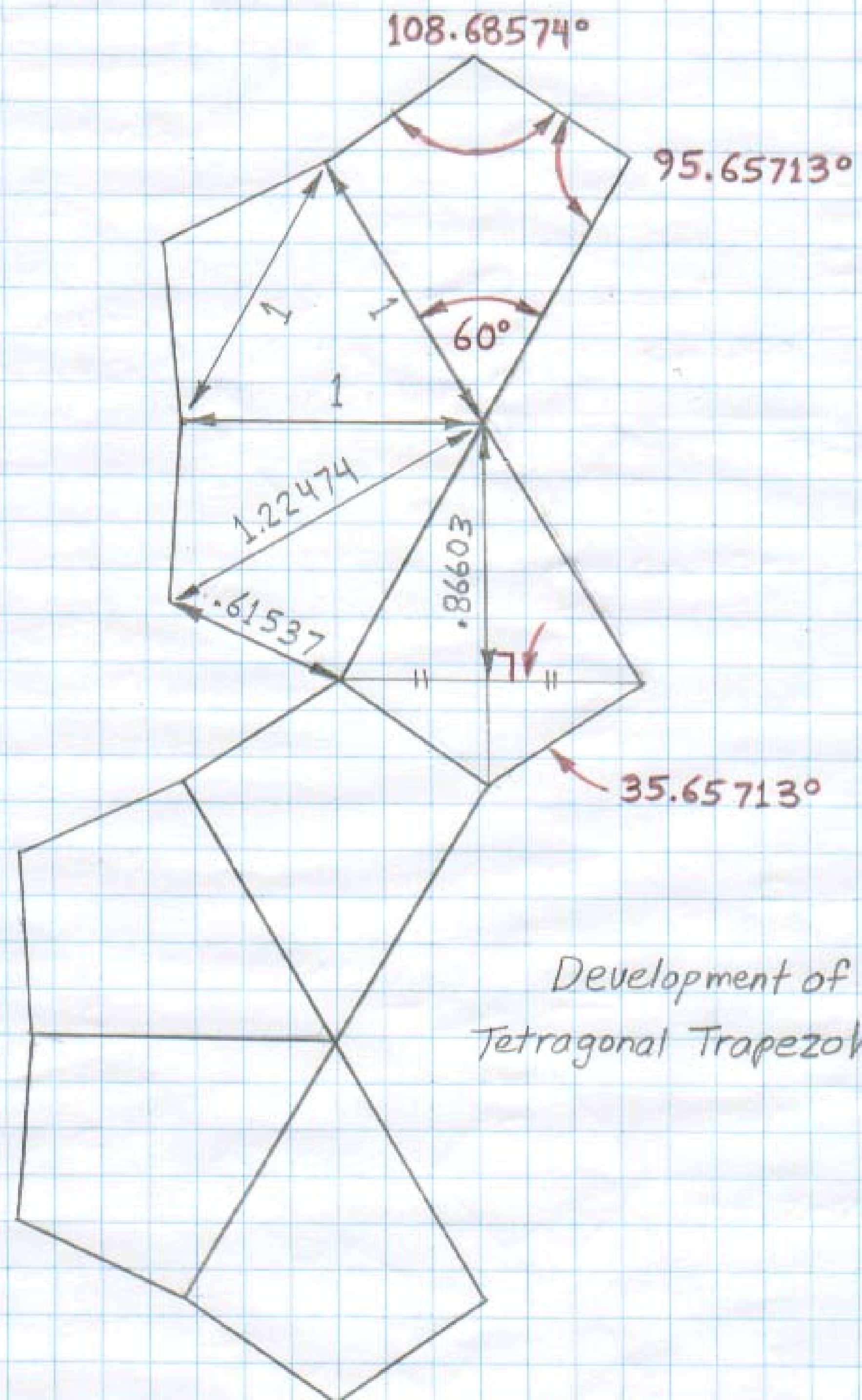
Studies of Trapezohedra

Tetragonal Trapezohedron: Irregular Kite Angles

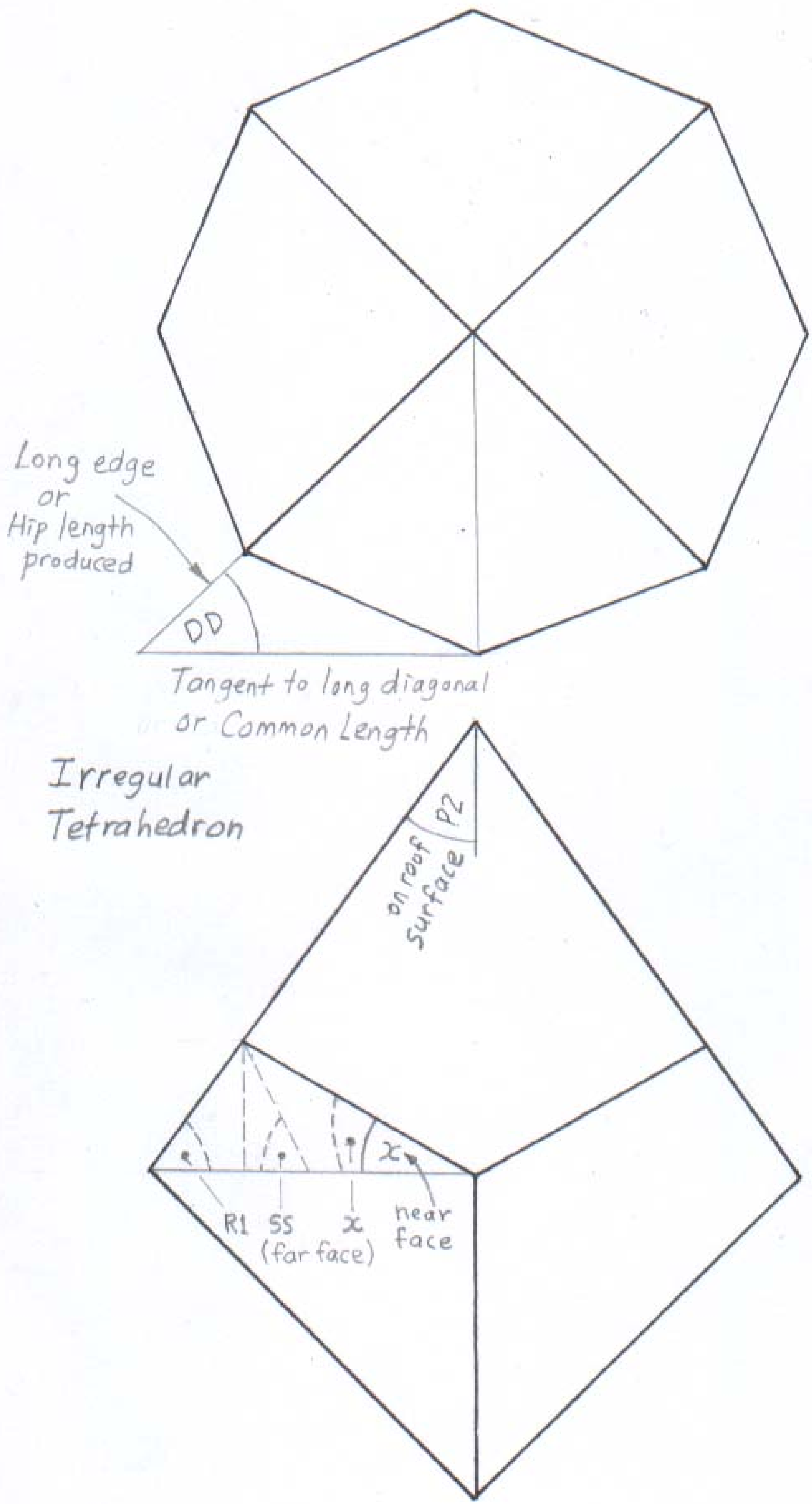
Trigonal Trapezohedron or Cube: 90° Kite Angles

