a mechanical explanation of the cause was to be had on some such principles as the following:—Vapour of sodium must possess by its molecular structure a tendency to vibrate in the periods corresponding to the degrees of refrangibility of the double line D. Hence the presence of sodium in a source of light must tend to originate light of that quality. On the other hand, vapour of sodium in an atmosphere round a source, must have a great tendency to retain in itself, *i. e.* to absorb and to have its temperature raised by light from the source, of the precise quality in question. In the atmosphere around the sun, therefore, there must be present vapour of sodium, which, according to the mechanical explanation thus suggested, being particularly opake for light of that quality, prevents such of it as is emitted from the sun from penetrating to any considerable distance through the surrounding atmosphere. The test of this theory must be had in ascertaining whether or not vapour of sodium has the special absorbing power anticipated. I have the impression that some Frenchman did make this out by experiment, but I can find no reference on the point.

"I am not sure whether Professor Stokes's suggestion of a mechanical theory has ever appeared in print. I have given it in my lectures regularly for many years, always pointing out along with it that solar and stellar chemistry were to be studied by investigating terrestrial substances giving bright lines in the spectra of artificial flames corresponding to the dark lines of the solar and stellar spectra."

II. *Illustrations of the Dynamical Theory of Gases. By* J. C. MAXWELL, *M.A., Professor of Natural Philosophy in Marischal College and University of Aberdeen.*

[Concluded from vol. xix. p. 32.]

PART II. *On the Process of Diffusion of two or more kinds of moving particles among one another.*

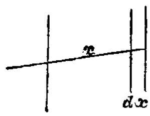
WE have shown, in the first part of this paper, that the motions of a system of many small elastic particles are of two kinds: one, a general motion of translation of the whole system, which may be called the motion in mass; and the other a motion of agitation, or molecular motion, in virtue of which velocities in all directions are distributed among the particles according to a certain law. In the cases we are considering, the collisions are so frequent that the law of distribution of the molecular velocities, if disturbed in any way, will be re-established in an inappreciably short time; so that the motion will always consist of this definite motion of agitation, combined with the general motion of translation.

When two gases are in communication, streams of the two gases might run freely in opposite directions, if it were not for the collisions which take place between the particles. The rate at which they actually interpenetrate each other must be investigated. The diffusion is due partly to the spreading of the particles by the molecular agitation, and partly to the actual motion of the two opposite currents in mass, produced by the pressure behind, and resisted by the collisions of the opposite stream. When the densities are equal, the diffusions due to these two causes respectively are as 2 to 3.

Prop. XIV. *In a system of particles whose density, velocity, &c. are functions of* x, *to find the quantity of matter transferred across the plane of* yz, *due to the motion of agitation alone.*

If the number of particles, their velocity, or their length of path is greater on one side of this plane than on the other, then more particles will cross the plane in one direction than in the other; and there will be a transference of matter across the plane, the amount of which may be calculated.

Let there be taken a stratum whose thickness is dx, and area unity, at a distance x from the origin. The number of collisions taking place in this stratum in unit of time will be



$$N\frac{v}{l}dx.$$

The proportion of these which reach a distance between nl and $(n+dn)l$ before they strike another particle is

$$e^{-n}dn.$$

The proportion of these which pass through the plane yz is

$$\frac{nl+x}{2nl} \text{ when } x \text{ is between } -nl \text{ and } 0,$$

and
$$-\frac{nl-x}{2nl} \text{ when } x \text{ is between } 0 \text{ and } +nl;$$

the sign being negative in the latter case, because the particles cross the plane in the negative direction. The mass of each particle is M; so that the quantity of matter which is projected from the stratum dx, crosses the plane yz in a positive direction, and strikes other particles at distances between nl and $(n+dn)l$ is

$$\frac{MNv(x\mp nl)}{2nl^2}dx\,e^{-n}dn, \quad . \quad . \quad . \quad . \quad . \quad (26)$$

where x must be between $\pm nl$, and the upper or lower sign is to be taken according as x is positive or negative.

In integrating this expression, we must remember that N, v, and l are functions of x, not vanishing with x, and of which the variations are very small between the limits $x=-nl$ and $x=+nl$.

As we may have occasion to perform similar integrations, we may state here, to save trouble, that if U and r are functions of x not vanishing with x, whose variations are very small between the limits $x=+r$ and $x=-r$,

$$\int_{-r}^{+r} \pm \mathrm{U}x^m dx = \frac{2}{m+2}\frac{d}{dx}(\mathrm{U}r^{m+2}). \quad . \quad . \quad . \quad (27)$$

When m is an odd number, the upper sign only is to be considered; when m is even or zero, the upper sign is to be taken with positive values of x, and the lower with negative values. Applying this to the case before us,

$$\int_{-nl}^{+nl} \frac{\mathrm{MN}vx\,dx}{2nl^2} = \tfrac{1}{3}\frac{d}{dx}(\mathrm{MN}vn^2l),$$

$$\int_{-nl}^{+nl} \mp \frac{\mathrm{MN}v}{2l}\,dx = -\tfrac{1}{2}\frac{d}{dx}(\mathrm{MN}vn^2l).$$

We have now to integrate

$$\int_0^\infty -\tfrac{1}{6}\frac{d}{dx}(\mathrm{MN}vl)n^2e^{-n}dn,$$

n being taken from 0 to ∞. We thus find for the quantity of matter transferred across unit of area by the motion of agitation in unit of time,

$$q = -\tfrac{1}{3}\frac{d}{dx}(\rho vl), \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

where $\rho=\mathrm{MN}$ is the density, v the mean velocity of agitation, and l the mean length of path.

