

AP CALCULUS TAKE HOME EXAM

Part #1: NO CALCULATOR, 28 QUESTIONS, 55 MINUTES

DATE: \_\_\_\_\_ TIME BEGAN: \_\_\_\_\_ TIME ENDED: \_\_\_\_\_

1.  $\int \cos(3x) dx =$

- (A)  $-\sin(3x) + C$
- (B)  $-\frac{1}{3}\sin(3x) + C$
- (C)  $\frac{1}{3}\sin(3x) + C$
- (D)  $\sin(3x) + C$
- (E)  $3\sin(3x) + C$

2.  $\lim_{x \rightarrow 0} \frac{2x^6 + 6x^3}{4x^5 + 3x^3}$  is

- (A) 0
- (B)  $\frac{1}{2}$
- (C) 1
- (D) 2
- (E) nonexistent

3.  $f(x) = \begin{cases} x^2 - 3x + 9 & \text{for } x \leq 2 \\ kx + 1 & \text{for } x > 2 \end{cases}$  The function  $f$  is defined. For what value of  $k$ , if any, is  $f$  continuous at  $x = 2$ ?

- (A) 1
- (B) 2
- (C) 3
- (D) 7
- (E) No value of  $k$  will make  $f$  continuous at  $x = 2$

4. If  $f(x) = \cos^3(4x)$ , then  $f'(x) =$

- (A)  $3\cos^2(4x)$
- (B)  $-12\cos^2(4x)\sin(4x)$
- (C)  $-3\cos^2(4x)\sin(4x)$
- (D)  $12\cos^2(4x)\sin(4x)$
- (E)  $-4\sin^3(4x)$

5. The function  $f$  given by  $f(x) = 2x^3 - 3x^2 - 12x$  has a relative minimum at  $x =$

- (A) -1
- (B) 0
- (C) 2
- (D)  $\frac{3 - \sqrt{105}}{4}$
- (E)  $\frac{3 + \sqrt{105}}{4}$

AP CALCULUS TAKE HOME EXAM

6. Let  $f$  be the function given by  $f(x) = (2x-1)^5(x+1)$ . Which of the following is an equation for the line tangent to the graph of  $f$  at the point where  $x = 1$ ?

- (A)  $y = 21x + 2$
- (B)  $y = 21x - 19$
- (C)  $y = 11x - 9$
- (D)  $y = 10x + 2$
- (E)  $y = 10x - 8$

7.  $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx =$

- (A)  $2e^{\sqrt{x}} + C$
- (B)  $\frac{1}{2}e^{\sqrt{x}} + C$
- (C)  $e^{\sqrt{x}} + C$
- (D)  $2xe^{\sqrt{x}} + C$
- (E)  $\frac{1}{2} \frac{e^{\sqrt{x}}}{\sqrt{x}} + C$

$x$	0	2	4	6
$f(x)$	4	$k$	8	12

8. The function  $f$  is continuous on the closed interval  $[0,6]$  and has the values given in the table above.

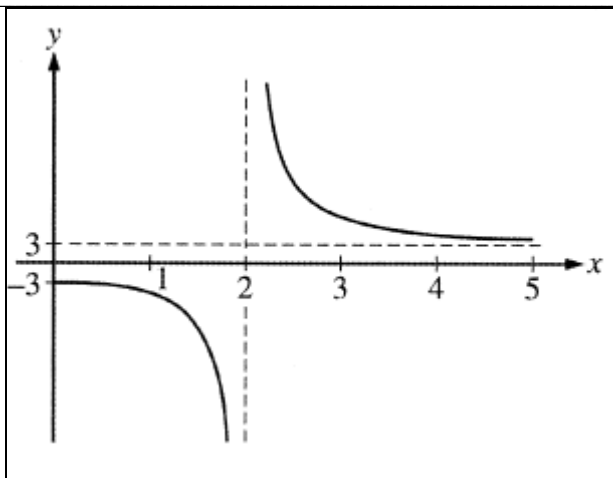
The trapezoidal approximation for  $\int_0^6 f(x)dx$  found with 3 subintervals of equal length is 52. What is the value of  $k$ ?

- (A) 2
- (B) 6
- (C) 7
- (D) 10
- (E) 14

9. A particle moves along the  $x$ -axis so that at any time  $t > 0$ , its velocity is given by  $v(t) = 4 - 6t^2$ . If the particle is at position  $x=7$  and at time  $t=1$ , what is the position of the particle at time  $t=2$ ?