Pentagonal Trapezohedron: Two 90° Kite Angles



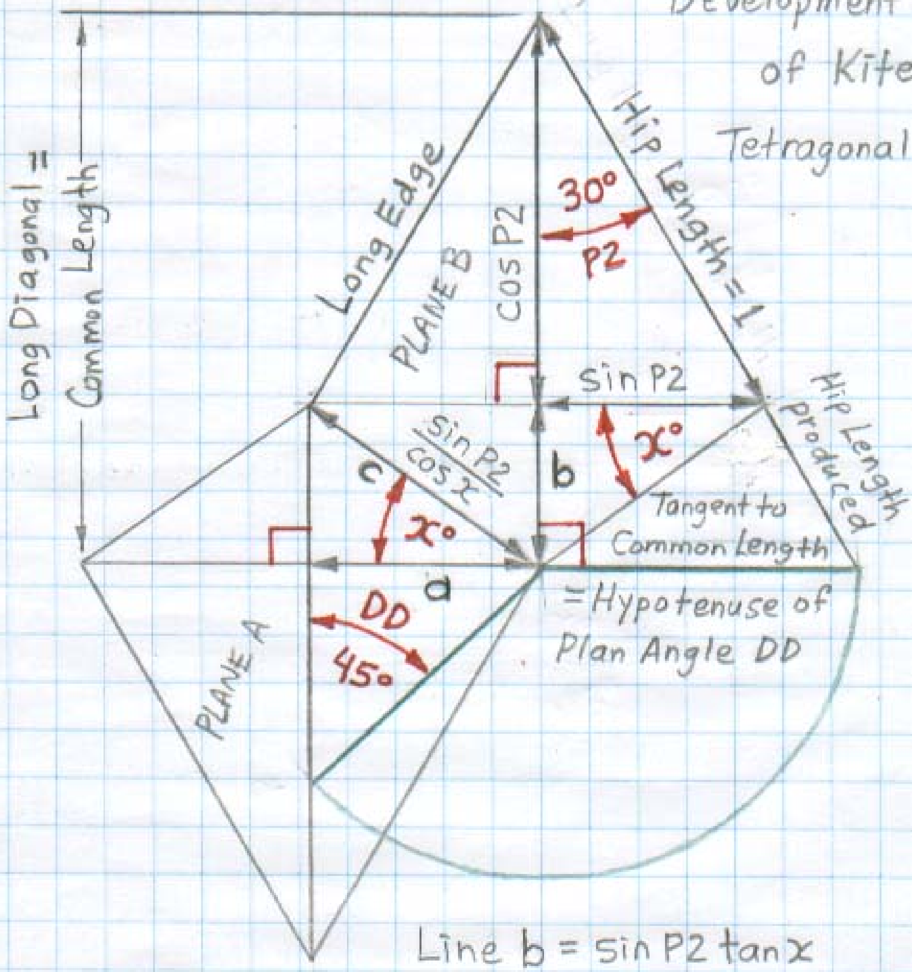
Development of
Tetragonal Trapezohedron

Plan and Elevation of Tetragonal Trapezohedron



Development and Analysis of Kite Angles

Tetragonal Trapezohedron



Line $b = \sin P2 \tan x$

Hypotenuse of Plan Angle $= (\sin P2 \tan x + \cos P2) \tan P2$

Line $a = (\sin P2 \tan x + \cos P2) \tan P2 \sin DD$

Line $c = (\sin P2 \tan x + \cos P2) \tan P2 \sin DD / \cos x$

Since Line c also equals $\sin P2 / \cos x$ we can write ...

$$\left[(\sin P2 \tan x + \cos P2) \tan P2 \sin DD - \sin P2\right] / \cos x = 0$$

For a graphical solution, substitute...

$P2 = \pi/6$ and $DD = \pi/4$ (angles are expressed in radians)

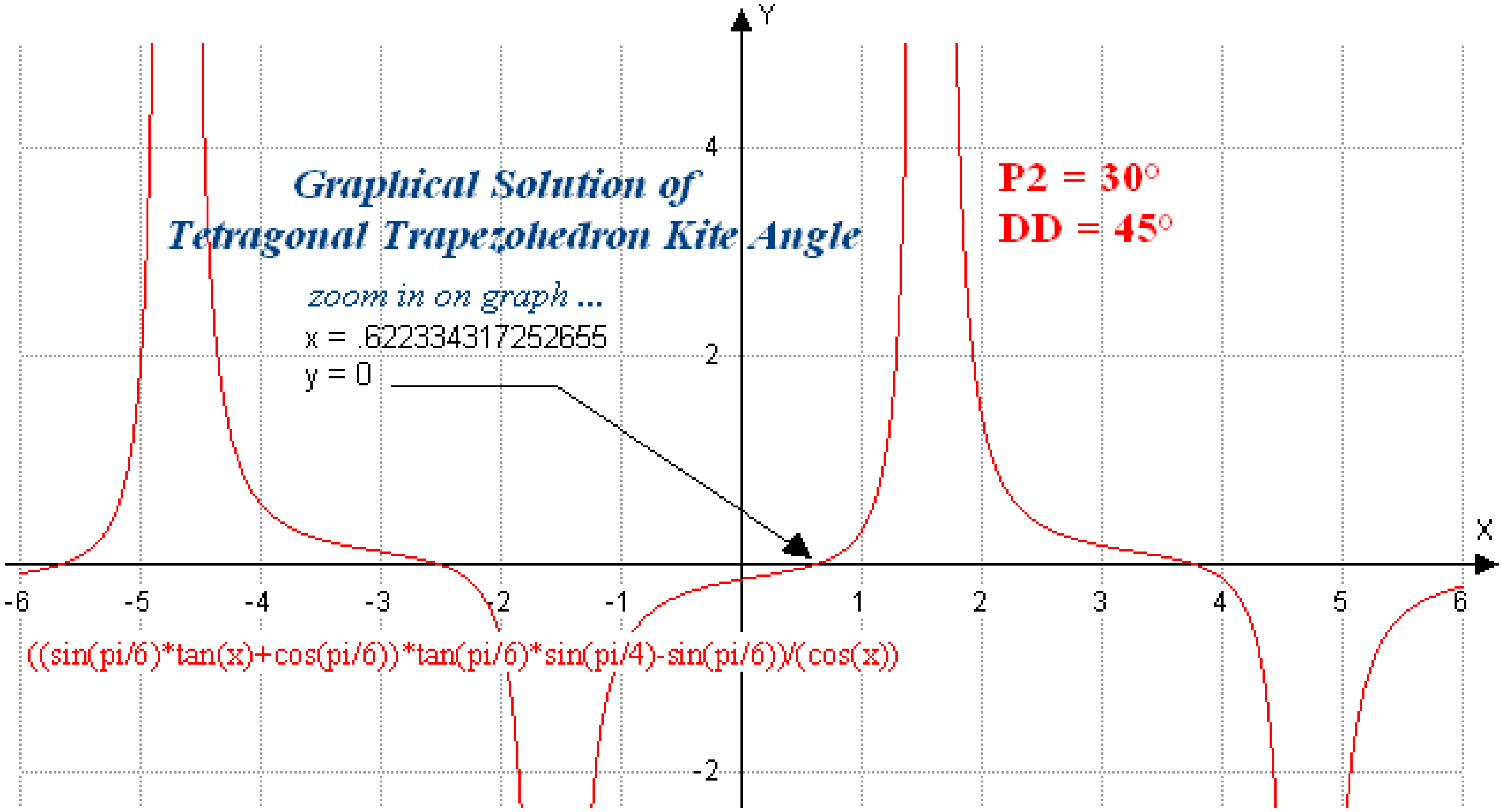
The formula reduces to the closed form...

