Hidden singles

				5				
			8		1			
	8	3				4	2	
	2			4			8	
6			3		8			5
	7			6			9	
	4	8				7	5	
			1		7			
				9				

In this puzzle there's already a 5 in row 1 and in columns 8 and 9, so the only square for 5 in the shaded box is row 2 column 7. This is a hidden single.

Here's another example from the same puzzle:

				5				
			8		1			
	8	3				4	2	
	2			4			8	
6			3		8			5
	7			6			9	
	4	8				7	5	
			1		7			
				9				

There's already a 5 in column 9 and also in the top, middle 3x3 box. So the only square for 5 in the shaded row is column 1.

Naked singles

				5				
			8		1			
	8	3				4	2	
	2			4			8	
6			3		8			5
	7			6			9	
	4	8				7	5	
			1		7			
				9				

Now consider what can go in row 3 column 5 (the shaded square) in the same puzzle. In row 3 we already have 2, 3, 4 and 8. In column 5 we already have 4, 5, 6 and 9. In the same 3x3 box we already have 1, 5 and 8. So there is only one candidate left for row 3 column 5, namely 7. This is an example of a naked single.

Locked candidates 1

8		1	6					
	9			8			3	6
6				7				
3			5					
9	5	8	7			4	2	
					9			7
				2				9
	8			3			7	
					7	6		

In this puzzle you can make no progress with hidden and naked singles. Let's look at where 6 can go in the shaded 3x3 box. There is already a 6 in row 9 and also in column 4, so 6 must go in column 6 in this box. This allows you to eliminate 6 as a candidate in column 6 in rows 4 and 5. This uncovers 6 as a hidden single in row 5 column 5.

In this example a candidate is restricted to one column in a 3x3 box but there's another type of locked candidate where a candidate is restricted to one 3x3 box within a row or column. The puzzle on the next page shows an example of this.

Locked candidates 2

	4	6	3	7	9	1		5
1		7		5	4	6		3
5	3		1			4	7	
4		3	6		5		1	7
		1	4	3			5	6
6	5			1	7	3	4	2
	1	5			3	7	6	4
		4	5			2	3	1
3	6	2	7	4	1	5		

In this puzzle you can make no progress with hidden and naked singles. Let's consider where 2 can go in row 3 (the shaded row). There's already a 2 in column 3 and also in column 9, so 2 is restricted to the middle 3x3 box in this row. As a result we can eliminate 2 as a candidate from other rows in this box, in particular from row 2 column 4. This leaves 8 as a naked single in row 2 column 4.

Naked pairs

9	5	7	6			1	4	2
1	3	8	2	4	5	9	6	7
4	6	2				8	3	5
		4	7		9		2	
	7			1			9	
		9	5		6	4	7	
8	2						1	4
7	9	1		6		2	5	3
	4						8	9

In this puzzle you can make no progress using singles and locked candidates. Now look at the squares in column 5. The only possible candidates for row 1 column 5 are 3 and 8. The same two candidates are the only possibilities for row 4 column 5. So if row 1 column 5 is 3 then row 4 column 5 will be 8, whereas if row 1 column 5 is 8 then row 4 column 5 will be 3. Either way we can eliminate 3 and 8 as candidates for the other squares in column 5. This leaves 2 as a naked single in row 6 column 5. This is an example of a naked pair. In this case the naked pair is in a column, but you can also get naked pairs in rows or 3x3 boxes.

Hidden pairs

4	5	1	8	9	3	7	6	2
		9	2		5	4		
3		2		4	6	9		5
	1	4	3	5	9		2	
	3		4	2				9
2	9	5	6		8	3	4	
5	4	3		8		2	9	6
1	2		9		4			
9		7	5		2	1		4

In this puzzle you can make no progress using singles, locked candidates or naked pairs. If you look at column 8 in this puzzle, there are only two squares where you can put 5, either row 5 or row 8. These two squares are also the only places where you can put 7 in this column. So if row 5 column 8 is 5 then row 8 column 8 is 7, whereas if row 5 column 8 is 7 then row 8 column 8 is 5. As a result you can eliminate other candidates from these two squares. This leaves 1 as a hidden single in row 5 and in the middle, right hand 3x3 box. This is an example of a hidden pair in a column. You can also get hidden pairs in rows or 3x3 boxes.