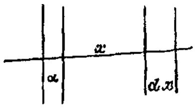
Prop. XV. The quantity transferred, in consequence of a mean motion of translation V, would obviously be

$$\mathrm{Q}=\mathrm{V}\rho. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

Prop. XVI. *To find the resultant dynamical effect of all the collisions which take place in a given stratum.*

Suppose the density and velocity of the particles to be functions of x, then more particles will be thrown into the given stratum from that side on which the density is greatest; and those particles which have greatest velocity will have the greatest effect, so that the stratum will not be generally in equilibrium, and the dynamical measure of the force exerted on the stratum will be the resultant momentum of all the particles which lodge in it during unit of time. We shall first take the case in which there is no mean motion of translation, and then consider the effect of such motion separately.

Let a stratum whose thickness is α (a small quantity compared with l), and area unity, be taken at the origin, perpendicular to the axis of x; and let another stratum, of thickness dx, and area unity, be taken at a distance x from the first.



If M_1 be the mass of a particle, N the number in unit of volume, v the velocity of agitation, l the mean length of path, then the number of collisions which take place in the stratum dx is

$$N\frac{v}{l}dx.$$

The proportion of these which reach a distance between nl and $(n+dn)l$ is

$$e^{-n}dn.$$

The proportion of these which have the extremities of their paths in the stratum α is

$$\frac{\alpha}{2nl}.$$

The velocity of these particles, resolved in the direction of x, is

$$-\frac{vx}{nl},$$

and the mass is M; so that multiplying all these terms together, we get

$$\frac{NMv^2\alpha x}{2n^2l^3}e^{-n}\,dx\,dn \quad . \quad . \quad . \quad . \quad . \quad (30)$$

for the momentum of the particles fulfilling the above conditions.

To get the whole momentum, we must first integrate with respect to x from $x=-nl$ to $x=+nl$, remembering that l may be a function of x, and is a very small quantity. The result is

$$\frac{d}{dx}\left(\frac{NMv^2}{3}\right)\alpha ne^{-n}dn.$$

Integrating with respect to n from $n=0$ to $n=\infty$, the result is

$$-\alpha\frac{d}{dx}\left(\frac{NMv^2}{3}\right)=\alpha X\rho \quad . \quad . \quad . \quad . \quad (31$$

as the whole resultant force on the stratum α arising from these collisions. Now $\frac{NMv^2}{3}=p$ by Prop. XII., and therefore we may write the equation

$$-\frac{dp}{dx}=X\rho, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

the ordinary hydrodynamical equation.

Prop. XVII. *To find the resultant effect of the collisions upon each of several different systems of particles mixed together.*

Let M_1, M_2, &c. be the masses of the different kinds of particles, N_1, N_2, &c. the number of each kind in unit of volume, v_1, v_2, &c. their velocities of agitation, l_1, l_2 their mean paths, p_1, p_2, &c. the pressures due to each system of particles; then

$$\left.\begin{aligned}\frac{1}{l_1}&=A\rho_1+B\rho_2+\&c.\\ \frac{1}{l_2}&=C\rho_1+D\rho_2+\&c.\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (33)$$

The number of collisions of the first kind of particles with each other in unit of time will be

$$N_1 v_1 A\rho_1.$$

The number of collisions between particles of the first and second kinds will be

$$N_1 v_1 B\rho_2, \text{ or } N_2 v_2 C\rho_1, \text{ because } v_1{}^3 B=v_2{}^3 C.$$

The number of collisions between particles of the second kind will be $N_2 v_2 D\rho_2$, and so on, if there are more kinds of particles.

Let us now consider a thin stratum of the mixture whose volume is unity.

The resultant momentum of the particles of the first kind which lodge in it during unit of time is

$$-\frac{dp_1}{dx}.$$

The proportion of these which strike particles of the first kind is

$$A\rho_1 l_1.$$

The whole momentum of these remains among the particles of the first kind. The proportion which strike particles of the second kind is

$$B\rho_2 l_1.$$

The momentum of these is divided between the striking particles in the ratio of their masses; so that $\frac{M_1}{M_1+M_2}$ of the whole goes to particles of the first kind, and $\frac{M_2}{M_1+M_2}$ to particles of the second kind.

The effect of these collisions is therefore to produce a force

$$-\frac{dp_1}{dx}\left(\mathrm{A}\rho_1 l_1+\mathrm{B}\rho_2 l_1\frac{\mathrm{M}_1}{\mathrm{M}_1+\mathrm{M}_2}\right)$$

on particles of the first system, and

$$-\frac{dp_1}{dx}\mathrm{B}\rho_2 l_1\frac{\mathrm{M}_2}{\mathrm{M}_1+\mathrm{M}_2}$$

on particles of the second system.

