Finite Quantum Dynamics

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Abstract

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1. Introduction

Quantum dynamics is quantization of classical dynamics. In quantum theory physicists despite several schemes of quantization. First by the restriction of smallest possible action in nature (quantum hypothesis, Planck, 1900) and then by the transformation of quantities to operators (Schrödinger, 1926). Transformation of classical (poisson) brackets to quantum commutators is a further step of quantization (Heisenberg, 1927). In present paper Schrödinger quantization rule is considered as a basic postulate for the quantization of Newtonian, Hamilton-Jacobi and Euler-Lagrange dynamics and for other quantum problems. Schrödinger proposed to quantize quantities to their operators by an interpretation of Schrödinger (psi) wave. Dirac later advanced this analysis by description of *bra-ket* notations. Heisenberg transformed poisson brackets to quantum commutators and putted quantum dynamics forward with *Canonical quantization*. Scheme of operator representation of dynamical properties instead of functional transformations is regarded Schrödinger quantization rule which would be used for the construction of future dynamics following above postulates.

2. Postulates of Quantum dynamics

Quantization of classical dynamics and formulation of quantum dynamics despite postulates regarding the regularized use of quantization schemes. Quantum relativity under Schrödinger description (of operators) may be postulated. Formulation of Quantum dynamics is given in following steps.

2.1. Step quantization. Classical dynamical aspects are quantized in several schemes of quantization. Firstly with Planck quantization rule, then with Schrödinger quantization rule and then with Heisenberg canonical quantization. However, we regularize these schemes of quantization by demonstrating that an aspect, to be more exact (modified), is quantized under every quantization schemes (if possible) stepwise. The deduction would be used to quantize quantum problems further.

2.2. Operators and Transformations. Dynamical properties of dynamical systems in theoretical physics have been described by transformations of several dynamical variables such as canonical transformation with p and q, and Ostrogradsky transformation with $\overset{(n)}{q}$ $(n = 0,1,2,3,....)$ etc. However, now with the need of modern (operator) representations and quantum convenience, we analysize the description of dynamical variables by non-transformed operators satisfying Schrödinger eigenvalue (eigenoperator) equation. Representation of quantum dynamical variables by non-transformed operators despite different axioms and analytical rules to be treated. For example, a system of two quantum operators is given in Heisenberg non-commutative representation by

$$\widehat{\mathcal{A}}\widehat{\mathcal{B}} = \frac{1}{2}[\widehat{\mathcal{A}}, \widehat{\mathcal{B}}]_+ + \frac{1}{2}[\widehat{\mathcal{A}}, \widehat{\mathcal{B}}]_-$$
$$\widehat{\mathcal{B}}\widehat{\mathcal{A}} = \frac{1}{2}[\widehat{\mathcal{A}}, \widehat{\mathcal{B}}]_+ - \frac{1}{2}[\widehat{\mathcal{A}}, \widehat{\mathcal{B}}]_- \qquad (2.2.1)$$

Operational methods for operators are not exactly governed by mathematical analysis as it governs transformations. However, we will use different analogies for the treatment of operators. For example, the operation of operator $\widehat{f}$ to operator system $\widehat{\mathcal{A}}\widehat{\mathcal{B}}$ is given by a new operator $\widehat{f}\,\widehat{\mathcal{A}}\widehat{\mathcal{B}}$, not $(\widehat{f}\,\widehat{\mathcal{A}})\widehat{\mathcal{B}} + \widehat{\mathcal{A}}(\widehat{f}\,\widehat{\mathcal{B}})$ (Liebnitz rule) as a system of transformation $\mathcal{A}\mathcal{B}$ should be obtained.

2.3. Quantum dynamical systems. Quantum dynamical systems are described under Schrödinger representation by quantum dynamical operators

$$\widehat{\mathcal{A}} := \widehat{p}_a, \widehat{q}_a, \widehat{\mathcal{F}}, \widehat{H}, \widehat{L}, \widehat{V}, \widehat{T}, \widehat{S}, \widehat{w}, \widehat{t} \qquad (2.3.1)$$

satisfying Schrödinger eigenvalue equation $\widehat{\mathcal{A}}\,|\,\psi\rangle = a\,|\,\psi\rangle$ and Schrödinger representation

$$a = \int \psi^* \widehat{\mathcal{A}} \psi \; dt = \psi^{-1} \widehat{\mathcal{A}}\; \psi \qquad (2.3.2)$$

Classical dynamical systems are described by eigenvalues

$$a := p_a, q_a, \mathcal{F}, H, L, V, T, S, w, t \qquad (2.3.3)$$

which are observed as the image of quantum operators under Schrödinger representation. It connects together classical and quantum (Schrödinger) dynamics.

2.4. States of Quantum dynamical systems. In classical dynamics, state of dynamical systems is given by the particular values of transformation variables while in quantum dynamics with non-transformed operators, state of quantum dynamical systems can not be defined exactly. To define the state of quantum dynamical systems, we first apply the operators to image eigenvalues under Schrödinger representation and we then form the particular state of classical dynamical variables in eigenvalues which describe the state of dynamical systems. Therefore, the state of quantum dynamical systems under Schrödinger representation is not directly defined. We would use to describe the state of dynamical systems by Schrödinger wave (Dirac's ket) $|\,\psi\rangle$ defined in Hilbert Space $\mathbb{H}$.