X-wings

		2	8		6	3		
6		1	3		9	7		2
	9	3	1				5	6
	3		4				6	
		7	6	3	5	2		
2			9	8	1		3	7
	2	4	5		8	6	7	
			2		4			
		8	7		3	1	2	4

In this puzzle you can make no progress using the techniques of the previous sections. Look at where 9 can go in column 3. There is already a 9 in row 6, so 9 can go in row 4 or row 8 in column 3. Now look at where 9 can go in column 7. There is already a 9 in row 3 and in row 6, so 9 can go in row 4 or row 8 in column 7. Now if there is a 9 in row 4 column 3 then there must be a 9 in row 8 column 7 and if there is a 9 in row 8 column 3 then there must be a 9 in row 4 column 7. Either way we can eliminate 9 as a candidate from all the other columns in rows 4 and 8. In particular we can remove 9 from row 8 column 8, which leaves 8 as the only candidate for row 8 column 8. This is an example of an x-wing. One candidate is restricted to two rows in two columns or vice versa.

Triples and quads

								2
	2		8		6	1		
		9	3		2	5	6	8
	9	8	5		1	6	3	7
						8		
	5	1	7		8	9	2	4
	6		1		9	2		
		3	2		5		8	

In this puzzle you can make no progress using the techniques from previous sections. Now look in row 1. The only two candidates for column 4 row 1 are 4 and 9, also the only two candidates for column 6 row 1 are 4 and 7, and the only three candidates for column 8 row 1 are 4, 7 and 9. If column 4 row 1 is 4 then column 6 row 1 is 7 and column 8 row 1 is 9. Alternatively if column 4 row 1 is 9 then column 6 row 1 and column 8 row 1 form a naked pair. Either way 4, 7 and 9 must appear in one of these three squares in the solution, so they can be ruled out as candidates for other squares in row 1. As a result 3 is the only candidate for row 1 column 7. This is an example of a naked triple.

Let's look at where 1, 5, 6 and 8 can go in row 1. The only squares where 1 can go in this row are columns 1, 2 and 5. The only squares where 5 can go in row 1 are columns 1, 3 and 5. The only squares where 6 can go in row 1 are columns 1 and 3. The only squares where 8 can go in row 1 are columns 1 and 2. So these four candidates are restricted to four squares. As a result you can eliminate other candidates from these four squares. This uncovers 3 as a hidden single in row 1, row 2 and column 2. This is an example of a hidden quad.

3		2						
		7		2	3			8
9	8		7					
		1	8				9	4
				1	4			
4	5				6	8	1	
					9		4	3
			4	6		5		9
						7		1

In this puzzle you can make no progress using the techniques from previous sections. Now look at where 1, 4 and 9 can go in the top right 3x3 box. The only squares where 1 and 4 can go in this box are rows 1, 2 and 3 in column 7. The only squares where 9 can go in this box are rows 1 and 2 in column 7. So these three candidates are restricted to three squares in this box. As a result you can eliminate other candidates from those three squares. This uncovers 3 as a hidden single in this box. This is an example of a hidden triple.

This same 3x3 box also contains a naked quad, four squares which contain only 2, 3 or 4 candidates from a set of four candidates.

Both the puzzles in these examples can be solved with triples, but there are puzzles which can't be solved with pairs or triples but can be solved with quads.

In a 9x9 sudoku you never have to look for hidden or naked sets with more than four candidates. For instance if a line or a box contained a naked quin, then there would be a hidden set with no more than four candidates in the same line or box.

