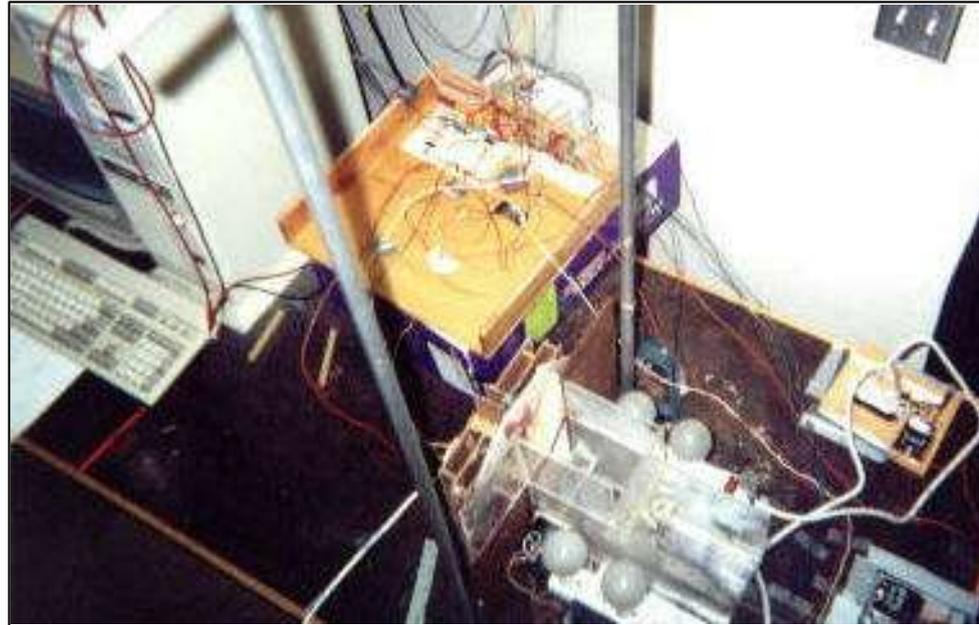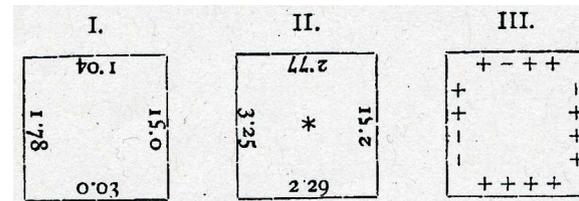# The Physics of Dice

Antonio Recuenco-Munoz

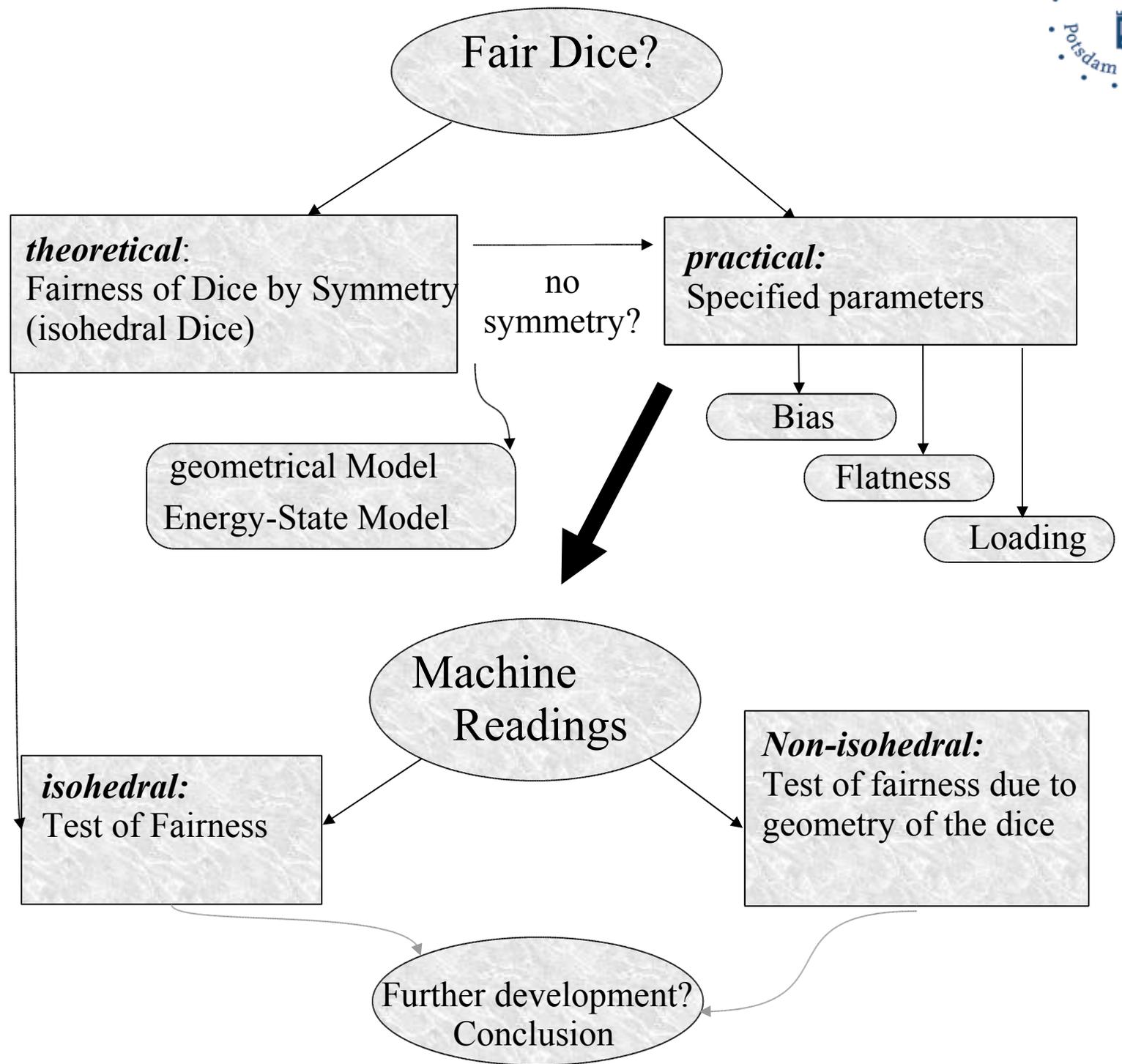*Experimental Physics-Seminar*
2. May 2006

Dice in Physics?

from GALTON (1890): Dice for Statistical Experiments.
Nature, No.1070, Vol.42, 13

- Arrangement of *random* data frequently used in science
  Statistical series using Dice first tried by Francis Galton, 1890

- Mechanical Generation of random data is easy, data close to real chance;
  albeit time- wrt. energy consumption. Today's preferred Methods:
  Data: Electrical Resistance, Nuclear Disintegration
  Hardware-Random Sample Generator :
  Quantum mechanical / thermal
  PRNGs (Algorithmus+Seed-Value;
  Chaitin-Kolmogorov-Chance)

- Mathematics for the geometrical models of any shape of dice not adequately
  developed; Currently, theory relies on only a few conclusions from
  Pegg *(1997): Fairness of ideally shaped isohedral Dice*

- Research of Dice for use in business (Manufacturing, Casino/Game Centres)
  or didactic (Simulation of Random Systems) applications
  Murray *(2001): Experimental Work with Dice Rolling Machine*

PHYSICS OF DICE

Fair Dice?

theoretical:
Fairness of Dice by Symmetry
(isohedral Dice)

no symmetry?

practical:
Specified parameters

geometrical Model

Energy-State Model

Bias

Flatness

Loading

Machine Readings

isohedral:
Test of Fairness

Non-isohedral:
Test of fairness due to geometry of the dice

Further development?
Conclusion

I. DICE

Fair Dice?