The effect of the collisions of those particles of the second system which strike into the stratum, is to produce a force

$$-\frac{dp_2}{dx}\mathrm{C}\rho_1 l_2\frac{\mathrm{M}_1}{\mathrm{M}_1+\mathrm{M}_2}$$

on the first system, and

$$-\frac{dp_2}{dx}\left(\mathrm{C}\rho_1 l_2\frac{\mathrm{M}_2}{\mathrm{M}_1+\mathrm{M}_2}+\mathrm{D}\rho_2 l_2\right)$$

on the second.

The whole effect of these collisions is therefore to produce a resultant force

$$-\frac{dp_1}{dx}\left(\mathrm{A}\rho_1 l_1+\mathrm{B}\rho_2 l_1\frac{\mathrm{M}_1}{\mathrm{M}_1+\mathrm{M}_2}\right)-\frac{dp_2}{dx}\mathrm{C}\rho_1 l_2\frac{\mathrm{M}_1}{\mathrm{M}_1+\mathrm{M}_2}+\text{\&c.} \quad (34)$$

on the first system,

$$-\frac{dp_1}{dx}\mathrm{B}\rho_2 l_1\frac{\mathrm{M}_2}{\mathrm{M}_1+\mathrm{M}_2}-\frac{dp_2}{dx}\left(\mathrm{C}\rho_1 l_2\frac{\mathrm{M}_2}{\mathrm{M}_1+\mathrm{M}_2}+\mathrm{D}\rho_2 l_2\right)+\text{\&c.} \quad (35)$$

on the second, and so on.

Prop. XVIII. *To find the mechanical effect of a difference in the mean velocity of translation of two systems of moving particles.*

Let V_1, V_2 be the mean velocities of translation of the two systems respectively, then $\frac{\mathrm{M}_1\mathrm{M}_2}{\mathrm{M}_1+\mathrm{M}_2}(\mathrm{V}_1-\mathrm{V}_2)$ is the mean momentum lost by a particle of the first, and gained by a particle of the second at collision. The number of such collisions in unit of volume is

$$\mathrm{N}_1\mathrm{B}\rho_2 v_1,\ \text{or}\ \mathrm{N}_2\mathrm{C}\rho_1 v_2;$$

therefore the whole effect of the collisions is to produce a force

$$=-\mathrm{N}_1\mathrm{B}\rho_2 v_1\frac{\mathrm{M}_1\mathrm{M}_2}{\mathrm{M}_1+\mathrm{M}_2}(\mathrm{V}_1-\mathrm{V}_2) \quad . \quad . \quad . \quad (36)$$

on the first system, and an equal and opposite force

$$=+\mathrm{N}_2\mathrm{C}\rho_1 v_2\frac{\mathrm{M}_1\mathrm{M}_2}{\mathrm{M}_1+\mathrm{M}_2}(\mathrm{V}_1-\mathrm{V}_2) \quad . \quad . \quad . \quad (37)$$

on unit of volume of the second system.

Prop. XIX. *To find the law of diffusion in the case of two gases diffusing into each other through a plug made of a porous material, as in the case of the experiments of Graham.*

The pressure on each side of the plug being equal, it was found by Graham that the quantities of the gases which passed in opposite directions through the plug in the same time were directly as the square roots of their specific gravities.

We may suppose the action of the porous material to be similar to that of a number of particles fixed in space, and obstructing the motion of the particles of the moving systems. If L_1 is the mean distance a particle of the first kind would have to go before striking a fixed particle, and L_2 the distance for a particle of the second kind, then the mean paths of particles of each kind will be given by the equations

$$\frac{1}{l_1}=A\rho_1+B\rho_2+\frac{1}{L_1},\quad \frac{1}{l_2}=C\rho_1+D\rho_2+\frac{1}{L_2}. \quad . \quad (38)$$

The mechanical effect upon the plug of the pressures of the gases on each side, and of the percolation of the gases through it, may be found by Props. XVII. and XVIII. to be

$$\frac{M_1N_1v_1V_1}{L_1}+\frac{M_2N_2v_2V_2}{L_2}-\frac{dp_1}{dx}\frac{l_1}{L_1}-\frac{dp_2}{dx}\frac{l_2}{L_2}=0; \quad . \quad (39)$$

and this must be zero, if the pressures are equal on each side of the plug. Now if Q_1, Q_2 be the quantities transferred through the plug by the mean motion of translation, $Q_1=\rho_1V_1=M_1N_1V_1$; and since by Graham's law

$$\frac{Q_1}{Q_2}=-\sqrt{\frac{M_1}{M_2}}=-\frac{v_2}{v_1},$$

we shall have

$$M_1N_1v_1V_1=-M_2N_2v_2V_2=U \text{ suppose};$$

and since the pressures on the two sides are equal, $\frac{dp_2}{dx}=-\frac{dp_1}{dx}$, and the only way in which the equation of equilibrium of the plug can generally subsist is when $L_1=L_2$ and $l_1=l_2$. This implies that $A=C$ and $B=D$. Now we know that $v_1^3B=v_2^3C$. Let $K=3\frac{A}{v_1^3}$, then we shall have

$$A=C=\tfrac{1}{3}Kv_1^3,\quad B=D=\tfrac{1}{3}Kv_2^3, \quad . \quad . \quad . \quad (40)$$

and

$$\frac{1}{l_1}=\frac{1}{l_2}=K(v_1p_1+v_2p_2)+\frac{1}{L}. \quad . \quad . \quad . \quad (41)$$

The diffusion is due partly to the motion of translation, and partly to that of agitation. Let us find the part due to the motion of translation.