Swordfish

6		5		1		3	9	2
2		1	5	9		8		6
8	9		2		6		5	1
5	6			2	7	9		
7		2		8	9			5
9	1	8	4	6	5	2	3	7
4	5	6						9
3	2	9	6		1	5		
1	8	7	9	5				3

In this puzzle you can make no progress using the techniques from previous sections. Look at where 4 can go in columns 3, 5 and 9. In column 3 it can only go in row 3 or row 4. In column 5 it can only go in row 3 or row 8. In column 9 it can only go in row 4 or row 8. In these three columns 4 is restricted to three rows. That means you can eliminate 4 as a candidate from other columns on rows 3, 4 and 8, in particular column 7 row 3. As a result 7 is a naked single in that square. This is an example of a swordfish. One candidate is restricted to three rows in three columns or vice versa.

Jellyfish

6		5		1		3	9	2
2		1	5	9		8		6
8	9		2		6		5	1
5	6			2	7	9		
7		2		8	9			5
9	1	8	4	6	5	2	3	7
4	5	6						9
3	2	9	6		1	5		
1	8	7	9	5				3

Let's look at the same puzzle from a different perspective. Where can 4 go on rows 1, 2, 5 and 9? On row 1 it can only go in column 2 or column 6. On row 2 it can only go in column 2, column 6 or column 8. On row 5 it can only go in column 2, column 7 or column 8. In row 9 it can only go in column 6, column 7 or column 8. In these four rows 4 is restricted to four columns. That means you can eliminate 4 as a candidate from other rows in columns 2, 6, 7 and 8, in particular row 3 column 7. Once again this results in a naked single in this square. This is an example of a jellyfish. One candidate is restricted to four columns in four rows or vice versa.

This puzzle can be solved with a swordfish, but there are puzzles which can't be solved with a swordfish but can be solved with a jellyfish.

In a 9x9 sudoku you never have to look for a candidate that is restricted to five rows on five columns or vice versa. For instance if a candidate is restricted to five rows on five columns then it will also be restricted to no more than four columns on four rows.

Simple colours

9	8	1	5	7	4	2	6	3
2	7	A			8			9
a	5	6	2	9		8	A	7
	2	5	7		9	6	3	
7	6	a	8		2	9		
	1	9				7	2	
6	9	7		2		3	8	
1	3	8	9				7	2
5	4	2	3	8	7	1	9	6

In this puzzle you can make no progress using the techniques from previous sections. If you look at row 3, column 3 and the top left 3x3 box, there are only two squares in each where 4 is a candidate. These squares are marked **a** and **A**. Either all the squares marked **a** are 4 or all the squares marked **A** are 4. Row 5 column 8 (the shaded square) shares a row with a square marked **a** and a column with a square marked **A**, so you can eliminate 4 as a candidate from this square. Now row 5 column 8 must be 5 and the puzzle is solved. This is an example of a colour trap. It happens when one candidate occurs in only two squares in a succession of lines and boxes.

		5	2		7	4		6
	2	6	5	9	4	3		
	4	a	6		a	2	9	5
2	7	4	3	6	8	1	5	9
a	8	9	7	5	A	6	4	
6	5	A	4	a	9	7		8
5	1		9			8	6	
	6	2	8	7	5	9		
		8	1		6	5		

In this puzzle you can make no progress using the techniques from previous sections. If you look at rows 5 and 6, columns 3 and 6, the middle left 3x3 box and the middle 3x3 box there are only two squares in each where 1 is a candidate. They are marked **a** and **A**. So either all the squares marked **a** are 1 or all the square marked **A** are 1. However in row 3 there are two squares marked **a**. They can't both be 1, so none of the squares marked **a** are 1 and as a result all the squares marked **A** must be 1. This is an example of a colour wrap.