2.5. Schrödinger quantization. All quantized and non-quantized aspects are further quantized under Schrödinger quantization scheme. It transforms quantities (functional transformation) to quantum operators by the help of quantum description of systems. The representation of all aspects with operators seems a useful quantization, since it confirms with basic quantum techniques and representations.

2.6. Quantum Relativity. Schrödinger Quantum dynamics is relativized with the postulation of Quantum relativity. We propose *Quantum frames* of references describing quantum dynamical properties (operators under Schrödinger scheme) by means of relative observation. Special relativization of Schrödinger dynamics despite four-wave interpretation and its (differential) treatment which would be deduced in subsequent section.

3. Quantum Relativity

"The formulation of Relativity could only be carried out by the spirit of that person who sees things from every view, from the eyes of everyone"

Now we come to Relativize Schrödinger quantum dynamics. Quantum relativization would be carried out in all ways of relativization: Simple relativization, Special relativization and General relativization. In Simple relativization we propose the observeability of quantum dynamical properties by means of relative observation. We propose *Quantum frames of references* $\mathcal{Q}$ and $\mathcal{Q}'$ which observe relatively quantum dynamical systems confirming with quantum axioms and representations. In Special relativization we deduce four-Schrödinger wave defined in four-dimensional relativized Hilbert Space under Lorentz transformation and Minkovskian geometry. It would further be generalized by the generalization of Einsteinian physics. In General relativization of Schrödinger dynamics we describe fields and Gravitation in Quantum manner of observation. We therefore deduce *Schrödinger Quantum Gravity* or *Schrödinger General Relativity* in this concern. We would formulate these Relativizations in subsequent sections.

3.1. Simple Quantum Relativity. Simple relativity is the means of relative observation. In Quantum (Schrödinger) concern we propose Quantum frames of references $\mathcal{Q}$, $\mathcal{Q}'$ to observe quantum dynamical systems by means of relative observation. Simple Quantum Relativity is pure theoretical deduction. However, in Quantum Relativity we do not propose mechanical model of Quantum frames. These are our relative views to achieve further understandings, to modify and extend quantum physics with its relativized analysis.

In Schrödinger representation, we relativize quantum physics demonstrating fixed Hilbert Spaces $\mathbb{H}$ and $\mathbb{H}'$ in $\mathcal{Q}$ and $\mathcal{Q}'$ quantum frames in its geometrization. The state of dynamical systems is described by relatively observed state vectors $|\psi\rangle$ and $|\psi'\rangle$ defined in Hilbert Spaces $\mathbb{H}$ and $\mathbb{H}'$ respectively. Quantum dynamical properties (operators and eigenvalues in Schrödinger representation) are obtained in relativized concern as follows.

The dynamical eigenvalues a are observed in $\mathcal{Q}$ frame and are imaged of Quantum dynamical properties $\widehat{\mathcal{A}}$ observed in $\mathcal{Q}$ frame of the dynamical state $|\psi\rangle$ defined in $\mathbb{H}$ (Hilbert Space) fixed to $\mathcal{Q}$-frame.

$$a = \int \psi^* \widehat{\mathcal{A}}\psi \, dt = \psi^{-1}\widehat{\mathcal{A}}\,\psi \tag{3.1.1}$$

$|\psi\rangle \in \mathbb{H}$ and $a, \widehat{\mathcal{A}}$ are observed in $\mathcal{Q}$-frame. We also assume configuration space t fixed to $\mathcal{Q}$-frame or relatively observed in this concern. The above deduction in $\mathcal{Q}'$-frame is followed.

$$a' = \int \psi'^* \widehat{\mathcal{A}'}\psi' \, dt' = \psi'^{-1}\widehat{\mathcal{A}'}\psi' \tag{3.1.2}$$

$|\psi'\rangle \in \mathbb{H}'$ and $a', \widehat{\mathcal{A}'}$ are observed in $\mathcal{Q}'$-frame. The configuration space t' is fixed to $\mathcal{Q}'$-frame. We strongly suggest the mathematical laws as same for all quantum frames. Not a deduction demonstrating different analytical rules for different quantum frames $\mathcal{Q}$ can be accepted in pure and usual analysis.

3.2. Special Quantum Relativity. Special Relativization of Schrödinger dynamics is exactly analysized by the deduction of four-Schrödinger wave defined in relativistic (relativized) four-dimensional Hilbert Space $\mathbb{H}(4)$ (described) under Lorentz transformation and Poincaré group in Minkovskian geometry for the dynamical (classical) properties in non-generalized form. Schrödinger four-wave under *proper time formalism* is given by

$$\psi_{\mathrm{Re}} := \psi(x^m, \tau) := \exp\left(\frac{i}{\hbar} S_{\mathrm{Re}}(x^m, \tau)\right) \tag{3.2.1}$$

The relativistic action defined in $\mathbb{M}(4)$ is given by

$$S_{\mathrm{Re}}(x^m, \tau) := -mc^2 \int_{\tau}^{\tau'} d\tau \tag{3.2.2}$$

with line element

$$ds^2 = c^2 d\tau^2 = g_{mn} x^m x^n \qquad (m,n = 0,1,2,3) \tag{3.2.3}$$

g_{mn} is fundamental metric-tensor in $\mathbb{M}(4)$

$$g_{mn} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{3.2.4}$$

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