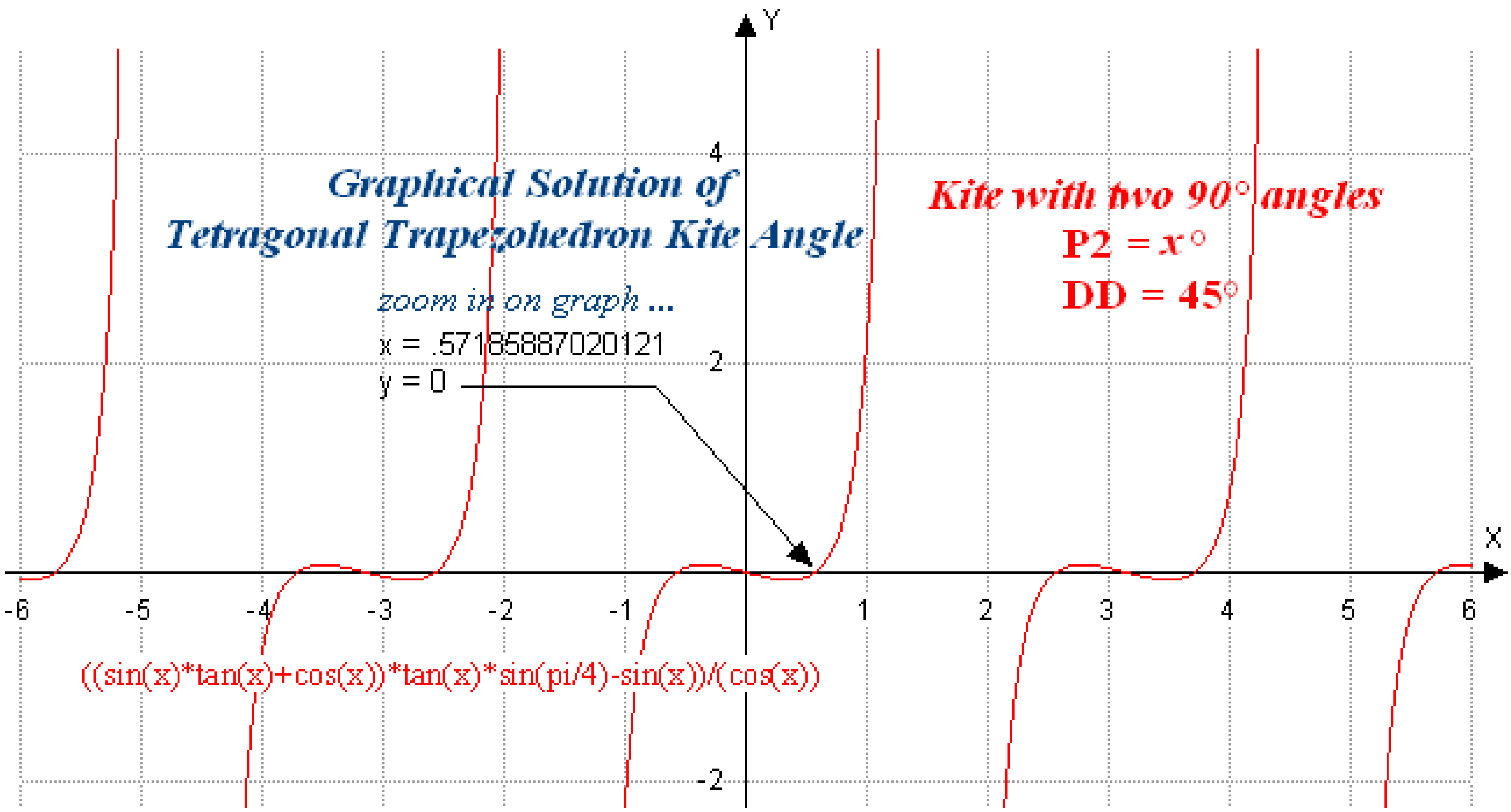
$$x = \arctan\left(\frac{1 - \sin DD}{\sin DD \tan P2}\right)$$

If the kite has two 90° corners then $P2 = x$ and...

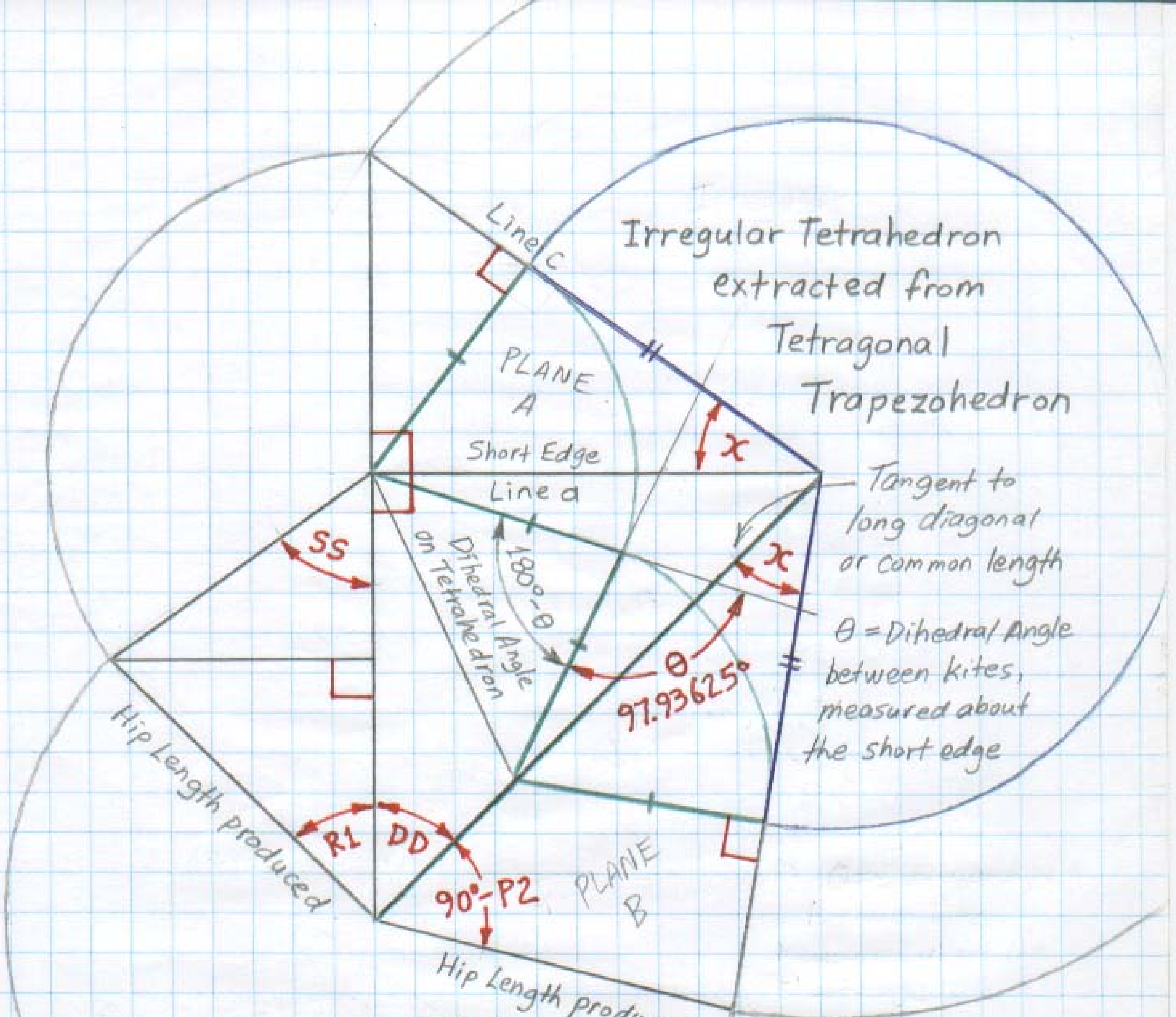
$$x = \arctan\sqrt{\frac{1 - \sin DD}{\sin DD}}$$

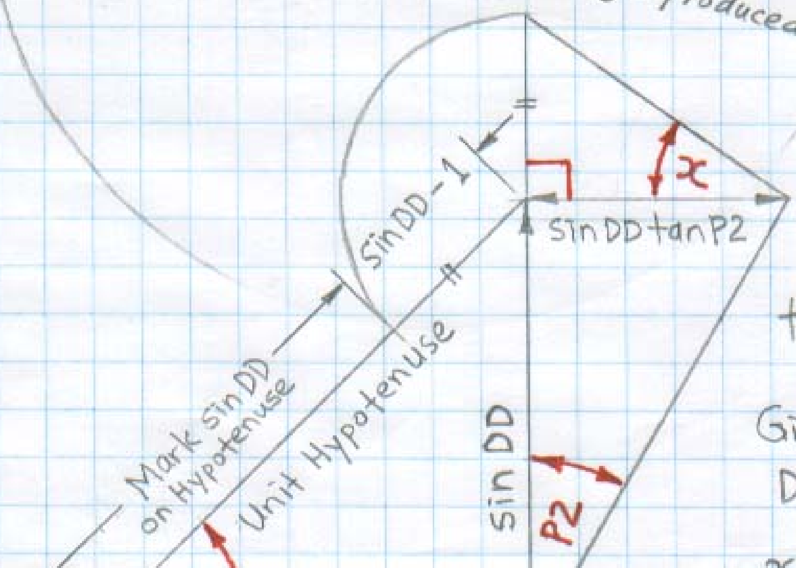
$$= \arctan\sqrt{\csc DD - 1}$$


Graphical Solution of
Tetragonal Trapezohedron Kite Angle
zoom in on graph ...
x = .622334317252655
y = 0
P2 = 30°
DD = 45°
((sin(pi/6)*tan(x)+cos(pi/6))*tan(pi/6)*sin(pi/4)-sin(pi/6))/(cos(x))