XY-wings

3	5			9		8	4	
9	6			8	4 5			7
1	8			3 4		3 5	6	9
7	9			1				
4	3	8	7	5	2	9	1	6
2	1				9	7		
6	7	9		2			3	
8	2	3	4	7	1	6	9	5
5	4	1	9	6	3	2	7	8

In this puzzle you can make no progress using the techniques from previous sections. Note that 4 and 5 are the only candidates for row 2 column 6. Similarly 3 and 4 are the only candidates for row 3 column 5 and 3 and 5 are the only candidates for row 3 column 7. Each of these squares must be one of two candidates from a set of three. One of these squares, row 3 column 5, shares a 3x3 box with one of the other squares and a row with the other one. This is called the pivot square. Now if row 3 column 5 is a 3 then row 3 column 7 must be 5, whereas if row 3 column 5 is a 4 then row 2 column 6 is a 5. Either way you can eliminate 5 as a candidate from the four shaded squares. As a result the only candidate for row 2 column 8 is 2, also 5 is a hidden single in the top right 3x3 box. This is an example of an xy-wing.

Xy-wings, with three 'connected' squares each with only two candidates, are the simplest form of xy-chains. Xy-chains can be four or more 'connected' squares long. The next page shows an example.

XY-chains

3	1 8	7	2		6		9	
1 8	4	2	1 5	A	9	3		
6	9	5		7			2	
2	7	B	5 6	9			1	3
9	1 6	8	B			7	5	
5	3		7					9
4	5	3	9	6	2	1		
7		9	8				4	
1 8		1 6	4	1 5	7	9	3	

This puzzle has examples of xy-chains with four and five squares. For a four link chain, suppose row 2 column 4 is 1 then row 2 column 1 must be 8, row 9 column 1 must be 1 and row 9 column 5 must be 5. Alternatively row 2 column 4 is 5. Either way the shaded square marked A cannot be 5.

For a five link chain, suppose row 5 column 2 is 1, then row 1 column 2 would be 8, row 2 column 1 would be 1, row 2 column 4 would be 5 and row 4 column 4 would be 6. Alternatively row 5 column 2 is 6. Either way the two shaded squares marked B cannot be 6.

There is another five link chain from row 9 column 3 to row 4 column 4 which also eliminates 6 from row 4 column 3.

XYZ-wings

			8	4	5	7		3
7	8	4	6	3	9	1	5	2
3	5		2	7	1	4		8
8	6	1 9	149			5		7
5		2	4 9	8		9		6
	3	7		5	6	8	2	1
1	9	8	3	6	4	2	7	5
	7	3	5		8	6	1	
6		5		1		3	8	

In this puzzle you can make no progress using the techniques from previous sections. Let's consider the possible candidates for row 4 column 4. If it's 1 then row 4 column 3 must be 9, if it's 4 then row 5 column 4 must be 9, otherwise row 4 column 4 itself must be 9. One of these three squares must be 9, which means that we can eliminate 9 as a candidate from any other squares that are in the intersection of row 4 and the middle 3x3 box. This is an example of an xyz-wing. There will always be at most two squares from which you can eliminate a candidate with an xyz-wing. In this case 9 can be eliminated from the shaded square, row 4 column 5. You can then place 2 and 9 in column 5. Note that with an xyz-wing the pivot square has three candidates and shares a 3x3 box with one of the other squares and a row or a column with the other.

Nice loops

		a	1	7	b		9	6
b		1	5	6	9		B	7
9	6	7	B	8	3	1	5	b
			3	4	1		6	
3	1	8	6	9	7	4	2	5
6	4	9	8	2	5	7	3	1
1		6	9	3		5		
	9	A		1		6		3
		3		5	6		1	

In this puzzle there are two squares in column 3 where 4 can go. They are marked **a** and **A**. There are also two squares in row 2 where 4 can go. Theses squares are marked **b** and **B**. (Note that row 1 column 1 could be 4, so there are three squares in the top left 3x3 box where 4 can go). The chain made of **b** and **B** continues in the top right 3x3 box and in row 3. Now **a** and **b** share the top left 3x3 box, so if **a** is 4 then **B** must be 4. Naturally if **a** is not 4 then **A** is 4. Either way the two shaded squares cannot be 4 since they share a row with a square marked **A** and a column with a square marked **B**. This is an example of a nice loop. Note that x-wings are another example of a nice loop.