I. DICE

Symmetries

Fair Dice?

theoretical:
Fairness of Dice by Symmetry
(isohedral Dice)
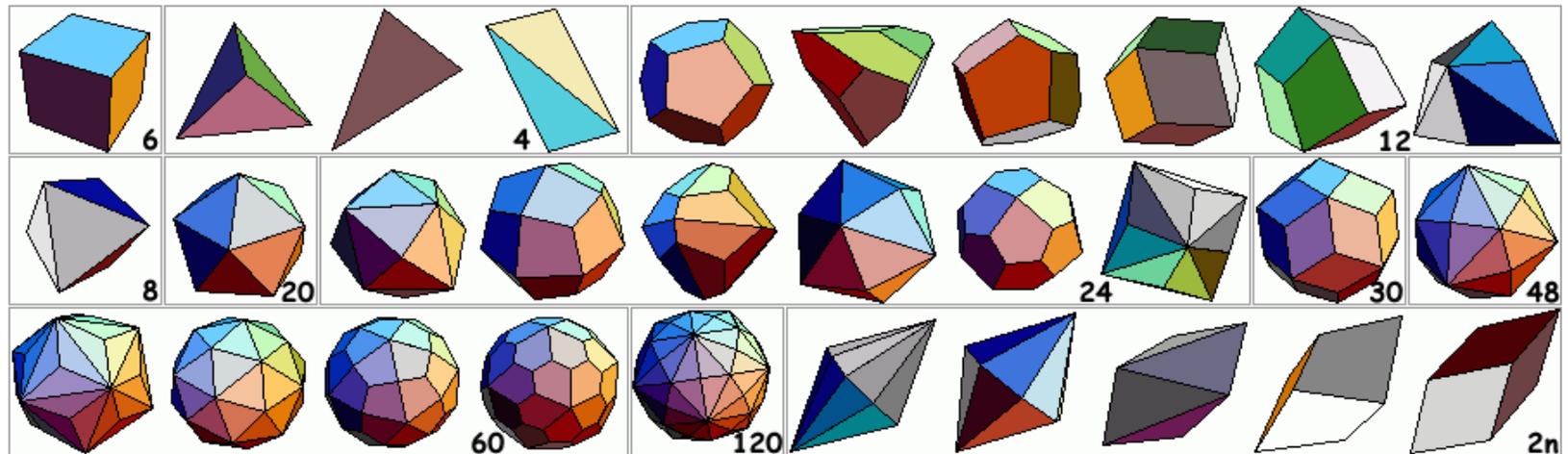
ISOHEDRAL DICE

- Each side of the Dice can be mapped to one other side through rotation and/or reflection

Aus PEGG (1997): A Complete List of Fair Dice.
University of Colorado (Master's Thesis)

- Isohedral Dice are *fair*: the likelihood that the Dice is laying on side n is equal for all sides

- Completeness of this argument is possible using Group Theory (Output: Fairness of platonic Polyhedron)

NON-ISOHEDRAL DICE

- Due to lack of Symmetry, there is no fairness of the Dice

- Results through external conditions affecting throws are easily biased; difficult reproducibility

  ↘ How can one take quantitative analysis into consideration?

  ↘ Can one prove the fairness of non-isohedral die?

PHYSICS OF DICE

I. DICE

Symmetries

  isohedral

   nonisohedral

Theory: Models

Fair Dice?

theoretical:
Fairness of Dice by Symmetry
(isohedral Dice)
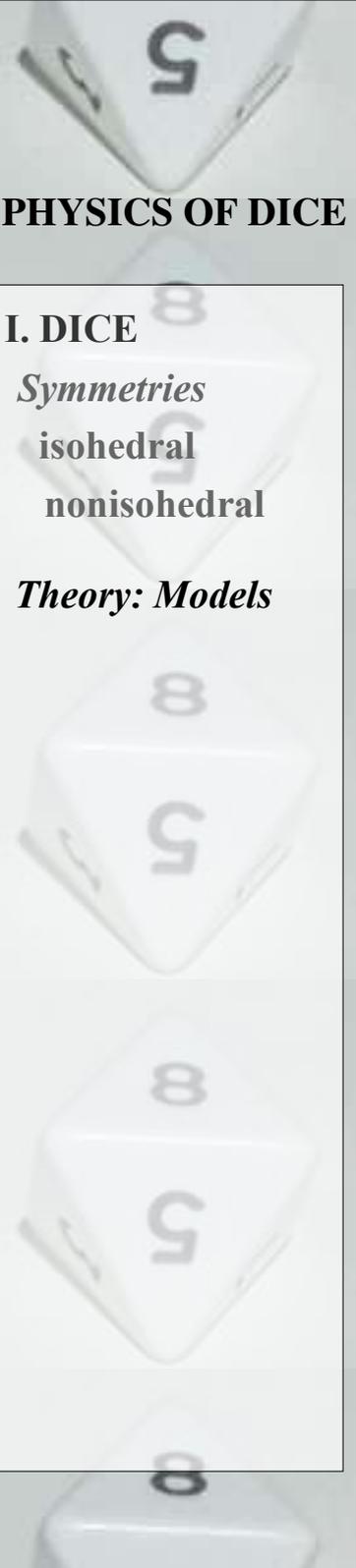
no
symmetry?

geometrical Model
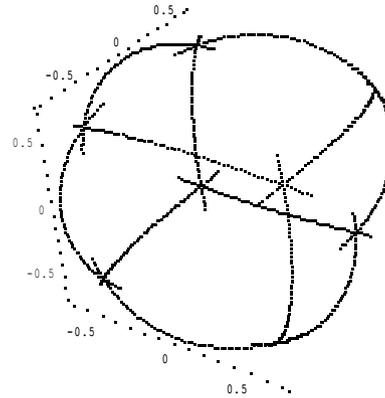
Energy-State Model

GEOMETRICAL MODEL

See PEGG (1997)

- The likelihood, of which side the dice will lay, depends
  on the positioning (area that the dice occupies) wrt. the angle
  that the side falls on.

Disadvantage
- Dynamics of the underlying process (Friction, Frequency...)
  not taken into consideration
  - Not valid for dice with *unstable sides*
    (= sides which don't enclose the projection of the
    gravity centre on their correspondent plane)

  Useful for rough approximations; further inspection leads to
  Dice Dynamics of the *Energy State Model*

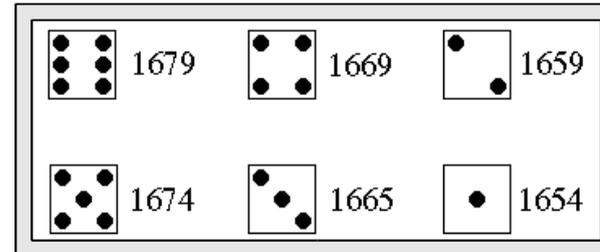# ENERGY STATE MODEL

I. DICE

Symmetries
  isohedral
   nonisohedral

Theory: Models
  Geometrical
  Energy-State

Results of a cubic dice with dots on the surface face
after 10000 throws (see PEGG, 1997)