Graphical Solution of
Tetragonal Trapezohedron Kite Angle
zoom in on graph ...
x = .57185887020121
y = 0
Kite with two 90° angles
P2 = x°
DD = 45°
((sin(x)*tan(x)+cos(x))*tan(x)*sin(pi/4)-sin(x))/(cos(x))

Irregular Tetrahedron extracted from Tetragonal Trapezohedron





Solution of Kite Angle x

$$\tan x = \frac{1 - \sin DD}{\sin DD \tan P2}$$

Given Angles:
$DD = 45°$, $P2 = 30°$

$x = 35.65713°$

Views from the Vertices

Footprints and Hip Runs

b: Unit Vector

⊥ to PLANE B

$x = \cos 35.26439°$

$y = 0$

$z = \sin 35.26439°$

Intersection of 60° Angles

Square Footprint

Symmetrical

SS = 54.73561°

DD = 45°

P2 = 30°

C5 = 35.26439°

R1 = 45°

R4P = 35.26439°

A5P = 30°

c: Unit Vector

⊥ to PLANE C

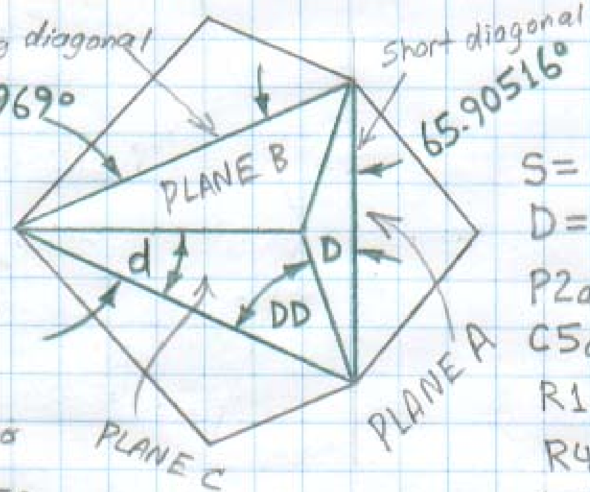
$x = 0$

$y = -\cos 35.26439°$

$z = \sin 35.26439°$

Angle between

b and **c** = 109.47122° = 180° − 2 × C5



s = 39.23152°

d = 24.09485°

p2 = 60°

c5 = 35.26439°

r1 = 18.43495°

r4p = 64.76060°

a5p = 16.77865°

S = 61.82944°

D = 18.71131°

P2a = 54.34287°

C5a = 56.61102°

R1 = 30.92280°

R4Pa = 68.45485°

A5Pa = 29.12538°

SS = 39.23152°

DD = 47.19385°

P2m = 35.65713°

C5m = 25.45273°

R1 = 30.92280°

R4Pm = 38.46920°

A5Pm = 20.43800°

Tetrahedron angles may be developed if desired

a: Unit Vector

⊥ to PLANE A

$x = \cos 35.26439° \cos 45°$

$y = -\cos 35.26439° \sin 45°$

$z = -\sin 35.26439°$

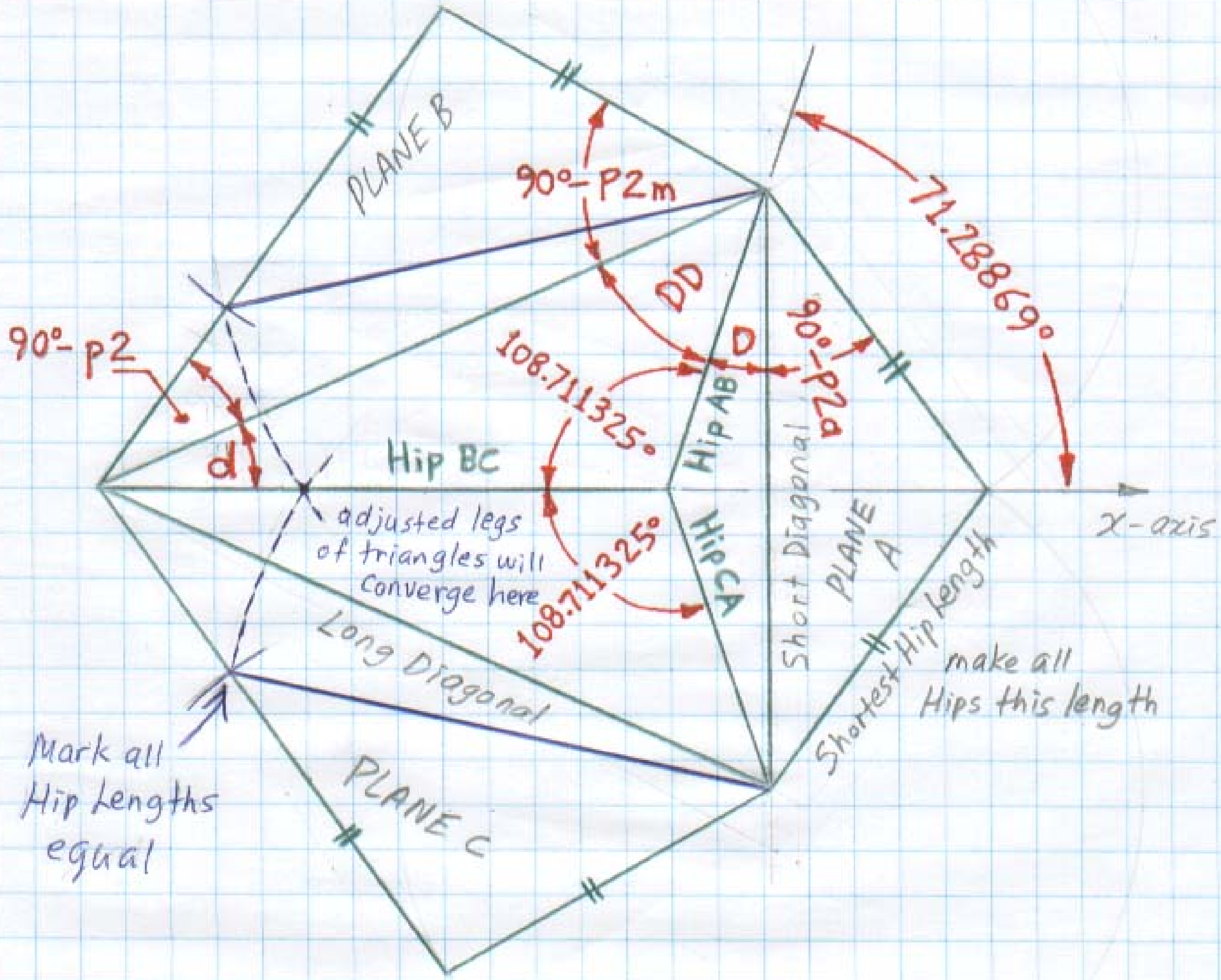
Angle between **a** and **b** = Angle between **a** and **c**

= 97.93625° = 180° − (C5m + C5a)