- (A) -10
- (B) -5
- (C) -3
- (D) 3
- (E) 17

AP CALCULUS TAKE HOME EXAM



10. The function  $f$  is given by  $f(x) = \frac{ax^2 + 12}{x^2 + b}$ . The figure above shows a portion of the graph of  $f$ . Which of the following could be the values of the constants  $a$  and  $b$ ?

- (A)  $a=-3, b=2$
- (B)  $a=2, b=-3$
- (C)  $a=2, b=-2$
- (D)  $a=3, b=-4$
- (E)  $a=3, b=4$

11. What is the slope of the line tangent to the graph of  $y = \frac{e^{-x}}{x+1}$  at  $x=1$ ?

- (A)  $\frac{1}{e}$
- (B)  $\frac{3}{4e}$
- (C)  $\frac{1}{4e}$
- (D)  $\frac{1}{4e}$
- (E)  $\frac{1}{e}$

12. If  $f'(x) = \frac{2}{x}$  and  $f(\sqrt{e}) = 5$ , then  $f(e) =$

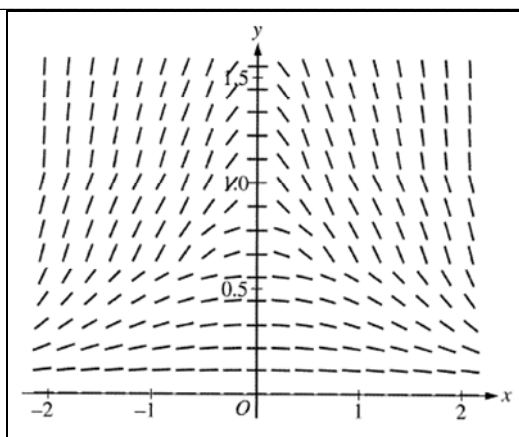
- (A) 2
- (B)  $\ln 25$
- (C)  $5 + \frac{2}{e} - \frac{2}{e^2}$
- (D) 6
- (E) 25

13.  $\int (x^3 + 1)^2 dx =$

- (A)  $\frac{1}{7}x^7 + x + C$
- (B)  $\frac{1}{7}x^7 + \frac{1}{2}x^4 + x + C$
- (C)  $6x^2(x^3 + 1) + C$
- (D)  $\frac{1}{3}(x^3 + 1)^3 + C$
- (E)  $\frac{(x^3 + 1)^3}{9x^2} + C$