Colours on multiple candidates

This extends the idea of simple colours to include squares which have only two candidates. Here's a simple example:

	7		4			3	1	
6				7	1			
4		1				7		
3A	6	7	5	4	3a 9A	1		2
3a	1	4	7		8	6	5	3A 9a
5			1	6			7	
	4	5	6		7	8	3	1
1		6	8	3	4	5		7
7	8	3		1	5			

Row 4 column 6 has only two candidates, 3 and 9. The 9 is marked with **A** and the 3 with **a**. Now there are only two squares for 3 in row 4, in column 1 and in row 5. They are marked with **a** or **A** as appropriate. Also in row 5 column 9, the only candidates are 3 and 9. The 3 is marked with **A** so the 9 is marked **a**. Now 9 can be eliminated as a candidate from the shaded squares as they share a row with a **9A** and a 3x3 box with a **9a**. The puzzle then solves easily.

1	8	5	4		9	3	7	
7	4	3	5			1		9
6	9	2	7	1	3	5	8	4
5			3	4	1			7
			6		7		4	
	7	4A	9				1	
	3	1A 4a	8	9	6	7	5	
	5	6a	1	7	4		3	
	1a 6A	7	2	3	5	4		

In this puzzle there are only two squares for 6 in the bottom left 3x3 box. These are marked **6a** and **6A**. There are only two candidates for row 9 column 2. The 6 is marked **A** so the 1 is marked with **a**. There are also only two squares for 1 in this 3x3 box, so the 1 in row 7 column 3 is marked with **A**. There are only two candidates for row 7 column 3, so the 4 is marked with **a**. There are also only two squares for 4 in column 3, so the 4 in the shaded square is marked **A**. Remember that either the candidates marked **a** are true or the candidates marked **A** are true. Now 6 can be eliminated from the shaded square because it shares a column with a **6a** and a square with a **4A**.

Nice loops on multiple candidates

	9	6						
	1		6		7	9		8
7	8			9			2	6
	2			8		7a 4A	6	9
6	7		2		9	8		
9	4	8		3	6	2	1	5a 7A
	5	9		6		4b	7	2
	6	7	9		4		8	
	3			7		6	9	4B

In this puzzle there are only two candidates for row 6 column 9. They are marked **5a** and **7A**. Also there are only two squares for 7 in the right middle 3x3 box. So the 7 in row 4 column 7 is marked with **a**. After making eliminations with the hidden pair on 4 and 7 in row 4, there are only two candidates for row 4 column 7. The 7 is marked with an **a** so the 4 is marked **4A**. Finally in the bottom right 3x3 box there are only two squares for 4. They are marked **4b** and **4B** (not **4a** and **4A** as there are other squares for 4 in column 7). Now if row 6 column 9 is 7, then row 4 column 7 must be 4. In that case row 7 column 7 can't be 4, so row 9 column 9 must be 4. Alternatively, if row 7 column 9 isn't 7, then it's 5. Either way row 9 column 9, the shaded square, can't be 5.

3	5	6	7			2	8	
4	9	1				7	3	6
7	8	2	6		3			9
9	1		5			6	7	
6	7		4					
2a 8A	4c		1	6	7	4B 8b	9	
							4	7
	4C	4c 9C	2C 9c	7		3		8
5		7			4		6	

Here's another example. In this puzzle, if row 6 column 1 is 8, then row 6 column 7 must be 4. In that case row 6 column 2 can't be 4 and all the candidates marked **C** are true, so row 8 column 4 would be 2. Alternatively, if row 6 column 1 isn't 8, then it must be 2. Either way the shaded square, row 8 column 1, can't be 2.

Note that hidden and naked pairs and xy-chains are examples of nice loops on multiple colours.