Phasors
Complex Numbers

D.ELANGOVAN

Lec /EEE

Lecture #2

Introduction

- Any steady-state voltage or current in a linear circuit with a sinusoidal source is a sinusoid.
  - This is a consequence of the nature of particular solutions for sinusoidal forcing functions.
  - All steady-state voltages and currents have the same frequency as the source.

Introduction (cont.)

- In order to find a steady-state voltage or current, all we need to know is its *magnitude* and its *phase* relative to the source (we already know its frequency).
- Usually, an AC steady-state voltage or current is given by the particular solution to a differential equation.

The Good News!

- We do not have to find this differential equation from the circuit, nor do we have to solve it.
- Instead, we use the concepts of *phasors* and complex impedances.
- Phasors and complex impedances convert problems involving differential equations into simple circuit analysis problems.

Phasors

- A *phasor* is a complex number that represents the magnitude and phase of a sinusoidal voltage or current.
- Remember, for AC steady-state analysis, this is all we need---we already know the frequency of any voltage or current.

Complex Impedance

- Complex *impedance* describes the relationship between the voltage across an element (expressed as a phasor) and the current through the element (expressed as a phasor).
- Impedance is a complex number.
- Impedance depends on frequency.

Complex Impedance (cont.)

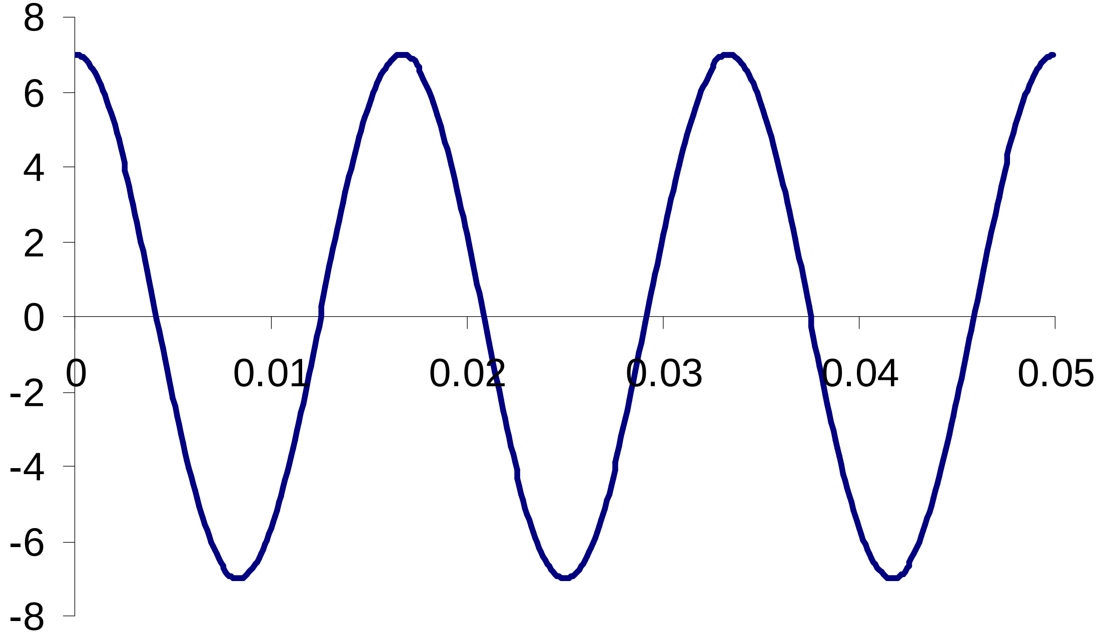
- Phasors and complex impedance allow us to use Ohm's law with complex numbers to compute current from voltage, and voltage from current.