The equation of motion of one of the gases through the plug is found by adding the forces due to pressures to those due to resistances, and equating these to the moving force, which in the case of slow motions may be neglected altogether. The result for the first is

$$\frac{dp_1}{dx}\left(\mathrm{A}\rho_1 l_1 + \mathrm{B}\rho_2 l_1 \frac{\mathrm{M}_1}{\mathrm{M}_1+\mathrm{M}_2}\right) + \frac{dp_2}{dx}\mathrm{C}\rho_1 l_2 \frac{\mathrm{M}_1}{\mathrm{M}_1+\mathrm{M}_2}$$

$$+\mathrm{N}_1\mathrm{B}\rho_2 v_1 \frac{\mathrm{M}_1\mathrm{M}_2}{\mathrm{M}_1+\mathrm{M}_2}(\mathrm{V}_1-\mathrm{V}_2) + \frac{\rho_1 v_1 \mathrm{V}_1}{\mathrm{L}} = 0. \quad . \quad . \quad (42)$$

Making use of the simplifications we have just discovered, this becomes

$$\frac{dp}{dx}\frac{\mathrm{K}l}{v_1^2+v_2^2}(v_1^3 p_1 + v_2^3 p_2) + \mathrm{K}\frac{v_1 v_2}{v_1^2+v_2^2}(p_1 v_2 + p_2 v_1)\mathrm{U} + \frac{1}{l}\mathrm{U}, \quad (43)$$

whence

$$\mathrm{U} = -\frac{dp}{dx}\frac{\mathrm{K}l(v_1^3 p_1 + v_2^3 p_2)}{\mathrm{K}v_1 v_2(p_1 v_2 + p_2 v_1) + \dfrac{v_1^2+v_2^2}{\mathrm{L}}}; \quad . \quad (44)$$

whence the rate of diffusion due to the motion of translation may be found; for

$$\mathrm{Q}_1 = \frac{\mathrm{U}}{v_1}, \text{ and } \mathrm{Q}_2 = -\frac{\mathrm{U}}{v_2}. \quad . \quad . \quad . \quad . \quad . \quad (45)$$

To find the diffusion due to the motion of agitation, we must find the value of q_1.

$$q_1 = -\tfrac{1}{3}\frac{d}{dx}(\rho_1 v_1 l_1),$$

$$= -\frac{\mathrm{L}}{v_1}\frac{d}{dx}\frac{p_1}{1+\mathrm{KL}(v_1 p_1 + v_2 p_2)},$$

$$q_1 = -\frac{l^2}{v_1\mathrm{L}}\frac{dp}{dx}\left(1+\mathrm{KL}v_2(p_1+p_2)\right). \quad . \quad . \quad (46)$$

Similarly,

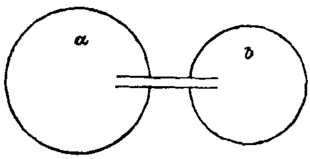
$$q_2 = +\frac{l^2}{v_2\mathrm{L}}\frac{dp}{dx}\left(1+\mathrm{KL}v_1(p_1+p_2)\right). \quad . \quad . \quad (47)$$

The whole diffusions are Q_1+q_1 and Q_2+q_2. The values of q_1 and q_2 have a term not following Graham's law of the square roots of the specific gravities, but following the law of equal volumes. The closer the material of the plug, the less will this term affect the result.

Our assumptions that the porous plug acts like a system of fixed particles, and that Graham's law is fulfilled more accurately the more compact the material of the plug, are scarcely sufficiently well verified for the foundation of a theory of gases; and even if we admit the original assumption that they are systems of moving elastic particles, we have not very good evidence as yet for the relation among the quantities A, B, C, and D.

Prop. XX. *To find the rate of diffusion between two vessels connected by a tube.*

When diffusion takes place through a large opening, such as a tube connecting two vessels, the question is simplified by the absence of the porous diffusion plug; and since the pressure is constant throughout the apparatus, the volumes of the two gases passing opposite ways through the tube at the same time must be equal. Now the quantity of gas which passes through the tube is due partly to the motion of agitation as in Prop. XIV., and partly to the mean motion of translation as in Prop. XV.



Let us suppose the volumes of the two vessels to be a and b, and the length of the tube between them c, and its transverse section s. Let a be filled with the first gas, and b with the second at the commencement of the experiment, and let the pressure throughout the apparatus be P.

