

## LIMITS WORKSHEET #4

### CONTINUITY NOTES

To determine whether a function is continuous at  $x = c$ , the following 3 statements must ALL be true:

1. $f(c)$ must exist
2. $\lim_{x \rightarrow c} f(x)$ must exist
3. $\lim_{x \rightarrow c} f(x)$ must equal $f(c)$

#### Examples:

<p>Is <math>f(x) = x+1</math> continuous at <math>x = 2</math>?</p> <p>1. <math>f(2)</math> exists <math>[f(2) = 3]</math></p> <p>2. <math>\lim_{x \rightarrow 2} f(x)</math> exists <math>[\lim_{x \rightarrow 2} x+1 = 3]</math></p> <p>3. <math>f(2) = \lim_{x \rightarrow 2} f(x) = 3</math></p> <p><span style="border: 1px solid black; padding: 2px;">Yes</span>, <math>f(x) = x+1</math> is continuous at <math>x = 2</math>.</p>	<p>2.</p> <p>Is <math>f(x) = \frac{1}{x}</math> continuous at <math>x = 0</math>?</p> <p>1. <math>f(0)</math> does not exist</p> <p><math>\therefore</math> <span style="border: 1px solid black; padding: 2px;">No</span>, <math>f(x) = \frac{1}{x}</math> is NOT continuous at <math>x = 0</math>.</p>
<p>3.</p> <p>Is <math>f(x) = \begin{cases} x-1 &amp; x &lt; 0 \\ x+1 &amp; x \geq 0 \end{cases}</math> continuous at <math>x = 0</math>?</p> <p>1. <math>f(0)</math> exists <math>[f(0) = 1]</math></p> <p>2. <math>\lim_{x \rightarrow 0} f(x)</math> does not exist.</p> <p><math>\therefore</math> <span style="border: 1px solid black; padding: 2px;">No</span>, <math>f(x) = x+1</math> is <u>not continuous</u> at <math>x = 0</math>.</p>	<p>4.</p> <p>Is <math>f(x) = \begin{cases} x^2 &amp; x &lt; 2 \\ x+2 &amp; x \geq 2 \end{cases}</math> continuous at <math>x = 2</math>?</p> <p>1. <math>f(2)</math> exists <math>[f(2) = 4]</math></p> <p>2. <math>\lim_{x \rightarrow 2} f(x)</math> exists. <math>[\lim_{x \rightarrow 2} f(x) = 4]</math></p> <p>3. <math>\lim_{x \rightarrow 2} f(x) = f(2) = 4</math></p> <p><math>\therefore</math> <span style="border: 1px solid black; padding: 2px;">YES</span>, <math>f(x)</math> is <u>continuous</u> at <math>x = 2</math>.</p>

#### Types of Discontinuity:

1. Point (or removable) discontinuity
2. Jump discontinuity
3. Infinite discontinuity
4. Oscillating discontinuity