Tetragonal Trapezohedron

View from 95.65713°, 95.65713°, 108.68574° Vertex

Development of Plan and Roof Surface Angles



Three-Branch Compound Joint Calculator Entries

Hip AB: Rise = .599028339 Run = 1 Deck Angle = 71.28869 x-axis to AB

Hip BC: Rise = 1 Run = 3 Deck Angle = 108.711325° AB to BC

Hip CA: Rise = .599028339 Run = 1 Deck Angle = 108.711325° BC to CA

Tetragonal Trapezohedron

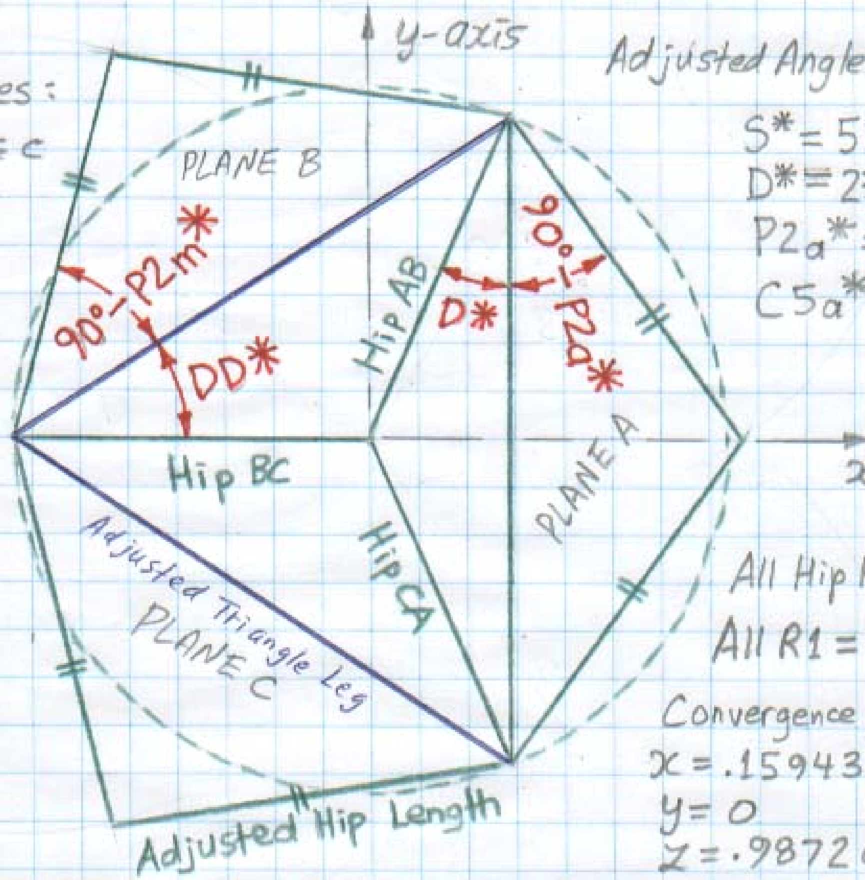
Inclined Deck View from 95.65713°, 95.65713°, 108.68574° vertex

Development of Convergent Plan and Roof Surface Angles

Adjusted Angles:
PLANE B and PLANE C

$SS^* = 43.653846°$
$DD^* = 33.240588°$
$P2_m^* = 47.828587°$
$C5_m^* = 35.264395°$

Sums of P2 angles and sums of C5 angles remain the same as for the level deck calculation.



Adjusted Angles: PLANE A

$S^* = 52.655482°$
$D^* = 23.518823°$
$P2_a^* = 54.342864°$
$C5_a^* = 46.799343°$

All Hip Runs are equal
All R1 = 27.608891°

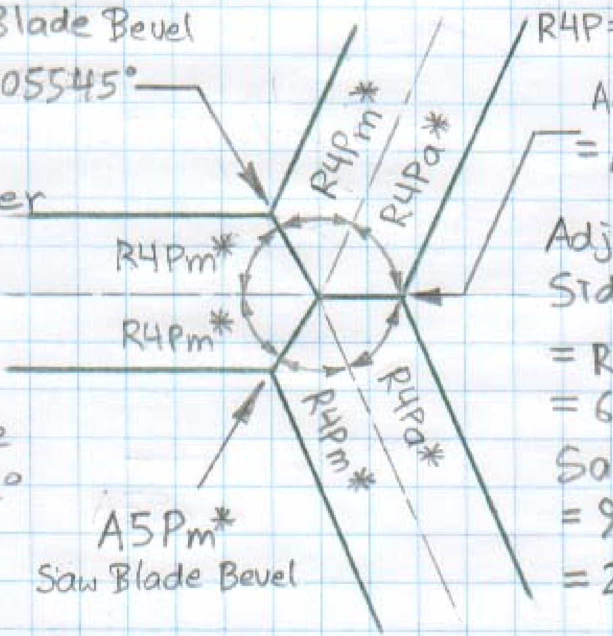
Convergence Vector:
$x = .159432$
$y = 0$
$z = .987209$

Angles for cutting dowels or unbacked Hip rafters

$A5P = \arcsin(\sin R1 \cos DD)$
$R4P = \arctan(\cos R1 / \tan DD)$

Adjusted Saw Blade Bevel
$= A5P_m^* = 22.805545°$

Adjusted Saw Blade Bevel
$= A5P_a^* = 25.146591°$



Adjusted Hip Rafter Side Cut Angle
$= R4P_m^*$
$= 53.51292°$

Saw Miter Angle
$= 90° - 53.51292°$
$= 36.48708°$

A5Pm*
Saw Blade Bevel

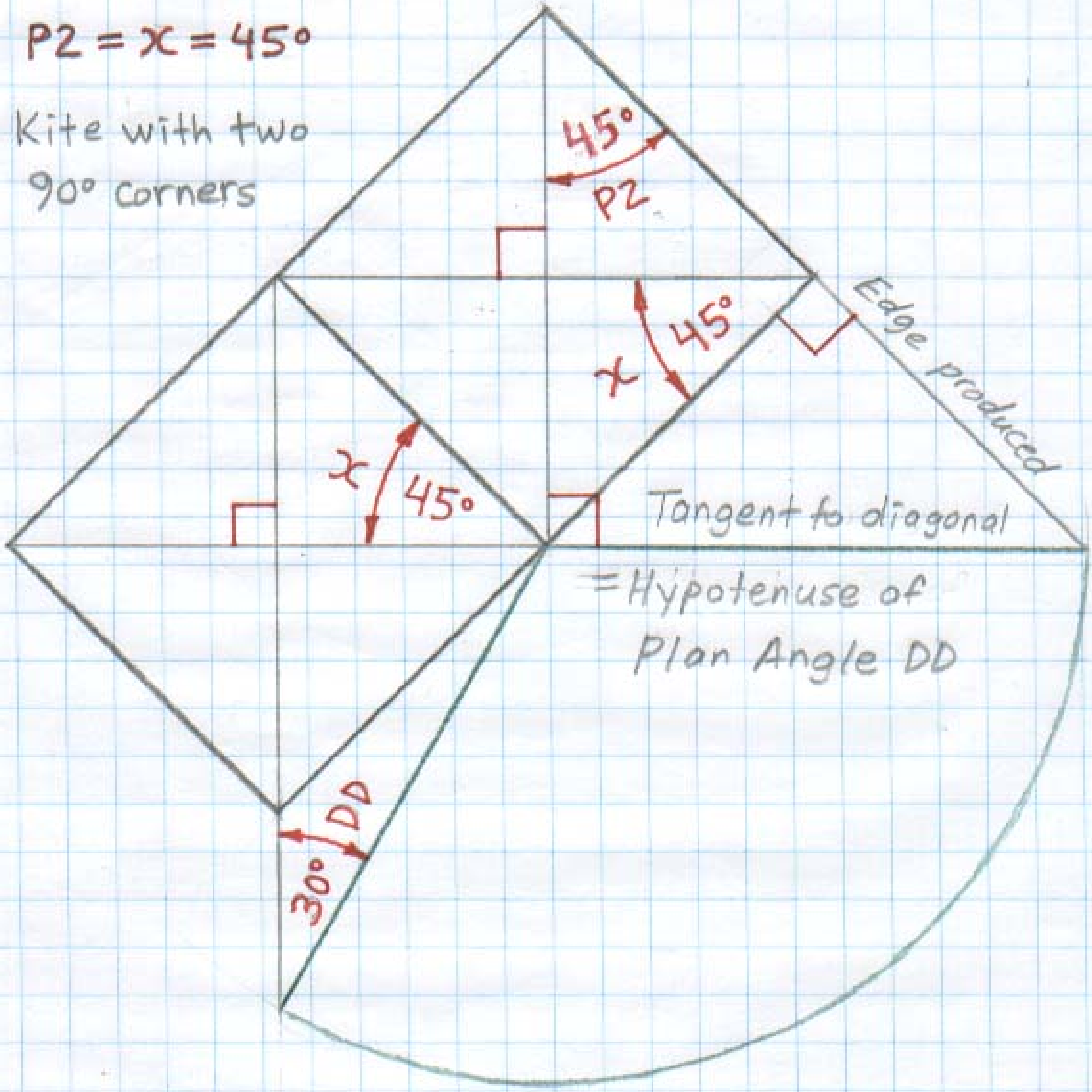
Adjusted Hip Rafter Side Cut Angle
$= R4P_a^*$
$= 63.84311°$

