mathbf{I}_{ca} - \mathbf{I}_{bc}$$



3.6 Three phase power measurement

Three Phase Circuit

Four wire system, Each phase measured separately



Three Phase Circuit

Three wire system,

The three phase power is the sum of the two watt-meters reading

$$P_T = P_{AB} + P_{CB}$$