Sinusoids

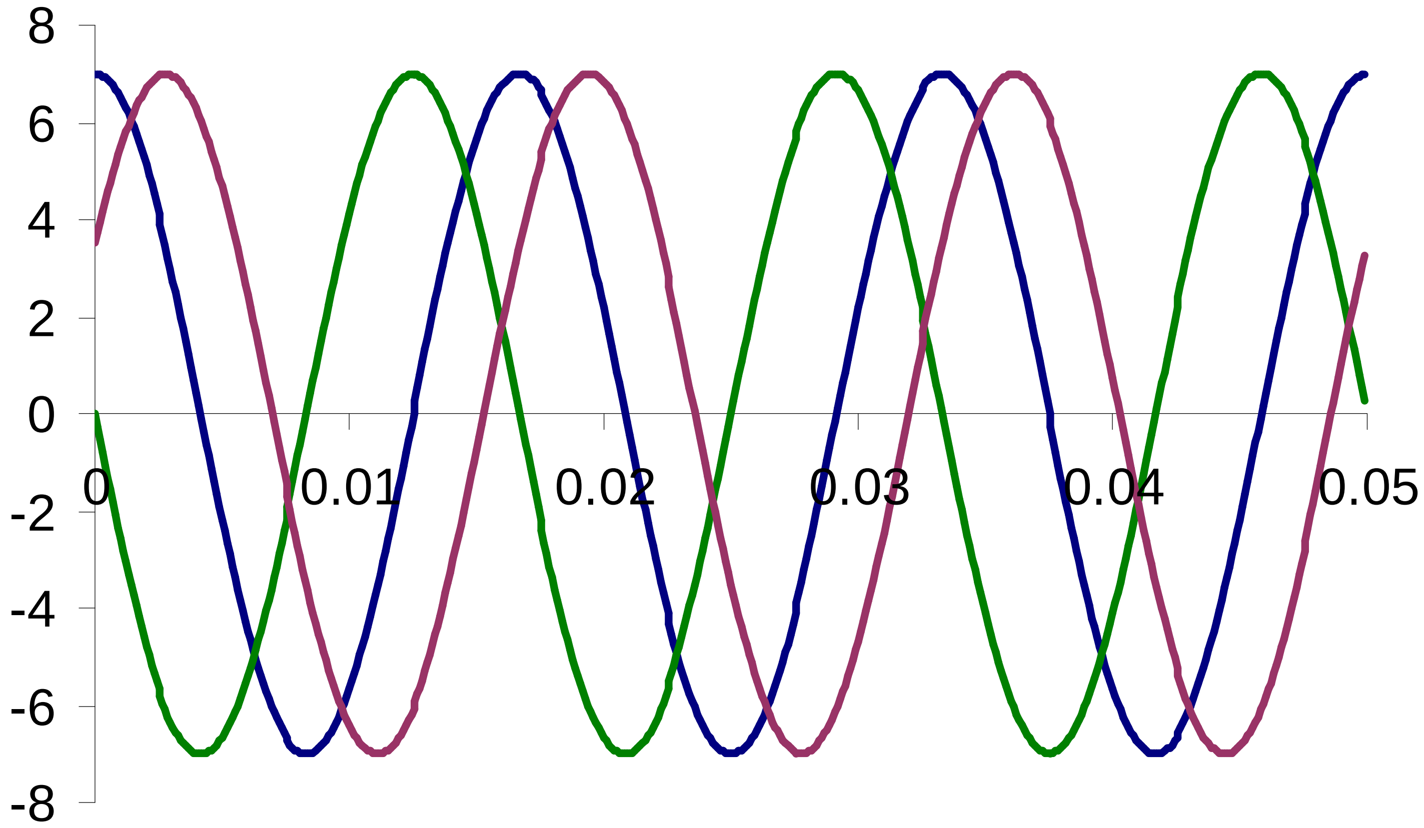
- Period: T
  - Time necessary to go through one cycle
- Frequency: $f = 1/T$
  - Cycles per second (Hz)
- Angular frequency (rads/sec): $\omega = 2\pi f$
- Amplitude: V_M

Example

What is the amplitude, period, frequency, and angular (radian) frequency of this sinusoid?



Phase



Leading and Lagging Phase

$$x_1(t) = X_{M_1} \cos(\omega t + \theta)$$

$$x_2(t) = X_{M_2} \cos(\omega t + \phi)$$

x_1(t) leads x_2(t) by θ-ϕ

x_2(t) lags x_1(t) by θ-ϕ

On the preceding plot, which signals lead and which signals lag?