• Based on Markov-Ketten:
  ● Initial State of the Dice = Vector
       (calculated by the Geometrical Model)
  ● n-th bounce of the Dice = Matrix A_N
       after consideration of System Dynamics
  ● a_{ij} = probability, the influence of side i
       on side j

Disadvantages
  • Dissipative System $\Rightarrow A_N$ varies, tasking!
    non-specifically $A_N \Rightarrow I$

Fair Dice?

theoretical:
Fairness of Dice by Symmetry
(isohedral Dice)

no
symmetry?

practical:
Specified parameters

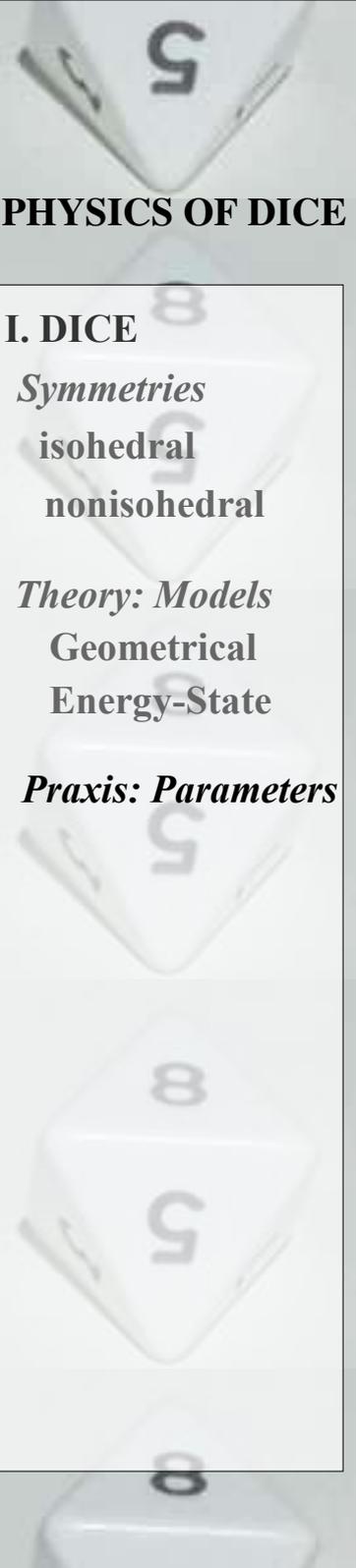geometrical Model

Energy-State Model

Bias

Flatness

Loading

PARAMETER SPECIFICATION

BIAS:

    Side: $j = 1,2,...,n$

    Probability: $p(j)$

  ↘Bias of Side j:

$$b(j) = n*p(j) - 1 \qquad \text{(n throws, where n/2=6} \Rightarrow b(6)=2)$$

FLATNESS *[cubic Dice]*:

    Intervals: $d1{:}6$, $d2{:}5$, $d3{:}4$

    "1-6-faces": $d1{:}6 < d2{:}5 = d3{:}4$

  ↘Flatness:

$$f1{:}6 = (d2{:}5+d3{:}4-2*d1{:}6) \qquad (18*20*20 \Rightarrow f1{:}6=0.1)$$

LOADING *[Sides of cubic Dice]*:

    Centre of gravity concerning side j: $CM(j)$

    (Emphasis on geom. Centre: CM(j)= 0)

    Length of all 3 Sides: d

  ↘Loading of Side *j* :

$$L(j) = 2*CM(j) / d \qquad = 0 \text{ (Fair) to 1 (emphasis on j)}$$

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Specified parameters

geometrical Model

Energy-State Model

Bias

Flatness

Loading

Machine Readings
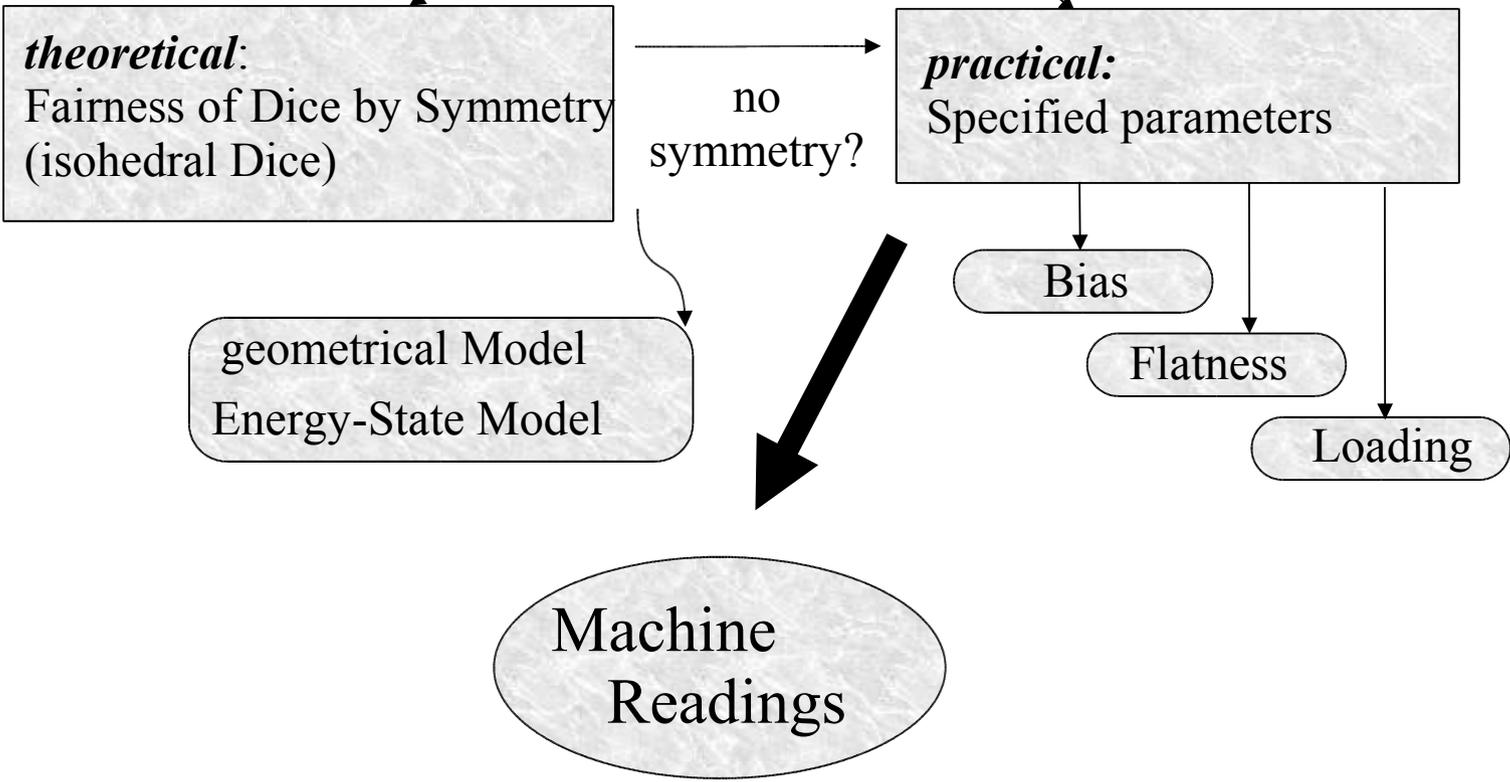
THE MACHINE
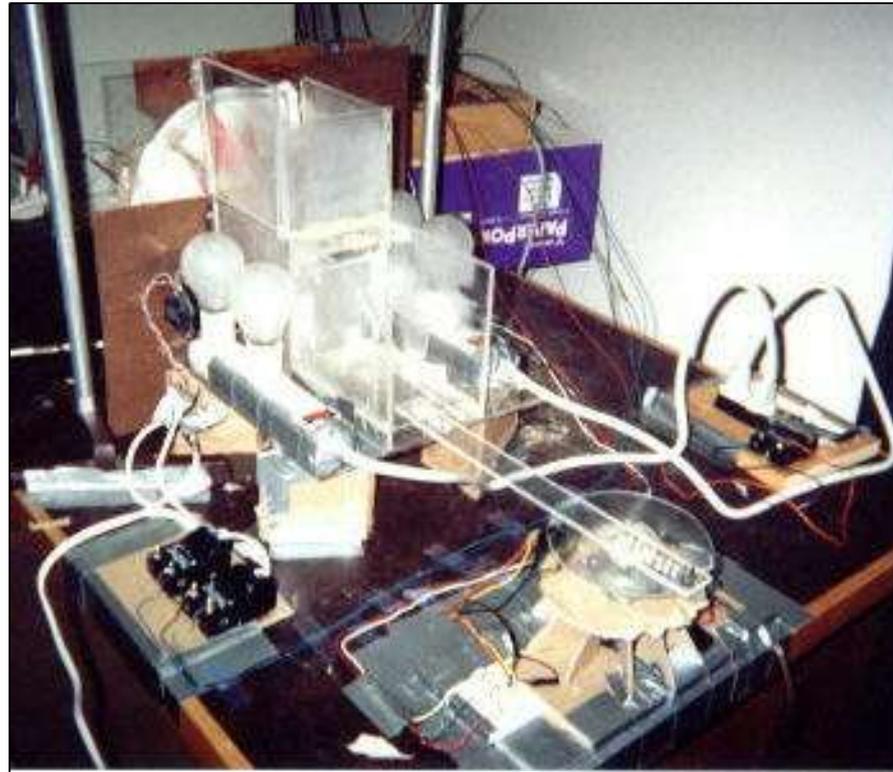
(Dr. Daniel Murray, University of British Columbia
Production: 2001)