Saw Miter Angle
$= 90° - 63.84311°$
$= 26.15689°$

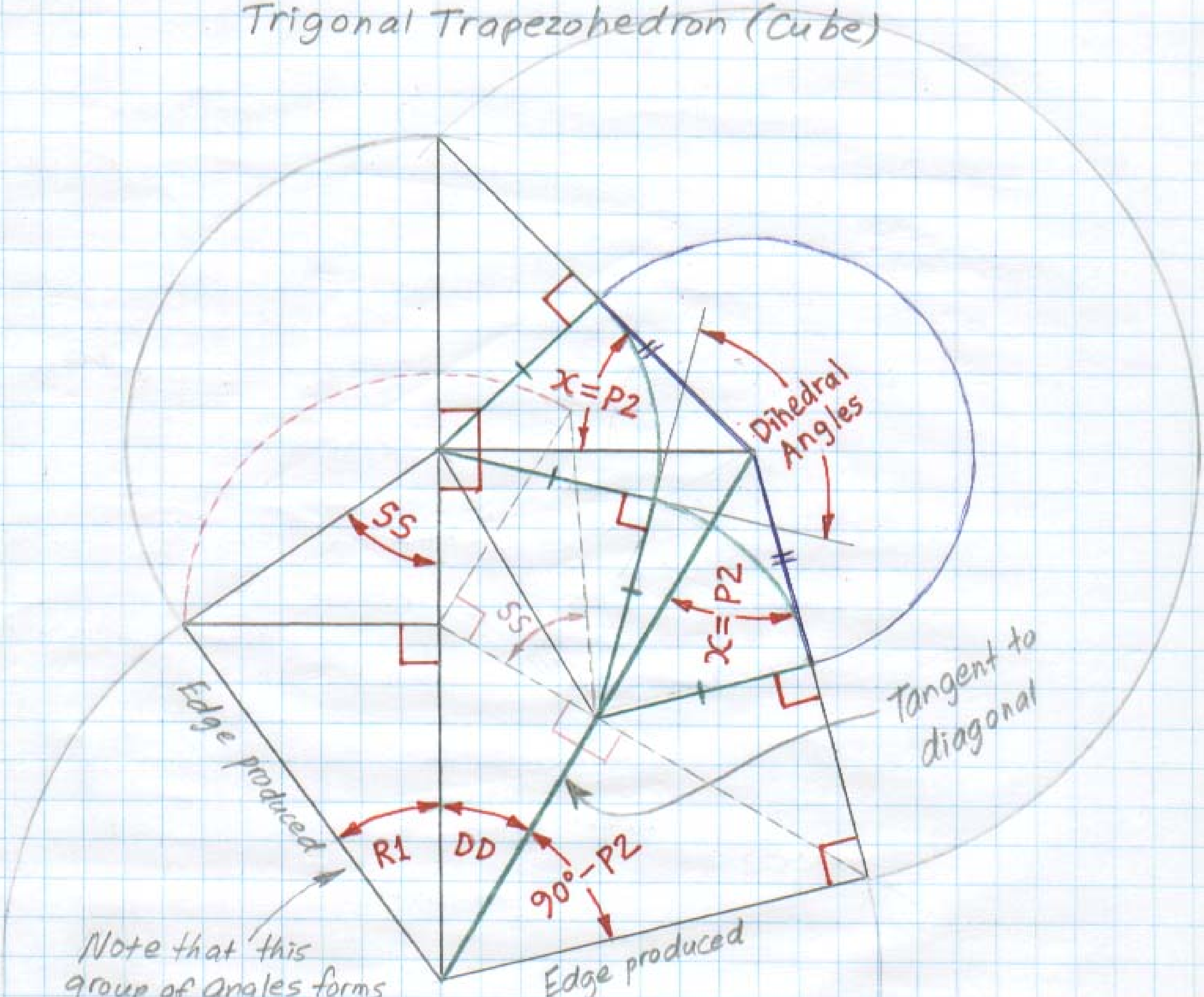
Development of Kite Angles
Trigonal Trapezohedron (Cube)

$P2 = x = 45°$

Kite with two
90° corners



Irregular Tetrahedron extracted from Trigonal Trapezohedron (Cube)



