Guided Practice

6. State the amplitude for $y = -2.5 \cos \theta$. Then graph the function.
7. State the period for $y = \sin 4\theta$. Then graph the function.

State the amplitude and period for each function. Then graph each function.

8. $y = 10 \sin 2\theta$
9. $y = 3 \cos 2\theta$
10. $y = 0.5 \sin \frac{\theta}{6}$
11. $y = -\frac{1}{5} \cos \frac{\theta}{4}$

Write an equation of the sine function with each amplitude and period.

12. amplitude $= 0.8$, period $= \pi$
13. amplitude $= 7$, period $= \frac{\pi}{3}$

Write an equation of the cosine function with each amplitude and period.

14. amplitude $= 1.5$, period $= 5\pi$
15. amplitude $= \frac{3}{4}$, period $= 6$

16. **Music** Write a sine equation that represents the initial behavior of the vibrations of the note D above middle C having an amplitude of 0.25 and a frequency of 294 hertz.

EXERCISES

Practice

State the amplitude for each function. Then graph each function.

17. $y = 2 \sin \theta$
18. $y = -\frac{3}{4} \cos \theta$
19. $y = 1.5 \sin \theta$

State the period for each function. Then graph each function.

20. $y = \cos 2\theta$
21. $y = \cos \frac{\theta}{4}$
22. $y = \sin 6\theta$

State the amplitude and period for each function. Then graph each function.

23. $y = 5 \cos \theta$
24. $y = -2 \cos 0.5\theta$
25. $y = -\frac{2}{5} \sin 9\theta$
26. $y = 8 \sin 0.5\theta$
27. $y = -3 \sin \frac{\pi}{2}\theta$
28. $y = \frac{2}{3} \cos \frac{3\pi}{7}\theta$
29. $y = 3 \sin 2\theta$
30. $y = 3 \cos 0.5\theta$
31. $y = -\frac{1}{3} \cos 3\theta$
32. $y = \frac{1}{3} \sin \frac{\theta}{3}$
33. $y = -4 \sin \frac{\theta}{2}$
34. $y = -2.5 \cos \frac{\theta}{5}$

35. The equation of the vibrations of the note F above middle C is represented by $y = 0.5 \sin 698\pi t$. Determine the amplitude and period for the function.

Write an equation of the sine function with each amplitude and period.

36. amplitude = 0.4, period = 10π

37. amplitude = 35.7, period = $\frac{\pi}{4}$

38. amplitude = $\frac{1}{4}$, period = $\frac{\pi}{3}$

39. amplitude = 0.34, period = 0.75π

40. amplitude = 4.5, period = $\frac{5\pi}{4}$

41. amplitude = 16, period = 30

Write an equation of the cosine function with each amplitude and period.

42. amplitude = 5, period = 2π

43. amplitude = $\frac{5}{8}$, period = $\frac{\pi}{7}$

44. amplitude = 7.5, period = 6π

45. amplitude = 0.5, period = 0.3π

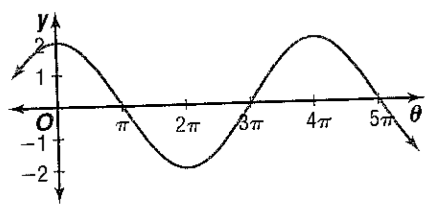
46. amplitude = $\frac{2}{5}$, period = $\frac{3}{5}\pi$

47. amplitude = 17.9, period = 16

48. Write the possible equations of the sine and cosine functions with amplitude 1.5 and period $\frac{\pi}{2}$.

Write an equation for each graph.

49.