Let a volume y of the first gas pass from a to b, and a volume y' of the second pass from b to a; then if p_1 and p_2 represent the pressures in a due to the first and second kinds of gas, and p'_1 and p'_2 the same in the vessel b,

$$p_1=\frac{a-y}{a}\mathrm{P},\quad p_2=\frac{y'}{a}\mathrm{P},\quad p'_1=\frac{y}{b}\mathrm{P},\quad p'_2=\frac{b-y'}{b}\mathrm{P}. \quad . \quad (48)$$

Since there is still equilibrium,

$$p_1+p_2=p'_1+p'_2,$$

which gives

$$y=y' \text{ and } p_1+p_2=\mathrm{P}=p'_1+p'_2. \quad . \quad . \quad . \quad (49)$$

The rate of diffusion will be $+\frac{dy}{dt}$ for the one gas, and $-\frac{dy}{dt}$ for the other, measured in volume of gas at pressure P.

Now the rate of diffusion of the first gas will be

$$\frac{dy}{dt}=s\frac{k_1 q_1+p_1\mathrm{V}_1}{\mathrm{P}}=s\frac{-\frac{1}{3}v_1\frac{d}{dx}(p_1 l_1)+p_1\mathrm{V}_1}{\mathrm{P}}; \quad . \quad . \quad (50)$$

and that of the second,

$$-\frac{dy}{dt}=s\frac{-\frac{1}{3}v_2\frac{d}{dx}(p_2l_2)+p_2V_2}{P}. \quad \ldots \ldots \quad (51)$$

We have also the equation, derived from Props. XVI. and XVII.,

$$\frac{dp_1}{dx}\left(A\rho_1l_1(M_1+M_2)+B\rho_2l_1M_1-C\rho_1l_2M_1\right)$$
$$+B\rho_1\rho_2v_1M_2(V_1-V_2)=0. \quad \ldots \ldots \quad (52)$$

From these three equations we can eliminate V_1 and V_2, and find $\frac{dy}{dt}$ in terms of p and $\frac{dp}{dx}$, so that we may write

$$\frac{dy}{dt}=f\left(p_1,\frac{dp_1}{dx}\right). \quad \ldots \ldots \quad (53)$$

Since the capacity of the tube is small compared with that of the vessels, we may consider $\frac{dy}{dt}$ constant through the whole length of the tube. We may then solve the differential equation in p and x; and then making $p=p_1$ when $x=0$, and $p=p'_1$ when $x=c$, and substituting for p_1 and p'_1 their values in terms of y, we shall have a differential equation in y and t, which being solved, will give the amount of gas diffused in a given time.

The solution of these equations would be difficult unless we assume relations among the quantities A, B, C, D, which are not yet sufficiently established in the case of gases of different density. Let us suppose that in a particular case the two gases have the same density, and that the four quantities A, B, C, D are all equal.

The volume diffused, owing to the motion of agitation of the particles, is then

$$-\frac{1}{3}\frac{s}{P}\frac{dp}{dx}vl,$$

and that due to the motion of translation, or the interpenetration of the two gases in opposite streams, is

$$-\frac{s}{P}\frac{dp}{dx}\frac{kl}{v}.$$

The values of v are distributed according to the law of Prop. IV., so that the mean value of v is $\frac{2\alpha}{\sqrt{\pi}}$, and that of $\frac{1}{v}$ is $\frac{2}{\sqrt{\pi\alpha}}$, that of k being $\frac{1}{2}\alpha^2$. The diffusions due to these two causes are therefore in the ratio of 2 to 3, and their sum is

$$\frac{dy}{dt}=-\tfrac{4}{3}\sqrt{\frac{2k}{\pi}}\frac{sl}{\mathrm{P}}\frac{dp}{dx}. \quad . \quad . \quad . \quad . \quad . \quad . \qquad (54)$$

If we suppose $\frac{dy}{dt}$ constant throughout the tube, or, in other words, if we regard the motion as *steady* for a short time, then $\frac{dp}{dx}$ will be constant and equal to $\frac{p'_1-p_1}{c}$; or substituting from (48),

$$\frac{dy}{dt}=-\tfrac{4}{3}\sqrt{\frac{2k}{\pi}}\frac{sl}{abc}\Big((a+b)y-ab\Big), \quad . \qquad (55)$$

whence

$$y=\frac{ab}{a+b}\left(1-e^{-\frac{4}{3}\sqrt{\frac{2k}{\pi}}\frac{sl}{abc}(a+b)t}\right). \quad . \quad . \qquad (56)$$

By choosing pairs of gases of equal density, and ascertaining the amount of diffusion in a given time, we might determine the value of l in this expression. The diffusion of nitrogen into carbonic oxide or of deutoxide of nitrogen into carbonic acid, would be suitable cases for experiment. The only existing experiment which approximately fulfils the conditions is one by Graham, quoted by Herapath from Brande's Quarterly Journal of Science, vol. xviii. p. 76.

A tube 9 inches long and 0·9 inch diameter, communicated with the atmosphere by a tube 2 inches long and 0·12 inch diameter; 152 parts of olefiant gas being placed in the tube, the quantity remaining after four hours was 99 parts.