Note that this group of angles forms a standard kernel or tetrahedron

SS = 54.73561°

DD = 30°

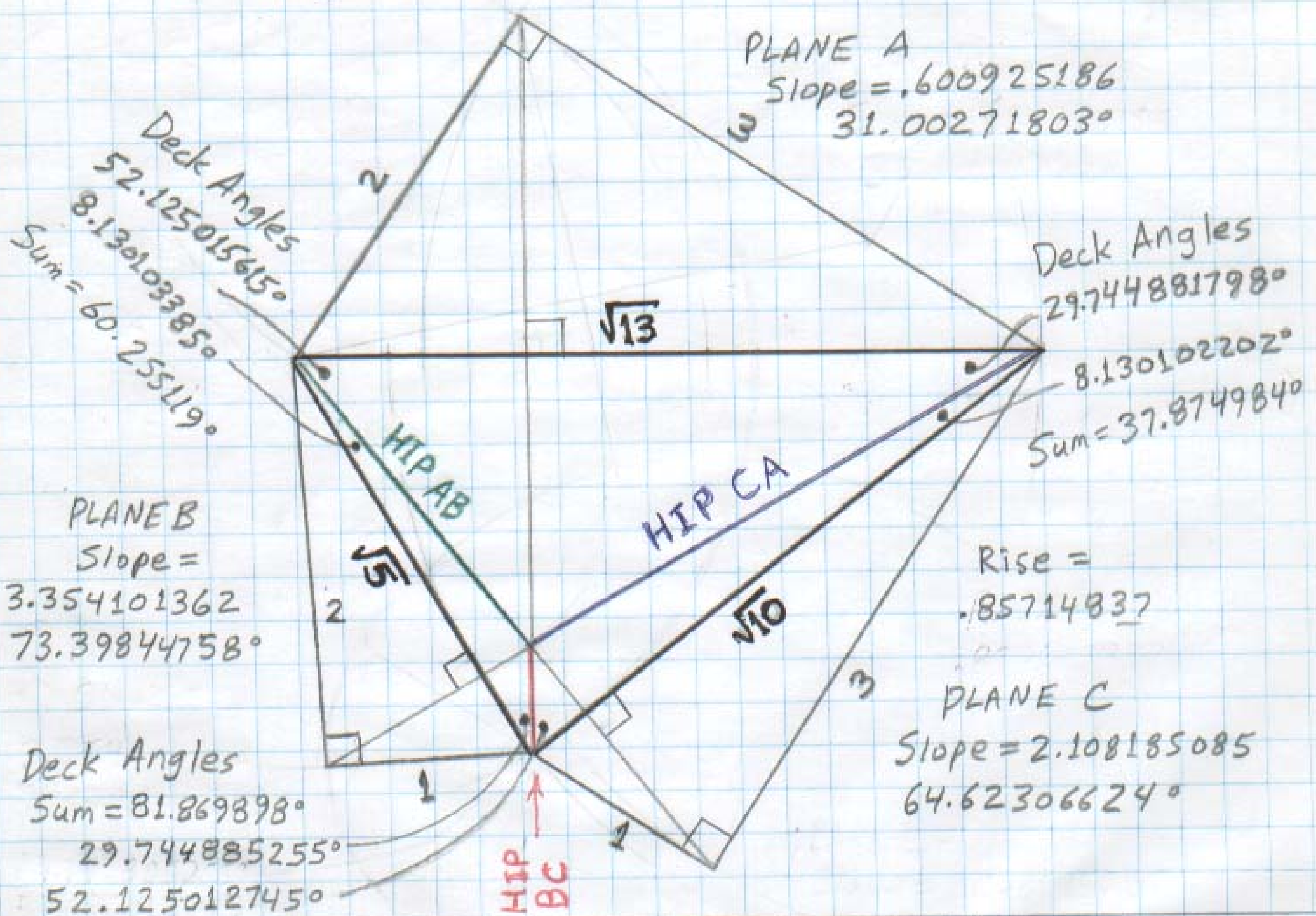
R1 = 35.26439°

P2 = x = 45°

C5 = 45°

Dihedral Angles = 180° - (2×C5) = 90°

Trirectangular Tetrahedron extracted from corner of Cube



Three-Branch Compound Joint Calculator Entries			Deck Angles
Hip AB	Slope = .474341632	25.37693273°	zero
Hip BC	Slope = 1.664100565	58.99728050°	142.1250114°
Hip CA	Slope = .29814239	16.60154924°	119.7448851°

Central Deck Angles... Hip AB to Hip BC = 142.1250114°

... Hip BC to Hip CA = 119.7448851°

... Hip CA to Hip AB = 98.1301026°

Convergence Vector

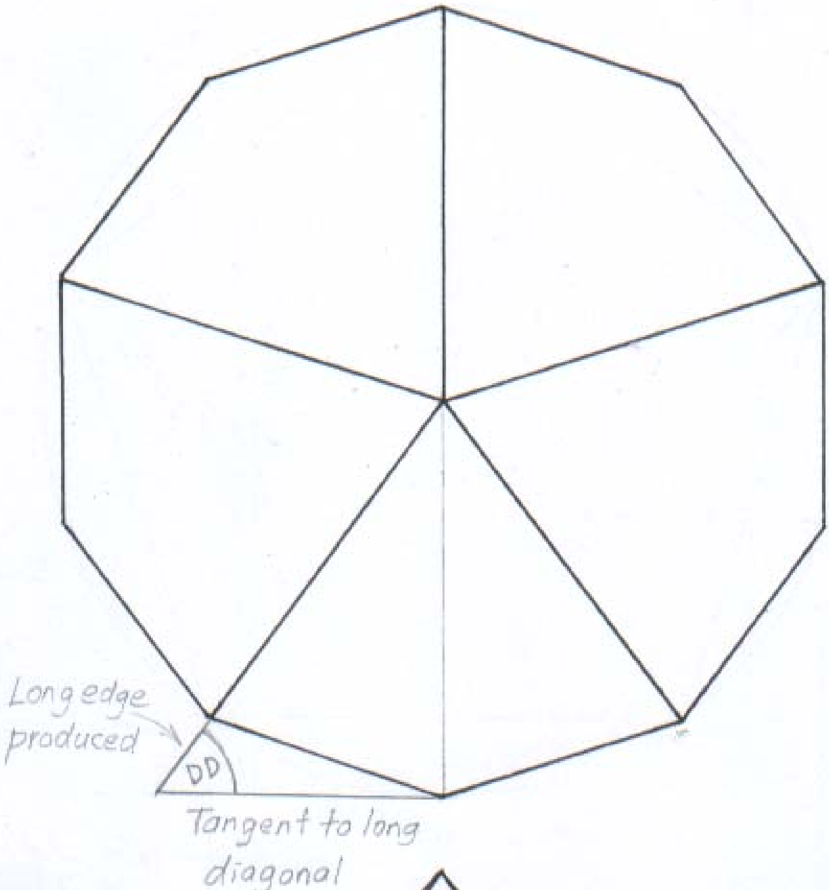
x = −.208656 y = .365148 z = .907265

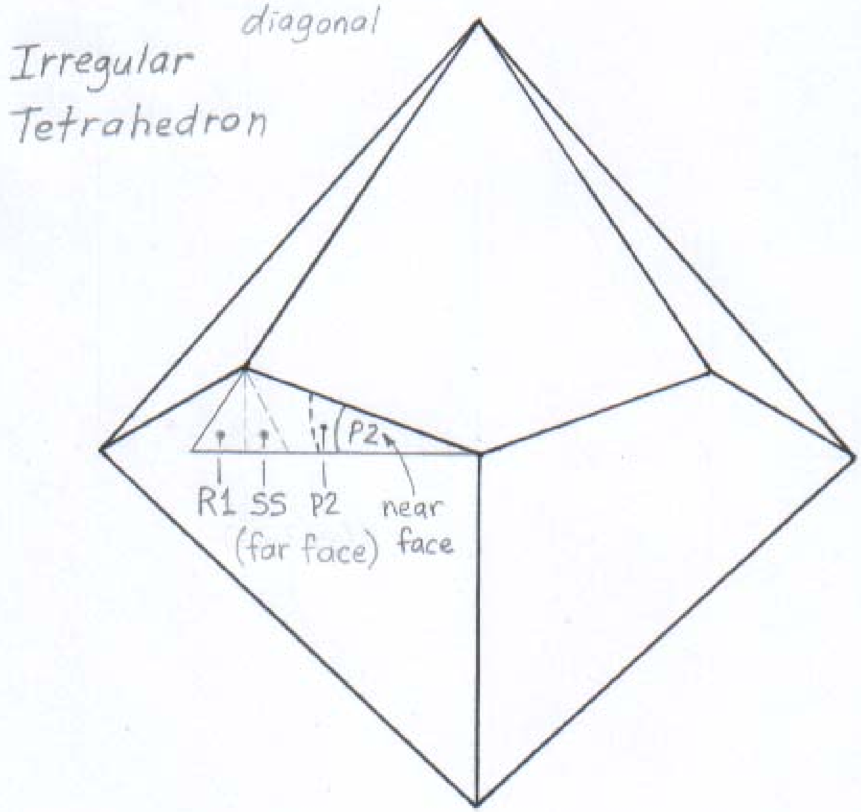
Angle to z-axis = 24.86995°

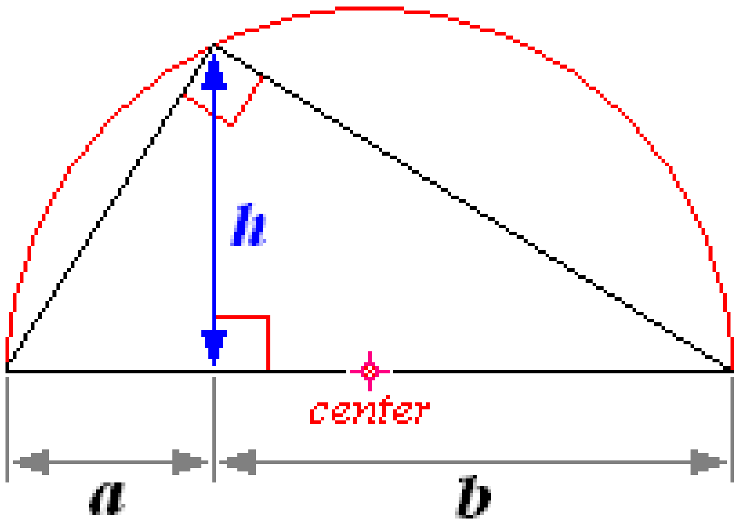
(Hip AB is on the x-axis)

SS = 54.73561°
DD = 30°
R1 = 35.26439°
P2 = 45°
CS = 45°

Plan and Elevation of Pentagonal Trapezohedron







An angle inscribed in a semicircle is a right angle

The two triangles formed by altitude h are similar

h is the geometric mean between a and b

$$\frac{a}{h} = \frac{h}{b}$$

$$h^2 = a\,b$$

$$h = \sqrt{a\,b}$$

$$\text{If } a = 1\text{, then } h = \sqrt{b}\text{ , and } b = h^2$$

$$\text{If } b = 1\text{, then } h = \sqrt{a}\text{ , and } a = h^2$$

Construction of Kite Angle P2 to produce Kites with two 90° angles for a

Pentagonal Trapezohedron

(1) Construct circle of unit radius and plan angle DD=54°

(2) Construct a tangent to the unit circle at point A

(3) Line segment OI, the distance from the origin to the intersect of the tangent with the y-axis, equals csc DD