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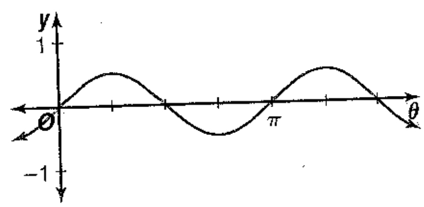

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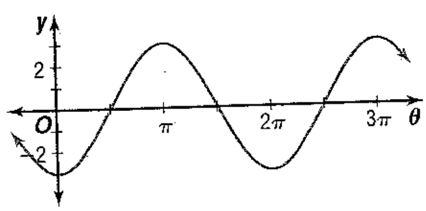

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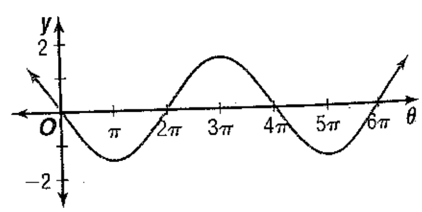


53. Write an equation for a sine function with amplitude 3.8 and frequency 120 hertz.

54. Write an equation for a cosine function with amplitude 15 and frequency 36 hertz.

Graphing Calculator

55. Graph these functions on the same screen of a graphing calculator. Compare the graphs.

a. $y = \sin x$ **b.** $y = \sin x + 1$ **c.** $y = \sin x + 2$

CHECK FOR UNDERSTANDING

Communicating Mathematics

Read and study the lesson to answer each question.

1. **Compare and contrast** the graphs $y = \sin x + 1$ and $y = \sin (x + 1)$.
2. **Name** the function whose graph is the same as the graph of $y = \cos x$ with a phase shift of $\frac{\pi}{2}$.
3. **Analyze** the function $y = A \sin (k\theta + c) + h$. Which variable could you increase or decrease to have each of the following effects on the graph?
   a. stretch the graph vertically
   b. translate the graph downward vertically
   c. shrink the graph horizontally
   d. translate the graph to the left.
4. **Explain** how to graph $y = \sin x + \cos x$.
5. **You Decide** Marsha and Jamal are graphing $y = \cos\left(\frac{\pi}{6}\theta - \frac{\pi}{2}\right)$. Marsha says that the phase shift of the graph is $\frac{\pi}{2}$. Jamal says that the phase shift is 3. Who is correct? Explain.

Guided Practice

6. State the phase shift for $y = 3 \cos\left(\theta - \frac{\pi}{2}\right)$. Then graph the function.
7. State the vertical shift and the equation of the midline for $y = \sin 2\theta + 3$. Then graph the function.

State the amplitude, period, phase shift, and vertical shift for each function. Then graph the function.

8. $y = 2 \sin (2\theta + \pi) - 5$
9. $y = 3 - \frac{1}{2} \cos\left(\frac{\theta}{2} - \frac{\pi}{4}\right)$
10. Write an equation of a sine function with amplitude 20, period 1, phase shift 0, and vertical shift 100.
11. Write an equation of a cosine function with amplitude 0.6, period 12.4, phase shift −2.13, and vertical shift 7.
12. Graph $y = \sin x - \cos x$.
13. **Health** If a person has a blood pressure of 130 over 70, then the person's blood pressure oscillates between the maximum of 130 and a minimum of 70.
    a. Write the equation for the midline about which this person's blood pressure oscillates.
    b. If the person's pulse rate is 60 beats a minute, write a sine equation that models his or her blood pressure using t as time in seconds.
    c. Graph the equation.

EXERCISES

Practice

State the phase shift for each function. Then graph each function.

14. $y = \sin (\theta - 2\pi)$
15. $y = \sin (2\theta + \pi)$
16. $y = 2 \cos\left(\frac{\theta}{4} + \frac{\pi}{2}\right)$

State the vertical shift and the equation of the midline for each function. Then graph each function.

17. $y = \sin \frac{\theta}{2} + \frac{1}{2}$

18. $y = 5 \cos \theta - 4$

19. $y = 7 + \cos 2\theta$

20. State the horizontal and vertical shift for $y = -8 \sin (2\theta - 4\pi) - 3$.

State the amplitude, period, phase shift, and vertical shift for each function. Then graph the function.