In this case there is not much difference of specific gravity between the gases, and we have $a=9\times(0{\cdot}9)^2\frac{\pi}{4}$ cubic inches, $b=\infty$, $c=2$ inches, and $s=(0{\cdot}12)^2\frac{\pi}{4}$ square inches;

$$l=\sqrt{\frac{\pi}{2k}}\,\tfrac{3}{4}\,\frac{ac}{s}\log_e 10\,.\,\frac{1}{t}\,.\,\log_{10}\left(\frac{a}{a-y}\right); \quad . \qquad (57)$$

$$\therefore\ l=0{\cdot}00000256 \text{ inch}=\tfrac{1}{389000}\text{ inch}. \quad . \quad . \quad . \qquad (58)$$

Prop. XXI. *To find the amount of energy which crosses unit of area in unit of time when the velocity of agitation is greater on one side of the area than on the other.*

The energy of a single particle is composed of two parts,—the *vis viva* of the centre of gravity, and the *vis viva* of the various motions of rotation round that centre, or, if the particle be capable of internal motions, the *vis viva* of these. We shall suppose that the whole *vis viva* bears a constant proportion to that due to the motion of the centre of gravity, or

$$E=\tfrac{1}{2}\beta M v^2,$$

where β is a coefficient, the experimental value of which is 1·634. Substituting E for M in Prop. XIV., we get for the transference of energy across unit of area in unit of time,

$$Jgq=-\tfrac{1}{3}\frac{d}{dx}(\tfrac{1}{2}\beta M v^2 N v l),$$

where J is the mechanical equivalent of heat in foot-pounds, and q is the transfer of heat in thermal units.

Now $MN=\rho$, and $l=\frac{1}{A\rho}$, so that $MNl=\frac{1}{A}$;

$$\therefore\ Jgq=-\tfrac{1}{2}\frac{\beta v^2}{A}\frac{dv}{dx}. \qquad (59)$$

Also, if T is the absolute temperature,

$$\frac{1}{T}\frac{dT}{dx}=\frac{2}{v}\frac{dv}{dx};$$

$$\therefore\ Jgq=-\tfrac{5}{4}\beta p l v\frac{1}{T}\frac{dT}{dx}, \qquad (60)$$

where p must be measured in dynamical units of force.

Let $J=772$ foot-pounds, $p=2116$ pounds to square foot, $l=\frac{1}{400000}$ inch, $v=1505$ feet per second, $T=522$ or 62° Fahrenheit; then

$$q=\frac{T'-T}{40000x}, \qquad (61)$$

where q is the flow of heat in thermal units per square foot of area; and T', and T are the temperatures at the two sides of a stratum of air x *inches* thick.

In Prof. Rankine's work on the Steam-engine, p. 259, values of the *thermal resistance*, or the reciprocal of the *conductivity*, are given for various substances as computed from a Table of conductivities deduced by M. Peclet from experiments by M. Despretz:—

	Resistance.
Gold, Platinum, Silver	0·0036
Copper	0·0040
Iron	0·0096
Lead	0·0198
Brick	0·3306
Air by our calculation	40000

It appears, therefore, that the resistance of a stratum of air to the conduction of heat is about 10,000,000 times greater than that of a stratum of copper of equal thickness. It would be almost impossible to establish the value of the conductivity of a gas by direct experiment, as the heat radiated from the sides of the vessel would be far greater than the heat conducted through the air, even if currents could be entirely prevented.

PART III. *On the Collision of Perfectly Elastic Bodies of any Form.*

When two perfectly smooth spheres strike each other, the force which acts between them always passes through their centres of gravity; and therefore their motions of rotation, if they have any, are not affected by the collision, and do not enter into our calculations. But, when the bodies are not spherical, the force of compact will not, in general, be in the line joining their centres of gravity; and therefore the force of impact will depend both on the motion of the centres and the motions of rotation before impact, and it will affect both these motions after impact.

In this way the velocities of the centres and the velocities of rotation will act and react on each other, so that finally there will be some relation established between them; and since the rotations of the particles about their three axes are quantities related to each other in the same way as the three velocities of their centres, the reasoning of Prop. IV. will apply to rotation as well as velocity, and both will be distributed according to the law

$$\frac{dN}{dx}=N\frac{1}{\alpha\sqrt{\pi}}e^{-\frac{x^2}{\alpha^2}}.$$

Also, by Prop. V., if x be the average velocity of one set of particles, and y that of another, then the average value of the sum or difference of the velocities is

$$\sqrt{x^2+y^2};$$

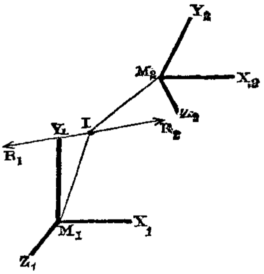
from which it is easy to see that, if in each individual case

$$u=ax+by+cz,$$

where x, y, z are independent quantities distributed according to the law above stated, then the *average values* of these quantities will be connected by the equation

$$u^2=a^2x^2+b^2y^2+c^2z^2.$$

Prop. XXII. *Two perfectly elastic bodies of any form strike each other: given their motions before impact, and the line of impact, to find their motions after impact.*



Let M_1 and M_2 be the centres of gravity of the two bodies. $M_1 X_1$, $M_1 Y_1$, and $M_1 Z_1$ the principal axes of the first; and $M_2 X_2$, M_2, Y_2, and $M_2 Z_2$ those of the second. Let I be the point of impact, and $R_1 I R_2$ the line of impact.