Class Examples

- Learning Extension E8.1
- Learning Extension E8.2

Phasors

- A phasor is a complex number that represents the magnitude and phase of a sinusoidal voltage or current:

$$X_M \cos(\omega t + \theta)$$

$$\Updownarrow$$

$$\mathbf{X} = X_M \angle \theta$$

Phasors (cont.)

- Time Domain:

$$X_M \cos(\omega t + \theta)$$

- Frequency Domain:

$$\mathbf{X} = X_M \angle \theta$$

Summary of Phasors

- Phasor (frequency domain) is a complex number:

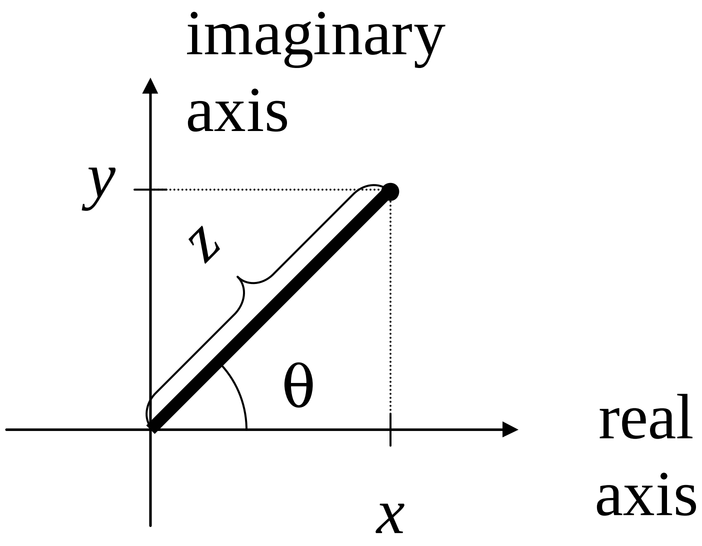
$$\mathbf{X} = z \angle \theta = x + jy$$

- Sinusoid is a time function:

$$x(t) = z \cos(\omega t + \theta)$$

Class Examples

Complex Numbers



- *x* is the real part
- *y* is the imaginary part
- *z* is the magnitude
- θ is the phase

More Complex Numbers

- Polar Coordinates: $\mathbf{A} = z \angle \theta$
- Rectangular Coordinates: $\mathbf{A} = x + jy$

$$x = z\cos\theta \qquad\qquad y = z\sin\theta$$

$$z = \sqrt{x^2 + y^2} \qquad\qquad \theta = \tan^{-1}\frac{y}{x}$$

Are You a Technology "Have"?

- There is a good chance that your calculator will convert from rectangular to polar, and from polar to rectangular.
- Convert to polar: $3 + j4$ and $-3 - j4$
- Convert to rectangular: $2 \angle 45°$ & $-2 \angle 45°$

Arithmetic With Complex Numbers

- To compute phasor voltages and currents, we need to be able to perform computation with complex numbers.
  - Addition
  - Subtraction
  - Multiplication
  - Division

Complex Number Addition and Subtraction

- Addition is most easily performed in rectangular coordinates:

$$\mathbf{A} = x + jy \qquad\qquad \mathbf{B} = z + jw$$

$$\mathbf{A} + \mathbf{B} = (x + z) + j(y + w)$$

- Subtraction is also most easily performed in rectangular coordinates:

$$\mathbf{A} - \mathbf{B} = (x - z) + j(y - w)$$

Complex Number Multiplication and Division

- Multiplication is most easily performed in polar coordinates:

$$\mathbf{A} = A_M \angle \theta \qquad\qquad \mathbf{B} = B_M \angle \phi$$

$$\mathbf{A} \times \mathbf{B} = (A_M \times B_M) \angle (\theta + \phi)$$

- Division is also most easily performed in polar coordinates:

$$\mathbf{A} / \mathbf{B} = (A_M / B_M) \angle (\theta - \phi)$$

Examples

- Find the time domain representations of

$$\mathbf{V} = 104\text{V} - j60\text{V}$$

$$\mathbf{I} = -1\text{mA} - j3\text{mA}$$

at 60 Hz

- If $\mathbf{Z} = -1 + j2\ \text{k}\Omega$, then find the value of

$$\mathbf{I}\ \mathbf{Z} + \mathbf{V}$$