14.  $\lim_{h \rightarrow 0} \frac{e^{(2+h)} - e^2}{h} =$

- (A) 0
- (B) 1
- (C)  $2e$
- (D)  $e^2$
- (E)  $2e^2$

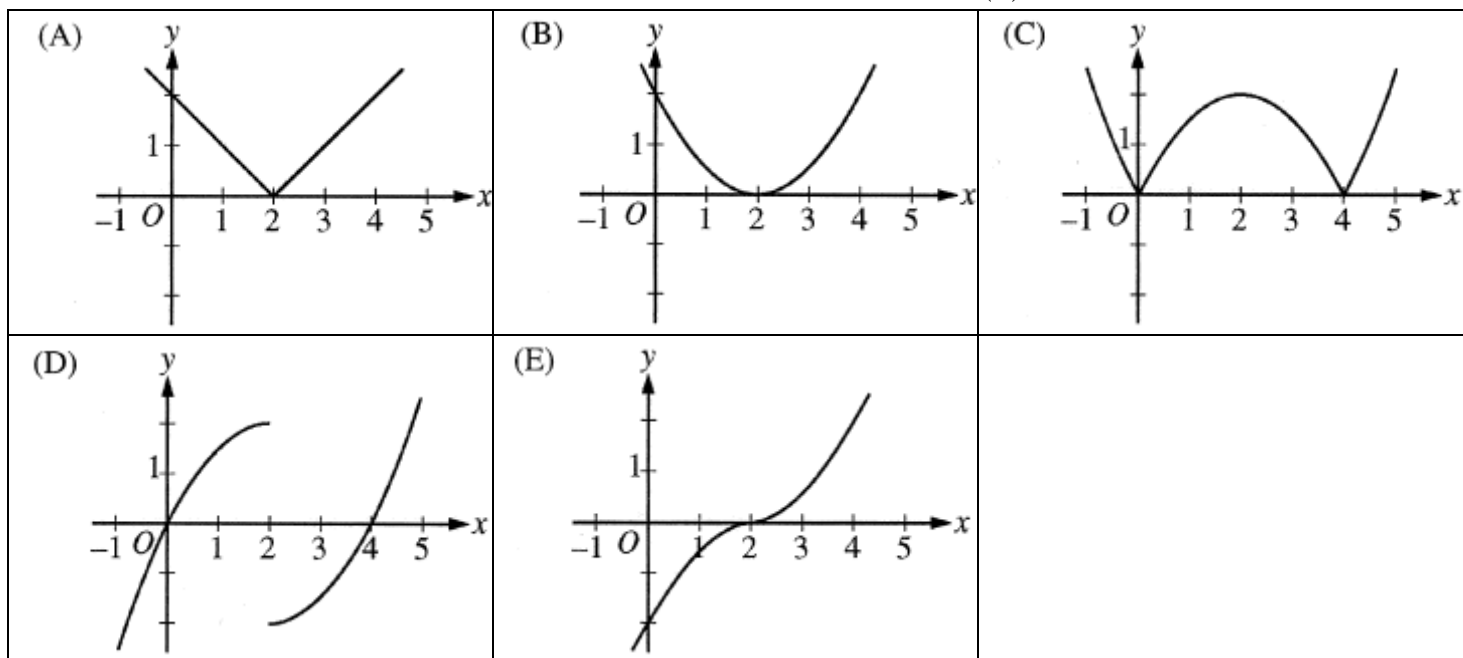


15. The slope field for a certain differential equation is shown above. Which of the following could be a solution to the differential equation with the initial condition  $y(0) = 1$  ?

- (A)  $y = \cos x$
- (B)  $y = 1 - x^2$
- (C)  $y = e^x$
- (D)  $y = \sqrt{1 - x^2}$
- (E)  $y = \frac{1}{1 + x^2}$

AP CALCULUS TAKE HOME EXAM

16. If  $f'(x) = |x - 2|$ , which of the following could be the graph of  $y = f(x)$ ?



17. What is the area of the region enclosed by the graphs of  $f(x) = x - 2x^2$  and  $g(x) = -5x$ ?

- (A)  $\frac{7}{3}$   
 (B)  $\frac{16}{3}$   
 (C)  $\frac{20}{3}$   
 (D) 9  
 (E) 36

18. For the function  $f$ ,  $f'(x) = 2x + 1$  and  $f(1) = 4$ . What is the approximation for  $f(1.2)$  found by using the line tangent to the graph of  $f$  at  $x = 1$ ?

- (A) 0.6  
 (B) 3.4  
 (C) 4.2  
 (D) 4.6  
 (E) 4.64

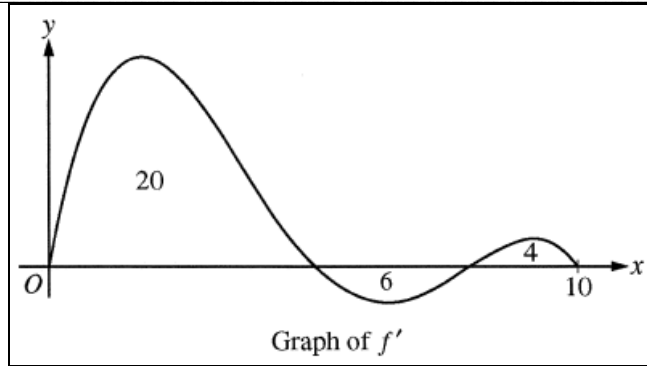
19. Let  $f$  be the function given by  $f(x) = x^3 - 6x^2$ . The graph of  $f$  is concave up when

- (A)  $x > 2$   
 (B)  $x < 2$   
 (C)  $0 < x < 4$   
 (D)  $x < 0$  or  $x > 4$  only  
 (E)  $x > 6$  only

AP CALCULUS TAKE HOME EXAM

20. If  $g(x) = x^2 - 3x + 4$  and  $f(x) = g'(x)$ , then  $\int_1^3 f(x) dx =$

- (A)  $-\frac{14}{3}$
- (B)  $-2$
- (C)  $2$
- (D)  $4$
- (E)  $\frac{14}{3}$



21. The graph of  $f'$ , the derivative of the function  $f$ , is shown above for  $0 \leq x \leq 10$ . The areas of the regions between the graph of  $f'$  and the  $x$ -axis are 20, 6, and 4 respectively. If  $f(0) = 2$ , what is the maximum value of  $f$  on the closed interval  $0 \leq x \leq 10$ ?