Let the coordinates of I with respect to M_1 be $x_1\, y_1\, z_1$, and with respect to M_2 let them be $x_2\, y_2\, z_2$.

Let the direction-cosines of the line of impact $R_1 I R_2$ be

$$l_1\, m_1\, n_1 \text{ with respect to } M_1, \text{ and } l_2\, m_2\, n_2 \text{ with respect to } M_2.$$

Let M_1 and M_2 be the masses, and $A_1\, B_1\, C_1$ and $A_2\, B_2\, C_2$ the moments of inertia of the bodies about their principal axes.

Let the velocities of the centres of gravity, resolved in the direction of the principal axes of each body, be

$$U_1\ V_1\ W_1 \text{ and } U_2\ V_2\ W_2 \text{ before impact,}$$

and

$$U'_1\ V'_1\ W'_1 \text{ and } U'_2\ V'_2\ W'_2 \text{ after impact.}$$

Let the angular velocities round the same axes be

$$p_1\ q_1\ r_1 \text{ and } p_2\ q_2\ r_2 \text{ before impact,}$$

and

$$p'_1\ q'_1\ r'_1 \text{ and } p'_2\ q'_2\ r'_2 \text{ after impact.}$$

Let R be the impulsive force between the bodies, measured by the momentum it produces in each.

Then, for the velocities of the centres of gravity, we have the following equations:

$$U'_1 = U_1 + \frac{Rl_1}{M_1}, \quad U'_2 = U_2 - \frac{Rl_2}{M_2}, \quad \ldots \quad (62)$$

with two other pairs of equations in V and W. The equations for the angular velocities are

$$p'_1 = p_1 + \frac{R}{A_1}(y_1 n_1 - z_1 m_1), \quad p'_2 = p_2 - \frac{R}{A_2}(y_2 n_2 - z_2 m_2), \quad (63)$$

with two other pairs of equations for q and r.

The condition of perfect elasticity is that the whole *vis viva* shall be the same after impact as before, which gives the equation

$$M_1(U'^2_1 - U^2_1) + M_2(U'^2_1 - U^2_2) + A_1(p'^2_1 - p^2_1) + A_2(p'^2_2 - p^2_2) + \&c. = 0. \quad (64)$$

The terms relating to the axis of x are here given; those relating to y and z may be easily written down.

Substituting the values of these terms, as given by equations

(62) and (63), and dividing by R, we find

$$l_1(U'_1+U_1)-l_2(U'_2+U_2)+(y_1n_1-z_1m_1)(p'_1+p_1)$$
$$-(y_2n_2-z_2m_2)(p'_2+p_2)+\&c.=0. \quad . \quad . \quad . \quad . \quad (65)$$

Now if v_1 be the velocity of the striking-point of the first body before impact, resolved along the line of impact,

$$v_1=l_1U_1+(y_1n_1-z_1m_1)p_1+\&c.;$$

and if we put v_2 for the velocity of the other striking-point resolved along the same line, and v'_1 and v'_2 the same quantities after impact, we may write equation (65),

$$v_1+v'_1-v_2-v'_2=0, \quad . \quad . \quad . \quad . \quad . \quad . \quad (66)$$

or

$$v_1-v_2=v'_2-v'_1, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (67)$$

which shows that the velocity of separation of the striking-points resolved in the line of impact is equal to that of approach.

Substituting the values of the accented quantities in equation (65) by means of equations (63) and (64), and transposing terms in R, we find

$$2\{U_1l_1-U_2l_2+p_1(y_1n_1-z_1m_1)-p_2(y_2n_2-z_2m_2)\}+\&c.$$
$$=-R\left\{\frac{l_1^2}{M_1}+\frac{l_2^2}{M_2}+\frac{(y_1n_1-z_1m_1)^2}{A_1}+\frac{(y_2n_2-z_2m_2)^2}{A_2}+\&c., \right. \quad (68)$$

the other terms being related to y and z as these are to x. From this equation we may find the value of R; and by substituting this in equations (63), (64), we may obtain the values of all the velocities after impact.

