

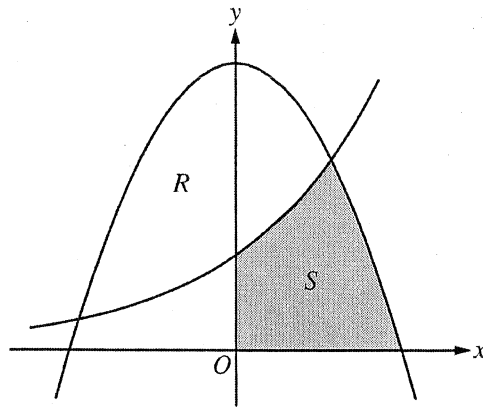
CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

1. The rate at which raw sewage enters a treatment tank is given by $E(t) = 850 + 715 \cos\left(\frac{\pi t^2}{9}\right)$ gallons per hour for $0 \leq t \leq 4$ hours. Treated sewage is removed from the tank at the constant rate of 645 gallons per hour. The treatment tank is empty at time $t = 0$.
- (a) How many gallons of sewage enter the treatment tank during the time interval $0 \leq t \leq 4$? Round your answer to the nearest gallon.
- (b) For $0 \leq t \leq 4$, at what time t is the amount of sewage in the treatment tank greatest? To the nearest gallon, what is the maximum amount of sewage in the tank? Justify your answers.
- (c) For $0 \leq t \leq 4$, the cost of treating the raw sewage that enters the tank at time t is $(0.15 - 0.02t)$ dollars per gallon. To the nearest dollar, what is the total cost of treating all the sewage that enters the tank during the time interval $0 \leq t \leq 4$?

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2. Let R and S in the figure above be defined as follows: R is the region in the first and second quadrants bounded by the graphs of $y = 3 - x^2$ and $y = 2^x$. S is the shaded region in the first quadrant bounded by the two graphs, the x -axis, and the y -axis.
- (a) Find the area of S .
- (b) Find the volume of the solid generated when R is rotated about the horizontal line $y = -1$.
- (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x -axis is an isosceles right triangle with one leg across the base of the solid. Write, but do not evaluate, an integral expression that gives the volume of the solid.

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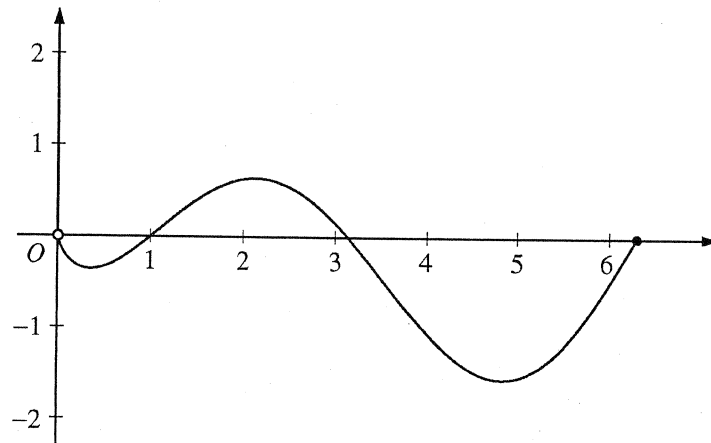
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t (minutes)	0	4	8	12	16
$H(t)$ ($^{\circ}\text{C}$)	65	68	73	80	90

3. The temperature, in degrees Celsius ($^{\circ}\text{C}$), of an oven being heated is modeled by an increasing differentiable function H of time t , where t is measured in minutes. The table above gives the temperature as recorded every 4 minutes over a 16-minute period.
- (a) Use the data in the table to estimate the instantaneous rate at which the temperature of the oven is changing at time $t = 10$. Show the computations that lead to your answer. Indicate units of measure.
- (b) Write an integral expression in terms of H for the average temperature of the oven between time $t = 0$ and time $t = 16$. Estimate the average temperature of the oven using a left Riemann sum with four subintervals of equal length. Show the computations that lead to your answer.
- (c) Is your approximation in part (b) an underestimate or an overestimate of the average temperature? Give a reason for your answer.
- (d) Are the data in the table consistent with or do they contradict the claim that the temperature of the oven is increasing at an increasing rate? Give a reason for your answer.

CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.



Graph of f

4. Let f be the function given by $f(x) = (\ln x)(\sin x)$. The figure above shows the graph of f for $0 < x \leq 2\pi$. The function g is defined by $g(x) = \int_1^x f(t) dt$ for $0 < x \leq 2\pi$.
- Find $g(1)$ and $g'(1)$.
 - On what intervals, if any, is g increasing? Justify your answer.
 - For $0 < x \leq 2\pi$, find the value of x at which g has an absolute minimum. Justify your answer.
 - For $0 < x < 2\pi$, is there a value of x at which the graph of g is tangent to the x -axis? Explain why or why not.

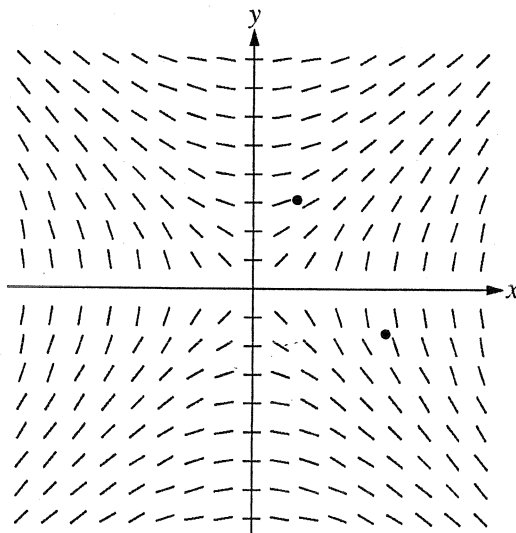
CALCULUS AB
SECTION II, Part B
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No calculator is allowed for these problems.

5. Consider the differential equation $\frac{dy}{dx} = \frac{x}{y}$, where $y \neq 0$.

- (a) The slope field for the given differential equation is shown below. Sketch the solution curve that passes through the point $(3, -1)$, and sketch the solution curve that passes through the point $(1, 2)$.

(Note: The points $(3, -1)$ and $(1, 2)$ are indicated in the figure.)



- (b) Write an equation for the line tangent to the solution curve that passes through the point $(1, 2)$.
- (c) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(3) = -1$, and state its domain.

CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.

6. Let $g(x) = xe^{-x} + be^{-x}$, where b is a positive constant.
- (a) Find $\lim_{x \rightarrow \infty} g(x)$.
- (b) For what positive value of b does g have an absolute maximum at $x = \frac{2}{3}$? Justify your answer.
- (c) Find all values of b , if any, for which the graph of g has a point of inflection on the interval $0 < x < \infty$. Justify your answer.