21. $y = 3 \cos \left(\theta - \frac{\pi}{2}\right)$

22. $y = 6 \sin \left(\theta + \frac{\pi}{3}\right) + 2$

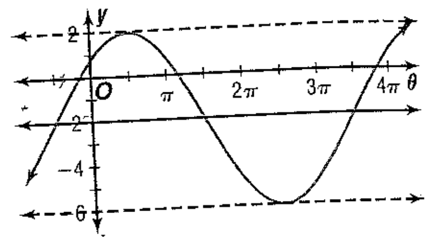
23. $y = -2 + \sin \left(\frac{\theta}{3} - \frac{\pi}{12}\right)$

24. $y = 20 + 5 \cos (3\theta + \pi)$

25. $y = \frac{1}{4} \cos \frac{\theta}{2} - 3$

26. $y = 10 \sin \left(\frac{\theta}{4} - 4\pi\right) -5$

27. State the amplitude, period, phase shift, and vertical shift of the sine curve shown at the right.



Write an equation of the sine function with each amplitude, period, phase shift, and vertical shift.

28. amplitude = 7, period = 3π, phase shift = π, vertical shift = -7

29. amplitude = 50, period = $\frac{3\pi}{4}$, phase shift = $\frac{\pi}{2}$, vertical shift = -25

30. amplitude = $\frac{3}{4}$, period = $\frac{\pi}{5}$, phase shift = π, vertical shift = $\frac{1}{4}$

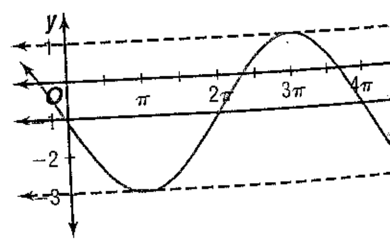
Write an equation of the cosine function with each amplitude, period, phase shift, and vertical shift.

31. amplitude = 3.5, period = $\frac{\pi}{2}$, phase shift = $\frac{\pi}{4}$, vertical shift = 7

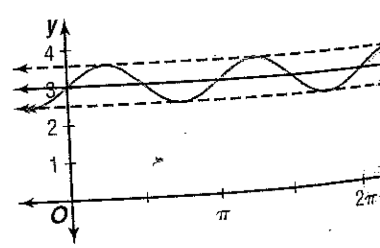
32. amplitude = $\frac{4}{5}$, period = $\frac{\pi}{6}$, phase shift = $\frac{\pi}{3}$, vertical shift = $\frac{7}{5}$

33. amplitude = 100, period = 45, phase shift = 0, vertical shift = -110

34. Write a cosine equation for the graph at the right.



35. Write a sine equation for the graph at the right.



Guided Practice

4. **Boating** If the equilibrium point is $y = 0$, then $y = -5 \cos\left(\frac{\pi}{6}t\right)$ models a buoy bobbing up and down in the water.
   a. Describe the location of the buoy when $t = 0$.
   b. What is the maximum height of the buoy?
   c. Find the location of the buoy at $t = 7$.

5. **Health** A certain person's blood pressure oscillates between 140 and 80. If the heart beats once every second, write a sine function that models the person's blood pressure.

6. **Meteorology** The average monthly temperatures for the city of Seattle, Washington, are given below.

Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
41°	44°	47°	50°	56°	61°	65°	66°	61°	54°	46°	42°

   a. Find the amplitude of a sinusoidal function that models the monthly temperatures.
   b. Find the vertical shift of a sinusoidal function that models the monthly temperatures.
   c. What is the period of a sinusoidal function that models the monthly temperatures?
   d. Write a sinusoidal function that models the monthly temperatures, using $t = 1$ to represent January.
   e. According to your model, what is the average monthly temperature in February? How does this compare to the actual average?
   f. According to your model, what is the average monthly temperature in October? How does this compare to the actual average?

EXERCISES

Applications and Problem Solving

Real World Application

7. **Music** The initial behavior of the vibrations of the note E above middle C can be modeled by $y = 0.5 \sin 660\pi t$.
   a. What is the amplitude of this model?
   b. What is the period of this model?
   c. Find the frequency (cycles per second) for this note.

8. **Entertainment** A rodeo performer spins a lasso in a circle perpendicular to the ground. The height of the knot from the ground is modeled by $h = -3 \cos\left(\frac{5\pi}{3}t\right) + 3.5$, where t is the time measured in seconds.
   a. What is the highest point reached by the knot?
   b. What is the lowest point reached by the knot?
   c. What is the period of the model?
   d. According to the model, find the height of the knot after 25 seconds.