We may, for example, find the value of U'_1 from the equation

$$\left.\begin{array}{l} U'_1\left\{\dfrac{l_1^2}{M_1}+\dfrac{l_2^2}{M_2}+\dfrac{(y_1n_1-z_1m_1)^2}{A_1}+\dfrac{(y_2n_2-z_2m_2)^2}{A_2}+\&c.\right\}\dfrac{M_1}{l_1} \\ =U_1\left\{-\dfrac{l_1^2}{M_1}+\dfrac{l_2^2}{M_2}+\dfrac{(y_1n_1-z_1m_1)^2}{A_1}+\dfrac{(y_2n_2-z_2m_2)^2}{A_2}+\&c.\right\}\dfrac{M_1}{l_1} \\ +2U_2l_2-2p_1(y_1n_1-z_1m_1)+2p_2(y_2n_2-z_2m_2)-\&c. \end{array}\right\}. \quad (69)$$

Prop. XXIII. *To find the relations between the average velocities of translation and rotation after many collisions among many bodies.*

Taking equation (69), which applies to an individual collision, we see that U'_1 is expressed as a linear function of U_1, U_2, p_1, p_2, &c., all of which are quantities of which the values are distributed among the different particles according to the law of Prop. IV. It follows from Prop. V., that if we square every term of the equation, we shall have a new equation between the *average values* of the different quantities. It is plain that, as soon as the required relations have been established, they will remain the same after collision, so that we may put $U_1'^2 = U_1^2$ in the equation of averages. The equation between the average values may then be written

$$(M_1U_1^2 - M_2U_2^2)\frac{l_2^2}{M_2} + (M_1U_1^2 - A_1p_1^2)\frac{(y_1n_1 - z_1m_1)^2}{A_1}$$
$$+ (M_1U_1^2 - A_2p_2^2)\frac{(y_2n_2 - z_2m_2)^2}{A_2} + \&c. = 0.$$

Now since there are collisions in every possible way, so that the values of l, m, n, &c. and x, y, z, &c. are infinitely varied, this equation cannot subsist unless

$$M_1U_1^2 = M_2U_2^2 = A_1p_1^2 = A_2p_2^2 = \&c.$$

The final state, therefore, of any number of systems of moving particles of any form is that in which the average *vis viva* of translation along each of the three axes is the same in all the systems, and equal to the average *vis viva* of rotation about each of the three principal axes of each particle.

Adding the *vires vivæ* with respect to the other axes, we find that the whole *vis viva* of translation is equal to that of rotation in each system of particles, and is also the same for different systems, as was proved in Prop. VI.

This result (which is true, however nearly the bodies approach the spherical form, provided the motion of rotation is at all affected by the collisions) seems decisive against the unqualified acceptation of the hypothesis that gases are such systems of hard elastic particles. For the ascertained fact that γ, the ratio of the specific heat at constant pressure to that at constant volume, is equal to 1·408, requires that the ratio of the whole *vis viva* to the *vis viva* of translation should be

$$\beta = \frac{2}{3(\gamma - 1)} = 1{\cdot}634;$$

whereas, according to our hypothesis, $\beta = 2$.

We have now followed the mathematical theory of the collisions of hard elastic particles through various cases, in which there seems to be an analogy with the phænomena of gases. We have deduced, as others have done already, the relations of pressure, temperature, and density of a single gas. We have also proved that when two different gases act freely on each other (that is, when at the same temperature), the mass of the single particles of each is inversely proportional to the square of the molecular velocity; and therefore, at equal temperature and pressure, *the number of particles in unit of volume is the same.*

We then offered an explanation of the internal friction of gases, and deduced from experiments a value of the mean length of path of a particle between successive collisions.

We have applied the theory to the law of diffusion of gases, and, from an experiment on olefiant gas, we have deduced a value of the length of path not very different from that deduced from experiments on friction.

Using this value of the length of path between collisions, we found that the resistance of air to the conduction of heat is 10,000,000 that of copper, a result in accordance with experience.

Finally, by establishing a necessary relation between the motions of translation and rotation of all particles not spherical, we proved that a system of such particles could not possibly satisfy the known relation between the two specific heats of all gases.

III. *On a New Theoretical Determination of the Velocity of Sound. By the* Rev. S. Earnshaw, *M.A., Sheffield.*

[Continued from vol. xix. p. 455.]

On the Velocity of the Sound of Thunder.

THERE yet remains to be considered a case of sound-velocity to which the investigations of Newton and the suggestion of Laplace are totally inadequate, which nevertheless is naturally suggested, by what has been done in the preceding articles, as necessary to complete the theory of sound-velocity: I allude to the propagation of the sound of a clap of thunder. The consideration of this case will strengthen the evidence of the soundness of the preceding investigations.

Before it was announced by myself at the Meeting of the British Association at Leeds in 1858, that according to theory violent sounds are propagated more rapidly than gentle sounds, I believe the fact was not suspected by philosophers. I was led to this result by a careful discussion of the integral of the well-known equation of motion of an elastic fluid in a horizontal tube. I was, however, not able to bring forward any instance of the fact having been observed, except a single one, recorded in one of Parry's Voyages to the North. The records of experimentalists agreed in stating, on the contrary, that all sounds travel at the same rate. Since that time the subject has rested. A few weeks ago, however, my attention was recalled to it by the receipt of a memoir printed in the *Bulletins de l'Académie Royale de Belgique*, kindly forwarded to me by its author, Professor Ch. Montigny of Antwerp, which has satisfied me that, in the case of a thunder-clap, sound is sometimes propagated with a velocity far greater than I had ever imagined, and that the problem of the propagation of sound is yet far from having been fully solved.