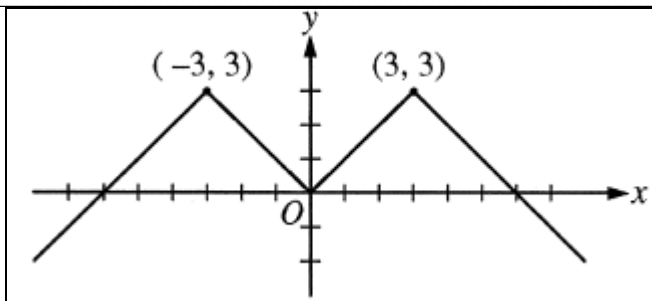
- (A) 16
- (B) 20
- (C) 22
- (D) 30
- (E) 32

22. If  $f'(x) = (x-2)(x-3)^2(x-4)^3$ , then  $f$  has which of the following relative extrema?

- I. A relative maximum at  $x = 2$
- II. A relative minimum at  $x = 3$
- III. A relative maximum at  $x = 4$

- (A) I only
- (B) III only
- (C) I and III only
- (D) II and III only
- (E) I, II, and III

AP CALCULUS TAKE HOME EXAM



23. The graph of the even function  $y = f(x)$  consists of 4 line segments, as shown above. Which of the following statements about  $f$  is false?

- (A)  $\lim_{x \rightarrow 0} (f(x) - f(0)) = 0$   
 (B)  $\lim_{x \rightarrow 0} \frac{f(x) - f(0)}{x} = 0$   
 (C)  $\lim_{x \rightarrow 0} \frac{f(x) - f(-x)}{2x} = 0$   
 (D)  $\lim_{x \rightarrow 2} \frac{f(x) - f(2)}{x - 2} = 1$   
 (E)  $\lim_{x \rightarrow 3} \frac{f(x) - f(3)}{x - 3}$  does not exist.

24. The radius of a circle is increasing. At a certain instant, the rate of increase in the area of the circle is numerically equal to twice the rate of increase in its circumference. What is the radius of the circle at that instant?

- (A)  $\frac{1}{2}$   
 (B) 1  
 (C)  $\sqrt{2}$   
 (D) 2  
 (E) 4

25. If  $x^2y - 3x = y^3 - 3$ , then at the point  $(-1, 2)$ ,  $\frac{dy}{dx} =$

- (A)  $-\frac{7}{11}$   
 (B)  $-\frac{7}{13}$   
 (C)  $-\frac{1}{2}$   
 (D)  $-\frac{3}{14}$   
 (E) 7

26. For  $x > 0$ ,  $f$  is a function such that  $f'(x) = \frac{\ln x}{x}$  and  $f''(x) = \frac{1 - \ln x}{x^2}$ . Which of the following is true?

- (A)  $f$  is decreasing for  $x > 1$ , and the graph of  $f$  is concave down for  $x > e$ .  
 (B)  $f$  is decreasing for  $x > 1$ , and the graph of  $f$  is concave up for  $x > e$ .  
 (C)  $f$  is increasing for  $x > 1$ , and the graph of  $f$  is concave down for  $x > e$ .  
 (D)  $f$  is increasing for  $x > 1$ , and the graph of  $f$  is concave up for  $x > e$ .  
 (E)  $f$  is increasing for  $0 < x < e$ , and the graph of  $f$  is concave down for  $0 < x < e^{3/2}$ .

AP CALCULUS TAKE HOME EXAM

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27. If  $f$  is a function given by  $f(x) = \int_4^{2x} \sqrt{t^2 - t} dt$ , then  $f'(2) =$

- (A) 0  
(B)  $\frac{7}{2\sqrt{12}}$   
(C)  $\sqrt{2}$   
(D)  $\sqrt{12}$   
(E)  $2\sqrt{12}$
- 

28. If  $y = \sin^{-1}(5x)$ , then  $\frac{dy}{dx} =$

- (A)  $\frac{1}{1+25x^2}$   
(B)  $\frac{5}{1+25x^2}$   
(C)  $\frac{-5}{\sqrt{1-25x^2}}$   
(D)  $\frac{1}{\sqrt{1-25x^2}}$   
(E)  $\frac{5}{\sqrt{1-25x^2}}$
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END OF PART A OF SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY  
CHECK YOUR WORK ON PART A ONLY.

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.