9. Biology In a certain region with hawks as predators and rodents as prey, the rodent population R varies according to the model $R = 1200 + 300 \sin\left(\frac{\pi}{2}t\right)$, and the hawk population H varies according to the model $H = 250 + 25 \sin\left(\frac{\pi}{2}t - \frac{\pi}{4}\right)$, with t measured in years since January 1, 1970.

a. What was the population of rodents on January 1, 1970?

b. What was the population of hawks on January 1, 1970?

c. What are the maximum populations of rodents and hawks? Do these max ever occur at the same time?

d. On what date was the first maximum population of rodents achieved?

e. What is the minimum population of hawks? On what date was the minim population of hawks first achieved?

f. According to the models, what was the population of rodents and hawks January 1 of the present year?

10. Waves A leaf floats on the water bobbing up and down. The distance bet its highest and lowest point is 4 centimeters. It moves from its highest poi down to its lowest point and back to its highest point every 10 seconds. W cosine function that models the movement of the leaf in relationship to the equilibrium point.

11. Tides Write a sine function which models the oscillation of tides in Savan Georgia, if the equilibrium point is 4.24 feet, the amplitude is 3.55 feet, the shift is −4.68 hours, and the period is 12.40 hours.

12. Meteorology The mean average temperature in Buffalo, New York, is 47. The temperature fluctuates 23.5° above and below the mean temperature. $t = 1$ represents January, the phase shift of the sine function is 4.

a. Write a model for the average monthly temperature in Buffalo.

b. According to your model, what is the average temperature in March?

c. According to your model, what is the average temperature in August?

13. **Meteorology** The average monthly temperatures for the city of Honolulu, Hawaii, are given below.

Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
73°	73°	74°	76°	78°	79°	81°	81°	81°	80°	77°	74°

a. Find the amplitude of a sinusoidal function that models the monthly temperatures.

b. Find the vertical shift of a sinusoidal function that models the monthly temperatures.

c. What is the period of a sinusoidal function that models the monthly temperatures?

d. Write a sinusoidal function that models the monthly temperatures, using $t = 1$ to represent January.

e. According to your model, what is the average temperature in August? How does this compare to the actual average?

f. According to your model, what is the average temperature in May? How does this compare to the actual average?

14. **Critical Thinking** Write a cosine function that is equivalent to $y = 3 \sin (x - \pi) + 5$.

15. **Tides** Burntcoat Head in Nova Scotia, Canada, is known for its extreme fluctuations in tides. One day in April, the first high tide rose to 13.25 feet at 4:30 A.M. The first low tide at 1.88 feet occurred at 10:51 A.M. The second high tide was recorded at 4:53 P.M.

a. Find the amplitude of a sinusoidal function that models the tides.

b. Find the vertical shift of a sinusoidal function that models the tides.

c. What is the period of a sinusoidal function that models the tides?

d. Write a sinusoidal function to model the tides, using t to represent the number of hours in decimals since midnight.

e. According to your model, determine the height of the water at 7:30 P.M.

16. **Meteorology** The table at the right contains the times that the sun rises and sets in the middle of each month in New York City, New York. Suppose the number 1 represents the middle of January, the number 2 represents the middle of February, and so on.

Month	Sunrise A.M.	Sunset P.M.
January	7:19	4:47
February	6:56	5:24
March	6:16	5:57
April	5:25	6:29
May	4:44	7:01
June	4:24	7:26
July	4:33	7:28
August	5:01	7:01
September	5:31	6:14
October	6:01	5:24
November	6:36	4:43
December	7:08	4:28

a. Find the amount of daylight hours for the middle of each month.

b. What is the amplitude of a sinusoidal function that models the daylight hours?

c. What is the vertical shift of a sinusoidal function that models the daylight hours?