Each throw requires 2.5 s (1 Day = 34560 Throws!)

PHYSICS OF DICE

I. DICE
Symmetries
 isohedral
 nonisohedral

Theory: Models
 Geometrical
 Energy-State

Praxis: Parameters

II. READINGS

The Machine
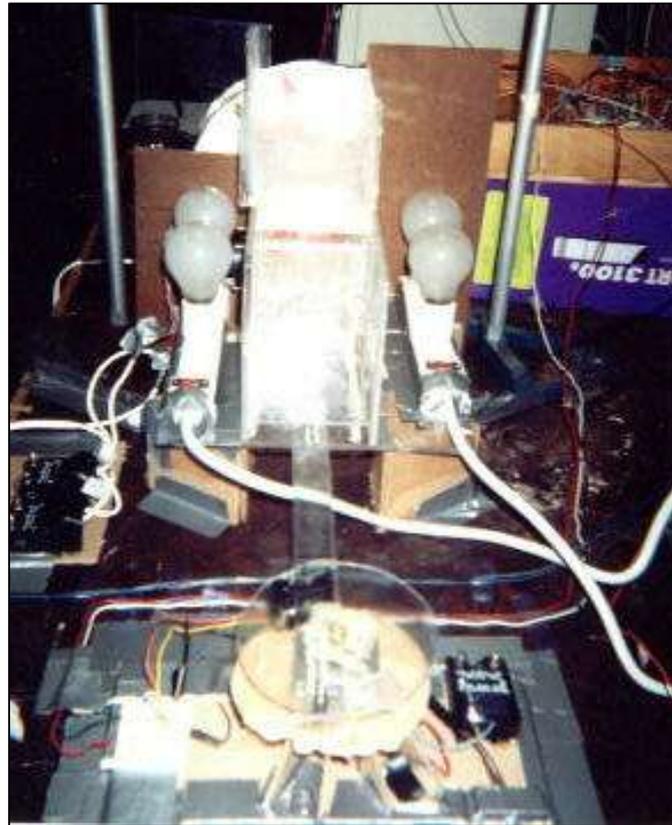
SWEEPER

Motor (72 rpm) throws the Dice off of the table the the wheel:

- Computer signal supplies a 120 V small Light + Phototransistor (PT)
 ↘ PT after 1cycle is blocked
 - Tension of the PT is read
 - Motor is shut off if needed

- Safety Door (so Dice cannot fall back into Spinner)
 2 Switches: Control of a Motor, that open (releases the dice) and closes (blocks dice) the 6 cm x 10 cm door

SCHLEUDERRAD
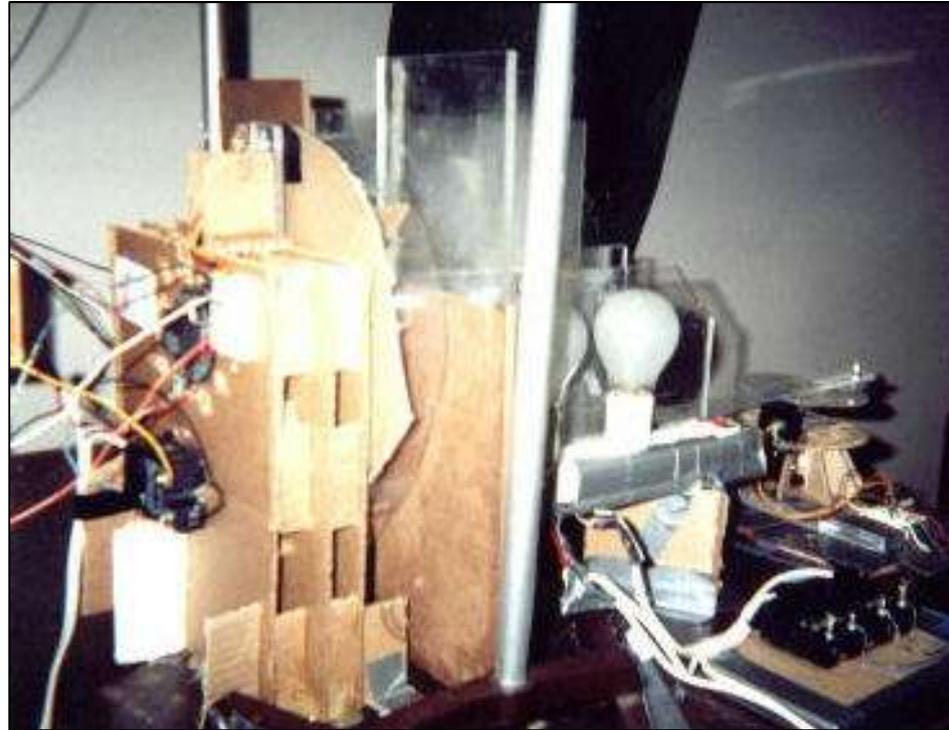
Plastic Wheel
(26.5 cm x 3.5 cm),
connected to a Motor

