

EXERCISES

Use a half-angle identity to find the exact value of each function.

14. $\cos 15^\circ$

15. $\sin 75^\circ$

16. $\tan \frac{5\pi}{12}$

17. $\sin \frac{3\pi}{8}$

18. $\cos \frac{7\pi}{12}$

19. $\tan 22.5^\circ$

20. If θ is an angle in the first quadrant and $\cos \theta = \frac{1}{4}$, find $\tan \frac{\theta}{2}$.

Use the given information to find $\sin 2\theta$, $\cos 2\theta$, and $\tan 2\theta$.

21. $\cos \theta = \frac{4}{5}$, $0^\circ < \theta < 90^\circ$

22. $\sin \theta = \frac{1}{3}$, $0 < \theta < \frac{\pi}{2}$

23. $\tan \theta = -2$, $\frac{\pi}{2} < \theta < \pi$

24. $\sec \theta = -\frac{4}{3}$, $90^\circ < \theta < 180^\circ$

25. $\cot \theta = \frac{3}{2}$, $180^\circ < \theta < 270^\circ$

26. $\csc \theta = -\frac{5}{2}$, $\frac{3\pi}{2} < \theta < 2\pi$

27. If α is an angle in the second quadrant and $\cos \alpha = -\frac{\sqrt{2}}{3}$, find $\tan 2\alpha$.

Verify that each equation is an identity.

28. $\csc 2\theta = \frac{1}{2} \sec \theta \csc \theta$

29. $\cos A - \sin A = \frac{\cos 2A}{\cos A + \sin A}$

30. $(\sin \theta + \cos \theta)^2 - 1 = \sin 2\theta$

31. $\cos x - 1 = \frac{\cos 2x - 1}{2(\cos x + 1)}$

32. $\sec 2\theta = \frac{\cos^2 \theta + \sin^2 \theta}{\cos^2 \theta - \sin^2 \theta}$

33. $\tan \frac{A}{2} = \frac{\sin A}{1 + \cos A}$

34. $\sin 3x = 3 \sin x - 4 \sin^3 x$

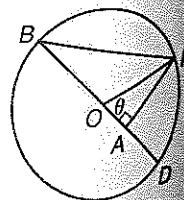
35. $\cos 3x = 4 \cos^3 x - 3 \cos x$

Applications
Problem
Solving



36. **Architecture** Refer to the application at the beginning of the lesson. If the angle of the water is doubled, what is the ratio of the new maximum height to the original maximum height?

37. **Critical Thinking** Circle O is a unit circle. Use the figure to prove that $\tan \frac{1}{2}\theta = \frac{\sin \theta}{1 + \cos \theta}$.



38. **Physics** Suppose a projectile is launched with velocity v at an angle θ to the horizontal from the base of a hill that makes an angle α with the horizontal ($\theta > \alpha$). Then the range of the projectile, measured along the slope of the hill, is given by $R = \frac{v^2 \cos \theta \sin(\theta - \alpha)}{g \cos^2 \alpha}$. Show that if $\alpha = 45^\circ$, then

$$R = \frac{v^2 \sqrt{2}}{g} (\sin 2\theta - \cos 2\theta - 1)$$

internet CONNECTION

Research
For the latitude and longitude of world cities, and the distance between them, visit: www.amc.glencoe.com



Mixed Review