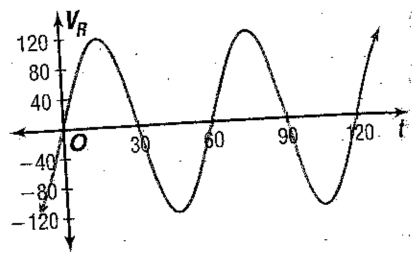
d. What is the period of a sinusoidal function that models the daylight hours?

e. Write a sinusoidal function that models the daylight hours.

17. **Critical Thinking** The average monthly temperature for Phoenix, Arizona can be modeled by $y = 70.5 + 19.5 \sin\left(\frac{\pi}{6}t + c\right)$. If the coldest temperature occurs in January ($t = 1$), find the value of c.

18. **Entertainment** Several years ago, an amusement park in Sandusky, Ohio, had a ride called the Rotor in which riders stood against the walls of a spinning cylinder. As the cylinder spun, the floor of the ride dropped out, and the riders were held against the wall by the force of friction. The cylinder of the Rotor had a radius of 3.5 meters and rotated counterclockwise at a rate of 14 revolutions per minute. Suppose the center of rotation of the Rotor was at the origin of a rectangular coordinate system.
   a. If the initial coordinates of the hinges on the door of the cylinder are $(0, -3.5)$, write a function that models the position of the door at t seconds.
   b. Find the coordinates of the hinges on the door at 4 seconds.

19. **Electricity** For an alternating current, the instantaneous voltage V_R is graphed at the right. Write an equation for the instantaneous voltage.



20. **Meteorology** Find the number of daylight hours for the middle of each month or the average monthly temperature for your community. Write a sinusoidal function to model this data.

Mixed Review

21. State the amplitude, period, phase shift, and vertical shift for $y = -3 \cos(2\theta + \pi) + 5$. Then graph the function. *(Lesson 6-5)*

22. Find the values of θ for which $\cos \theta = 1$ is true. *(Lesson 6-3)*

23. Change 800° to radians. *(Lesson 6-1)*

24. **Geometry** The sides of a parallelogram are 20 centimeters and 32 centimeters long. If the longer diagonal measures 40 centimeters, find the measures of the angles of the parallelogram. *(Lesson 5-8)*

25. Decompose $\frac{2m + 16}{m^2 - 16}$ into partial fractions. *(Lesson 4-6)*

26. Find the value of k so that the remainder of $(2x^3 + kx^2 - x - 6) \div (x + 2)$ is zero. *(Lesson 4-3)*

27. Determine the interval(s) for which the graph of $f(x) = 2|x + 1| - 5$ is increasing and the intervals for which the graph is decreasing. *(Lesson 3-5)*

28. **SAT/ACT Practice** If one half of the female students in a certain school eat in the cafeteria and one third of the male students eat there, what fractional part of the student body eats in the cafeteria?

   A $\frac{5}{12}$ B $\frac{2}{5}$ C $\frac{3}{4}$ D $\frac{5}{6}$

   E not enough information given

Extra Practice See p. A

CHECK FOR UNDERSTANDING

Communicating Mathematics

Read and study the lesson to answer each question.

1. **Name** three values of θ that would result in $y = \cot \theta$ being undefined.
2. **Compare** the asymptotes and periods of $y = \tan \theta$ and $y = \sec \theta$.
3. **Describe** two different phase shifts of the secant function that would make it appear to be the cosecant function.

Guided Practice

Find each value by referring to the graphs of the trigonometric functions.

4. $\tan 4\pi$
5. $\csc\left(-\frac{7\pi}{2}\right)$

Find the values of θ for which each equation is true.

6. $\sec \theta = -1$
7. $\cot \theta = 1$

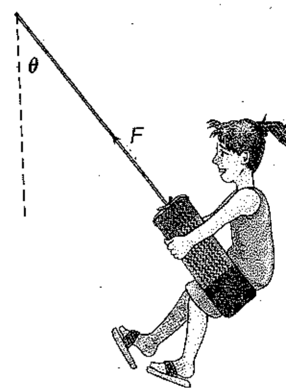
Graph each function.