- Starting the wheel with a Computer signal
- Sweeper forces the Dice into Spinner
- Increases momentum until Dice reaches the highest point of the wheel
- Motor stops, Dice falls in trough and rolls into Observation Platform
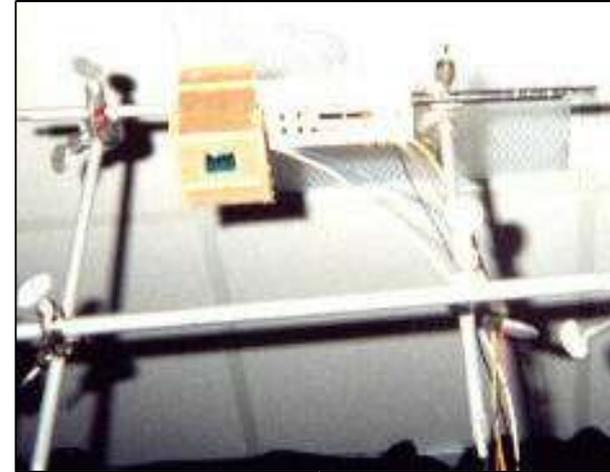
CAMERA

- Dice land on Plexi-Glass Table
   (10 cm x 10 cm in 30x30)
- CCD-Camera take a photo from
   Light Source: 4 Bulbs (100 W)
- Photo is transferred to a data processor
   (Electronic Counter)
- Point count of the side is recorded

PHYSICS OF DICE

I. DICE

Symmetries
  isohedral
  nonisohedral

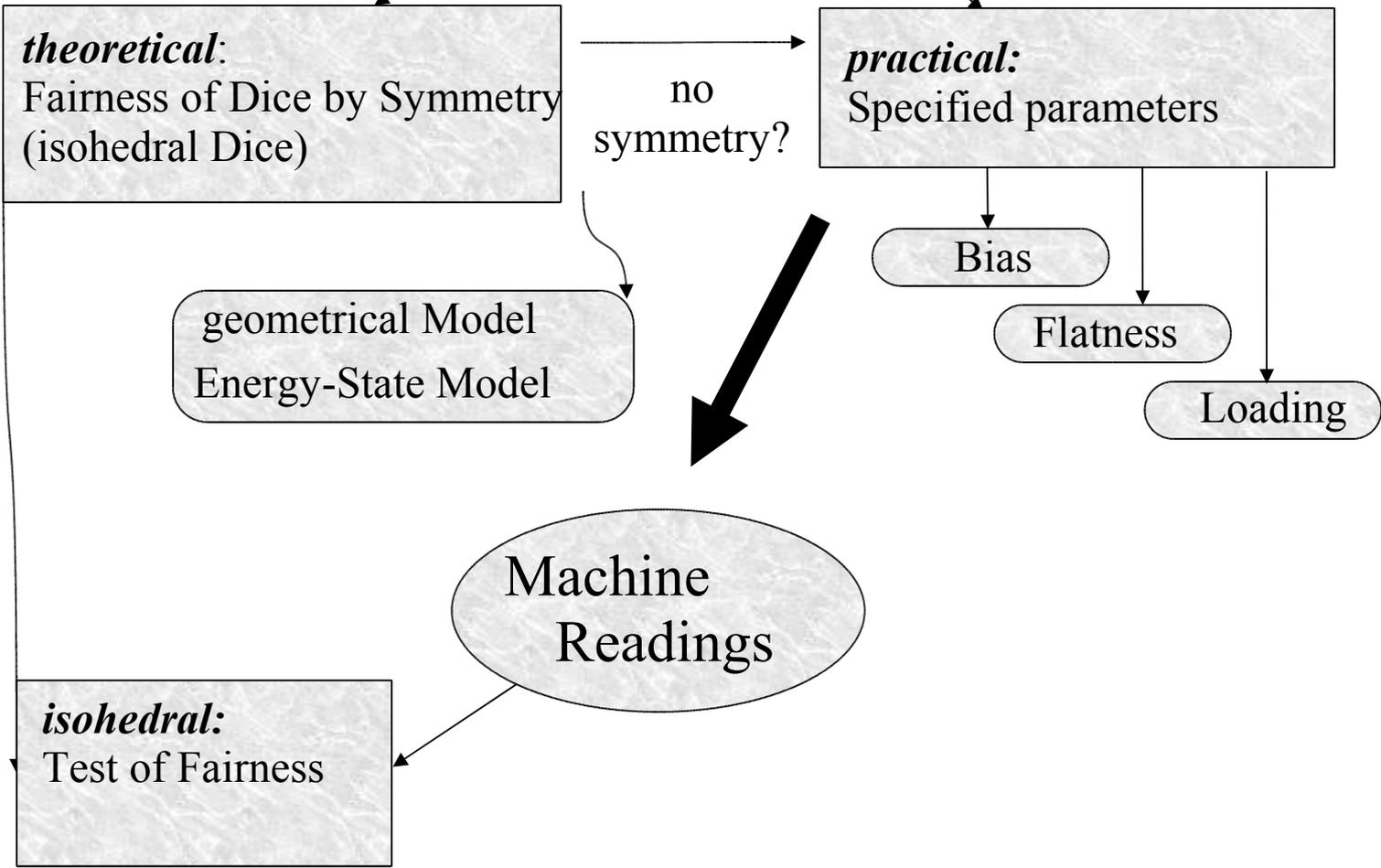*Theory: Models*
  Geometrical
  Energy-State

Praxis: Parameters

II. READINGS

The Machine

Experiments

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theoretical:
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practical:
Specified parameters

geometrical Model

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Flatness

Loading

Machine
Readings

isohedral:
Test of Fairness

ISOHEDRAL:
EXPERIMENT WITH CUBIC DIE

EHEM?

ASSUMPTIONS

• Dice thrown forcefully enough, so that the detailed Dynamics (Friction, Angular Momentum, Moment of Inertia...) possibly have little effect on the fairness of the dice

• Counterexample: Dice thrown into a liquid
  ↘ very high Friction -marked Loading!-
For viscose mediums the Mass and Shooting Parameter become meaningless (Solid Angle?)

Findings/Outcomes:

Dice from Nevada-Casinos:

Identical Side lengths within 0.01 mm (Micrometer)
Dice left for commercial use after 8 hours of throws

- no Bias after 640934 throws on Plexiglas
- all 6 sides had $p = 0.1667 \pm 0.001$

Dice of lower Quality:

- **0.10\$** from Kelowna, Canada (21543 throws on felt):

1:2895	*2*:42214	*3*:3389
4:3347	*5*:4383	*6*:3315

- **Flat** Dice ⇒ Bias proportional to deviation of the cubical form:
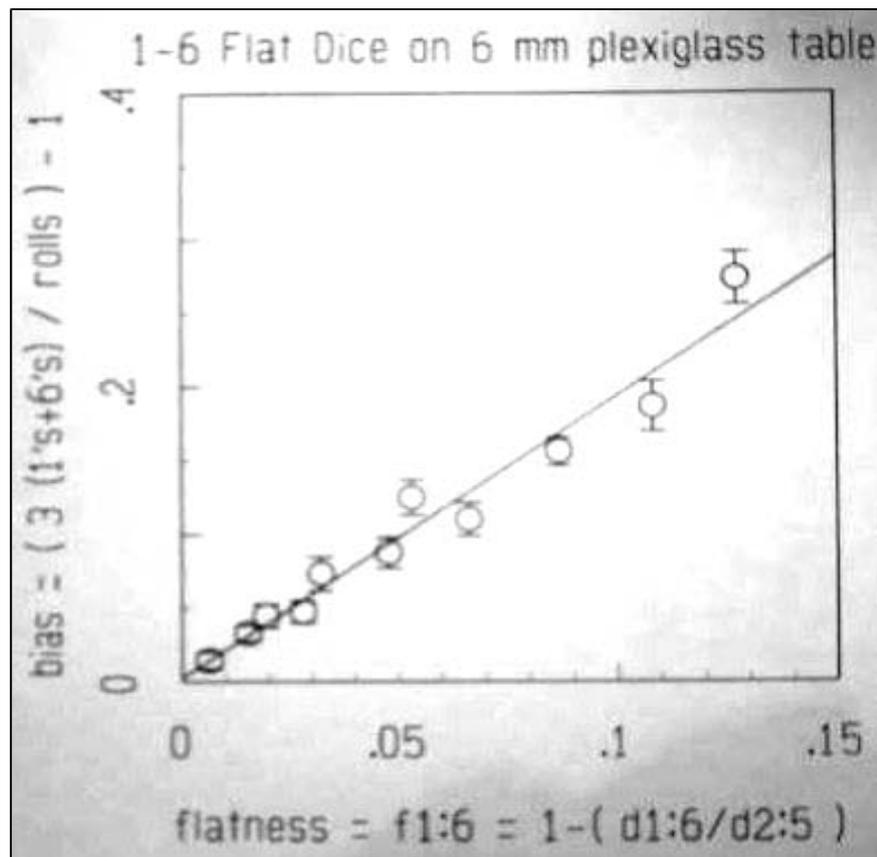
Shortened *(3% betw. sides 1 & 6), 36000 Throws*
  1 & 6:   6% more often than 1/6