8. $y = \tan\left(\theta + \frac{\pi}{4}\right)$
9. $y = \sec(2\theta + \pi) - 1$

Write an equation for the given function given the period, phase shift, and vertical shift.

10. cosecant function, period $= 3\pi$, phase shift $= \frac{\pi}{3}$, vertical shift $= -4$
11. cotangent function, period $= 2\pi$, phase shift $= -\frac{\pi}{4}$, vertical shift $= 0$

12. **Physics** A child is swinging on a tire swing. The tension on the rope is equal to the downward force on the end of the rope times $\sec \theta$, where θ is the angle formed by a vertical line and the rope.



   a. The downward force in newtons equals the mass of the child and the swing in kilograms times the acceleration due to gravity (9.8 meters per second squared). If the mass of the child and the tire is 73 kilograms, find the downward force.
   b. Write an equation that represents the tension on the rope as the child swings back and forth.
   c. Graph the equation for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$.
   d. What is the least amount of tension on the rope?
   e. What happens to the tension on the rope as the child swings higher and higher?

EXERCISES

Practice

Find each value by referring to the graphs of the trigonometric functions.

13. $\cot\left(\frac{5\pi}{2}\right)$
14. $\tan(-8\pi)$
15. $\sec\left(\frac{9\pi}{2}\right)$
16. $\csc\left(-\frac{5\pi}{2}\right)$
17. $\sec 7\pi$
18. $\cot(-5\pi)$

19. What is the value of csc (-6π)?

20. Find the value of tan (10π).

Find the values of θ for which each equation is true.

21. $\tan \theta = 0$

22. $\sec \theta = 1$

23. $\csc \theta = -1$

24. $\tan \theta = 1$

25. $\tan \theta = -1$

26. $\cot \theta = -1$

27. What are the values of θ for which sec θ is undefined?

28. Find the values of θ for which cot θ is undefined.

Graph each function.

29. $y = \cot\left(\theta - \frac{\pi}{2}\right)$

30. $y = \sec \frac{\theta}{3}$

31. $y = \csc \theta + 5$

32. $y = \tan\left(\frac{\theta}{2} - \frac{\pi}{4}\right) + 1$

33. $y = \csc(2\theta + \pi) - 3$

34. $y = \sec\left(\frac{\theta}{3} + \frac{\pi}{6}\right) - 2$

35. Graph $y = \cos \theta$ and $y = \sec \theta$. In the interval of -2π and 2π, what are the values of θ where the two graphs are tangent to each other?

Write an equation for the given function given the period, phase shift, and vertical shift.

36. tangent function, period $= 2\pi$, phase shift $= 0$, vertical shift $= -6$

37. cotangent function, period $= \frac{\pi}{2}$, phase shift $= \frac{\pi}{8}$, vertical shift $= 7$

38. secant function, period $= \pi$, phase shift $= -\frac{\pi}{4}$, vertical shift $= -10$

39. cosecant function, period $= 3\pi$, phase shift $= \pi$, vertical shift $= -1$

40. cotangent function, period $= 5\pi$, phase shift $= -\pi$, vertical shift $= 12$

41. cosecant function, period $= \frac{\pi}{3}$, phase shift $= -\frac{\pi}{2}$, vertical shift $= -5$

42. Write a secant function with a period of 3π, a phase shift of π units to the left, and a vertical shift of 8 units downward.

43. Write a tangent function with a period of $\frac{\pi}{2}$, a phase shift of $\frac{\pi}{4}$ to the right, and a vertical shift of 7 units upward.

Applications and Problem Solving

44. **Security** A security camera is scanning a long straight fence along one side of a military base. The camera is located 10 feet from the center of the fence. If d represents the distance along the fence from the center and t is time in seconds, then $d = 10 \tan \frac{\pi}{40} t$ models the point being scanned.

a. Graph the equation for $-20 \le t \le 20$.

b. Find the location the camera is scanning at 3 seconds.

c. Find the location the camera is scanning at 15 seconds.

45. **Critical Thinking** Graph $y = \csc \theta$, $y = 3 \csc \theta$, and $y = -3 \csc \theta$. Compare and contrast the graphs.