PARAMETER-PLOTS
(see MURRAY & TEARE, 1993)

- <u>Bias-Flatness</u> (Beam with Standard Deviation)

Casino – Dice, 1:6-Sides become
gradually cut-up

- *alternative Plot*: Bias-Loading

PHYSICS OF DICE

I. DICE

Symmetries
  isohedral
  nonisohedral

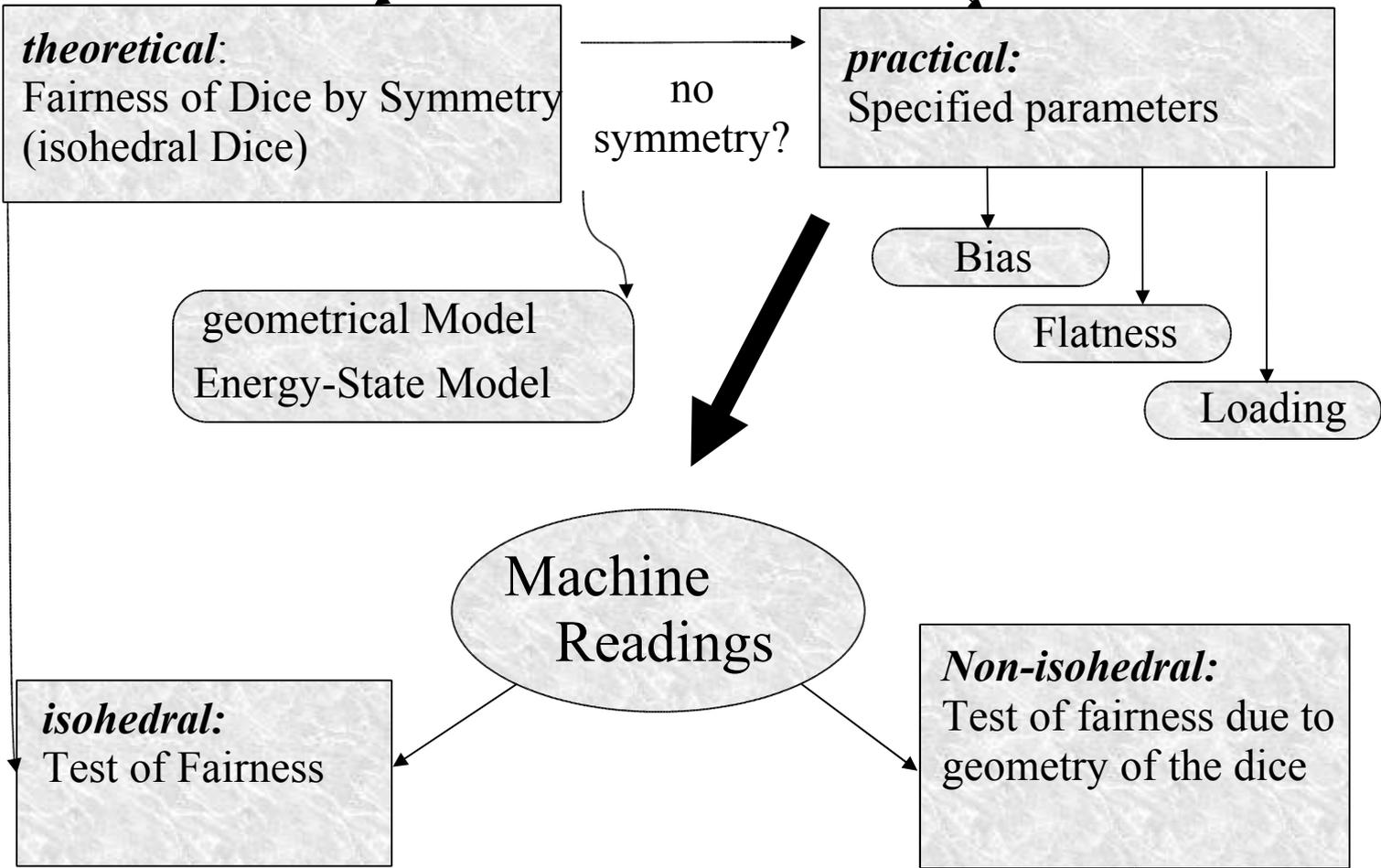*Theory: Models*
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  Energy-State

Praxis: Parameters

II. READINGS

The Machine

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  isohedral

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isohedral:
Test of Fairness

Non-isohedral:
Test of fairness due to
geometry of the dice

Non-Isohedral :
# EXPERIMENT WITH CYLINDRICAL DIE

Approximation:
- Coin of non-negligible thickness

Material, Dimensions:
- Nylon (19.1 mm diameter, Length: 3.5-47 mm)

[other alternatives for the material:]
- Wood (not isotropic corn! Yet showed no dramatic changes in experimental Data, and the preparation is cleaner)
- Foam, Aluminium, Polystyrene or Cellulose Acetate (Material for Casino Dice)
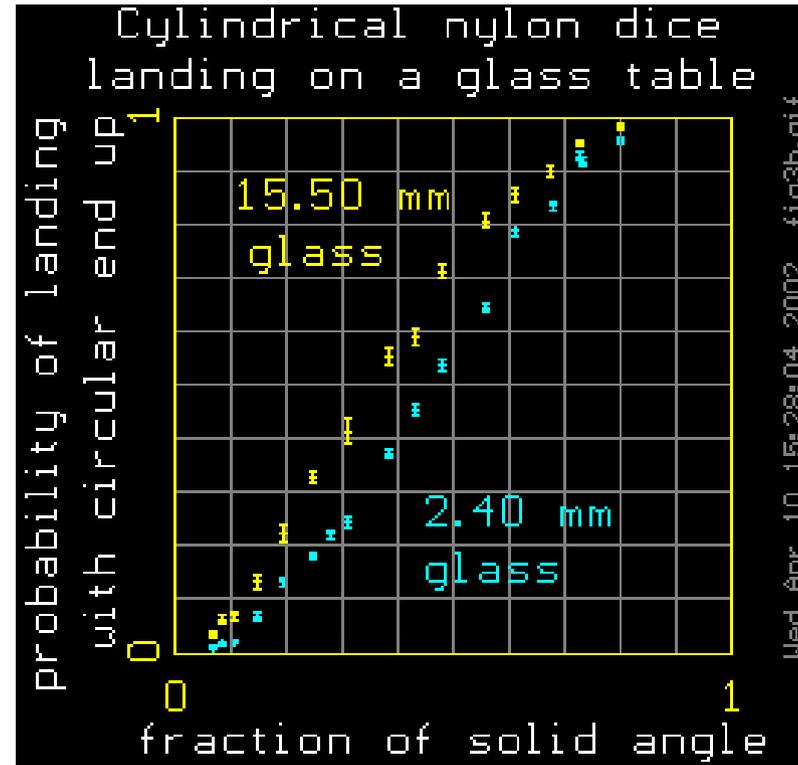
PHYSICS OF DICE

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    nonisohedral

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    Energy-State

Praxis: Parameters
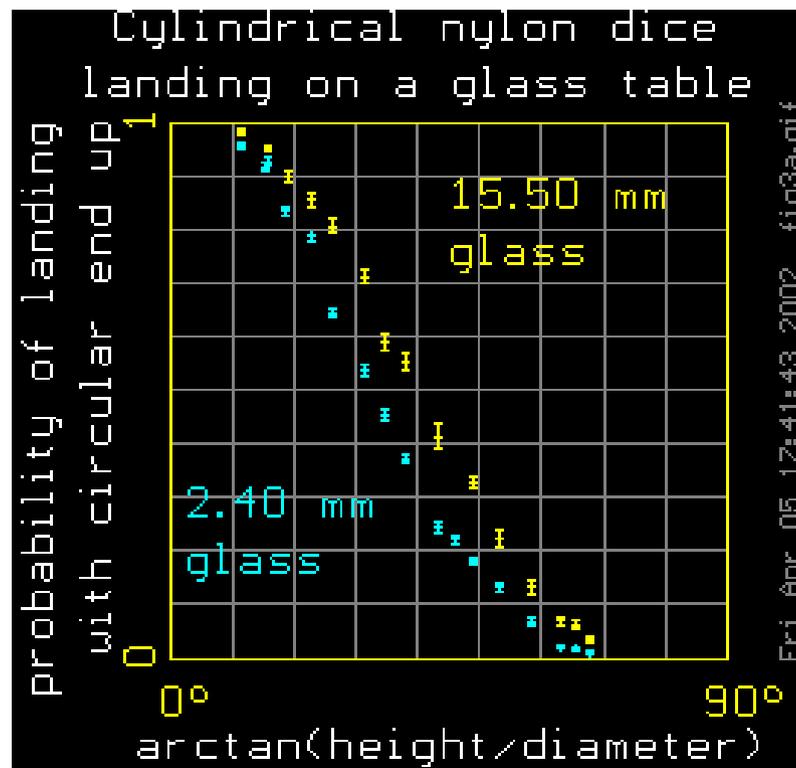
II. READINGS

The Machine

Experiments
    isohedral
    non-isohedral

- approx. 70000 throws on a well-defined 10 cm x 10 cm-
  -Glass Plate (Thickness: 2.40 and 15.50 mm)

- *2.40 mm thick Glass plate:*
    probability of landing on the base is 12%
    and 76% for the side
  ⇒ *15.50 mm thick Glass plate:*
    probability of landing is 19%, 19% and 62%

Cylindrical nylon dice landing on a glass table

- approx. 70000 throws on a well-defined 10 cm x 10 cm-
    -Glass Plate (Thickness: 2.40 and 15.50 mm)

- *Plot of Height-Diameter-Ratio:*
        more representative than plot with solid angle!

PHYSICS OF DICE

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    Geometrical
    Energy-State

Praxis: Parameters
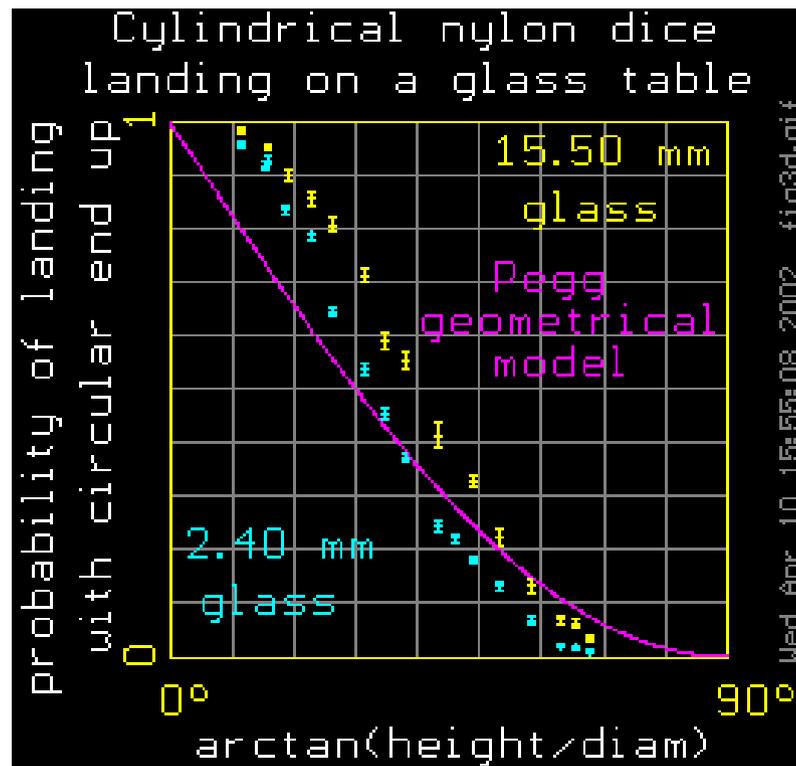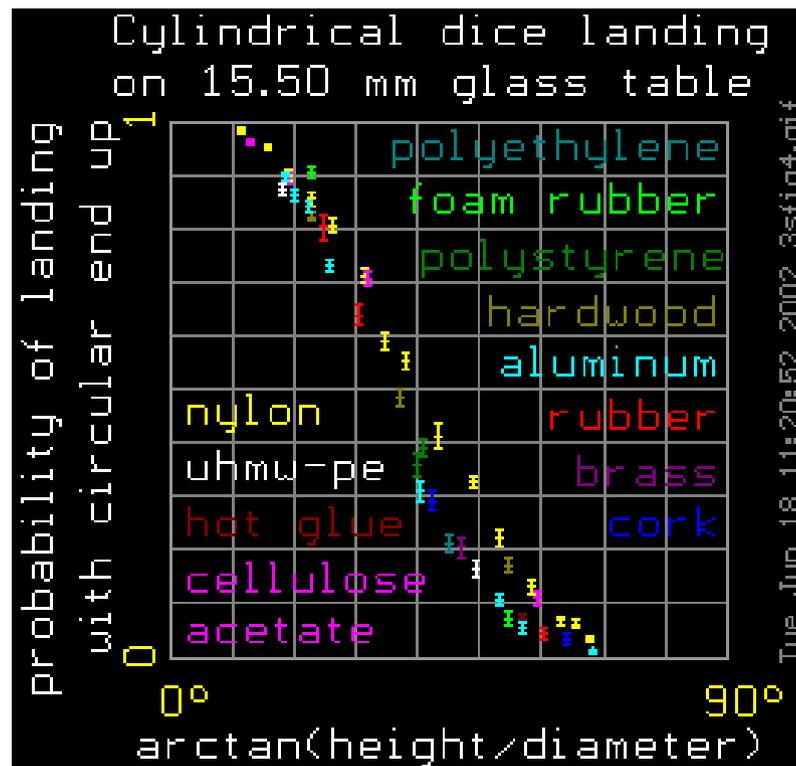
II. READINGS

The Machine

Experiments
    isohedral
    non-isohedral

- approx. 70000 throws on a well-defined 10 cm x 10 cm-
  -Glass Plate (Thickness: 2.40 and 15.50 mm)

- *Plot of Height-Diameter-Ratio:*
      more representative than plot with solid angle!

- Geometrical Model does not agree
      (particularly when using "longer" dice)

- approx. 70000 throws on a well-defined 10 cm x 10 cm-
  -Glass Plate (Thickness: 2.40 and 15.50 mm)

- *Plot of Height-Diameter-Ratio:*
    more representative than plot with solid angle!

- Influence of the material of the dice is clearly reflected
   in the shooting parameter!

PHYSICS OF DICE

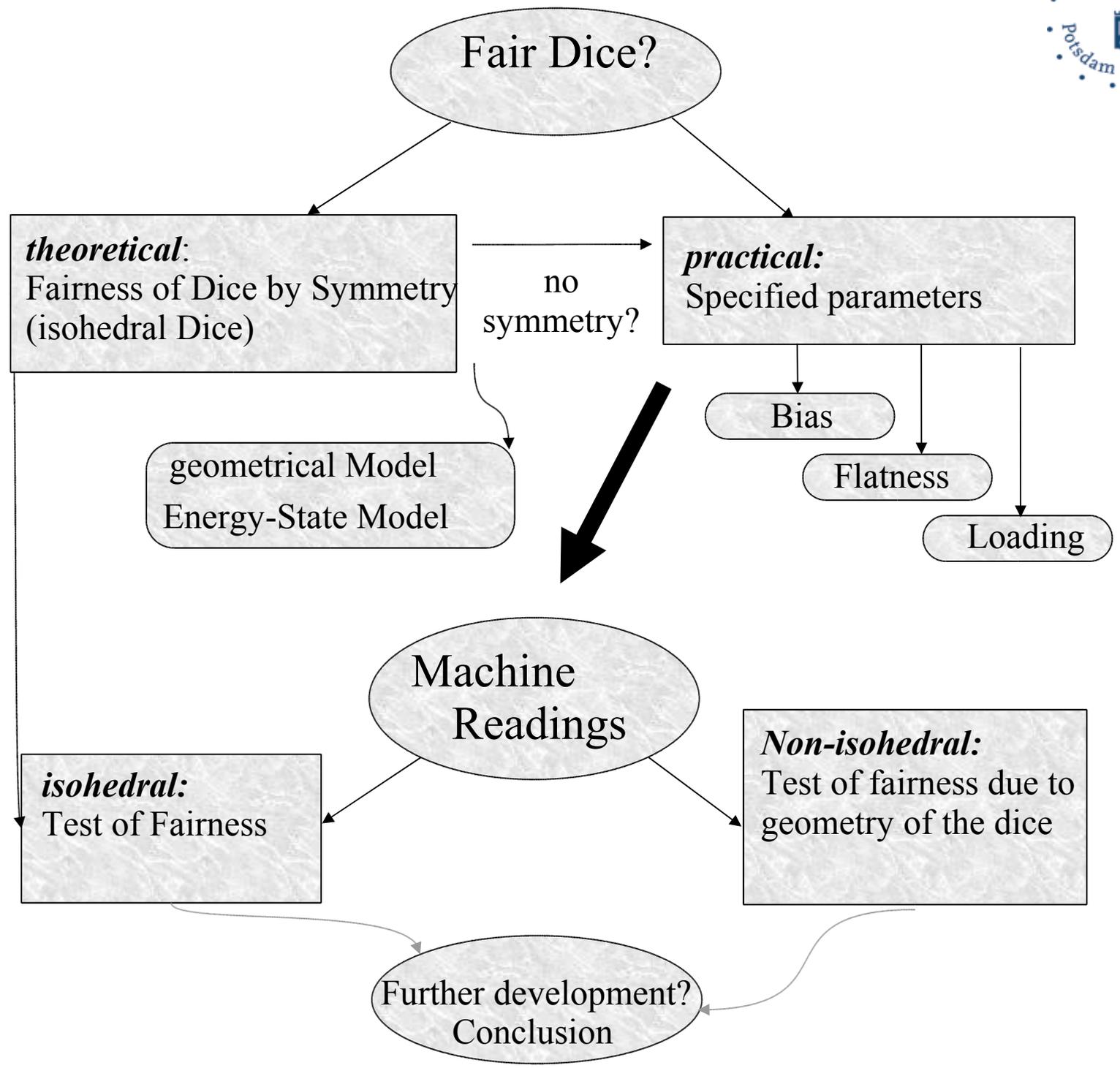**I. DICE**

Symmetries
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  nonisohedral

Theory: Models
  Geometrical
  Energy-State

Praxis: Parameters

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  non-isohedral

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Test of Fairness

Non-isohedral:
Test of fairness due to geometry of the dice

Further development?
Conclusion

Conclusion

- *Expected outcomes for cubic Dice*
  (fair Dice with centre of gravity is in the middle, linear dependance on the Bias for Deviation from the cube-shape)

- *Non-Isohedral Dice*
  - Possible Variations for cylindrical dice due to Geometry/ Effective Length (dependence on solid angle?)
  - Geometrical Models hold for small length values (small major-axis values)

- *Remaining open questions*
  - Other Dice: Predictions/Hypotheses a priori very difficult
  - Calculation of curvature from Energy-State Models and comparison with experimental Data

Bibliography

About the Dice Experiments:

LEVIN, Eugene M. (1983): Experiments with loaded dice.
    American Journal of Physics, volume 51, pages 149-152

MURRAY, Daniel B. and TEARE, Scott W. (1993): Probability
    of a tossed coin landing on edge. Physical Review E, volume
    48, 1993, pages 2547-2552

CHRISTIE, Dean; GLASHEEN, Ryan; HAMILTON, Chris;
IMOTO, Masashiro; MATTHEWS, Philip; MOFFAT, James;
MONAJEMI, Thalat; MURRAY, Daniel B.; NELSON, John &
STURM, Arne (2001): Experimentally obtained statistics of dice
    rolls. Poster at the 6th Experimental Chaos Conference,
    July 22-26, 2001, Potsdam, Germany

PHYSICS OF DICE

I. DICE

Symmetries
  isohedral
    nonisohedral

Theory: Models
    Geometrical
    Energy-State

Praxis: Parameters

II. READINGS

The Machine

Experiments
    isohedral
    non-isohedral

Conclusion

Bibliography

Contact:
        Dan Murray, Associate Professor
        Department of Physics and Astronomy
        University of British Columbia Okanagan
        daniel.murray@ubc.ca

Further information on Theory:
PEGG Jr., Edward T. (1997): A Complete List of Fair Dice.
        Master's Thesis

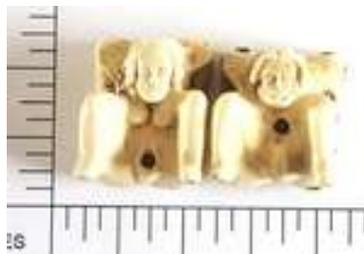*A popular science article:*
PETERSON, Ivar: Fun and Games in Nevada.
                                MAA Online, 28.4.1997
        "   : Unfair Dice. MAA Online, 26.10.1998
        "   : Tricky Dice Revisited. Science News, 13.04.2002