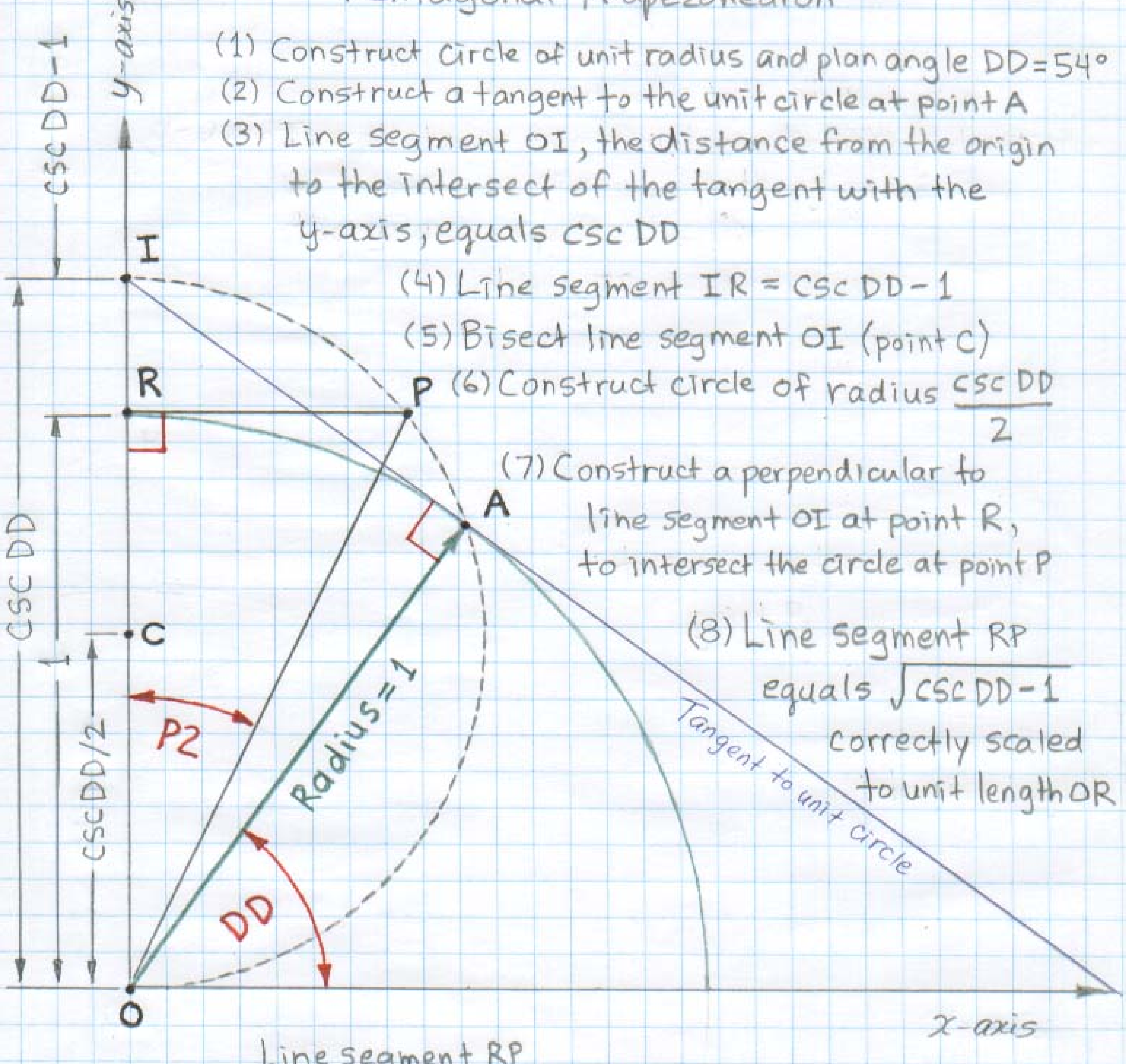
(4) Line segment IR = csc DD − 1

(5) Bisect line segment OI (point C)

(6) Construct circle of radius $\frac{\csc DD}{2}$

(7) Construct a perpendicular to line segment OI at point R, to intersect the circle at point P

(8) Line segment RP equals $\sqrt{\csc DD - 1}$ correctly scaled to unit length OR



$$\frac{\text{Line segment RP}}{\text{Line segment OR}} = \tan P2$$

$$P2 = \arctan\sqrt{\csc DD - 1} = 25.91365°$$

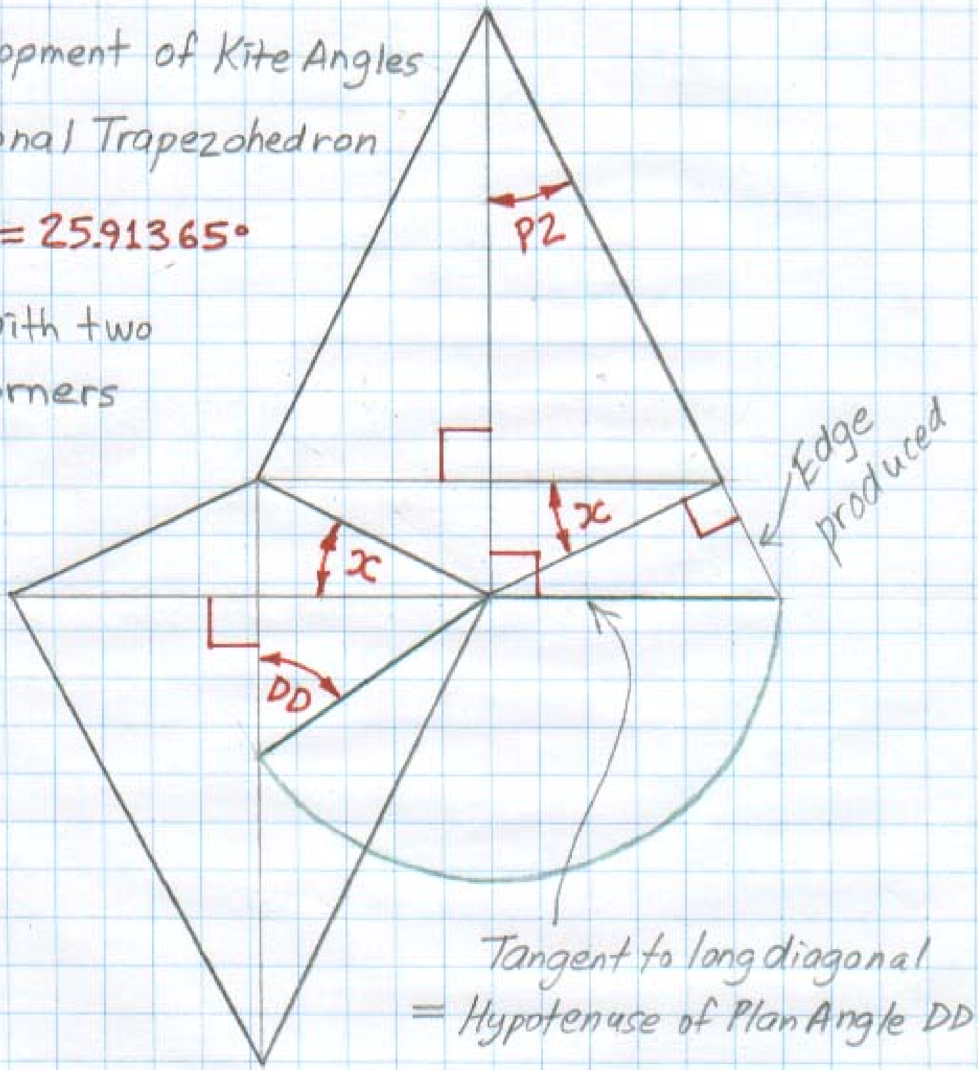
Development of Kite Angles

Pentagonal Trapezohedron

$P2 = x = 25.91365°$

Kite with two

90° corners

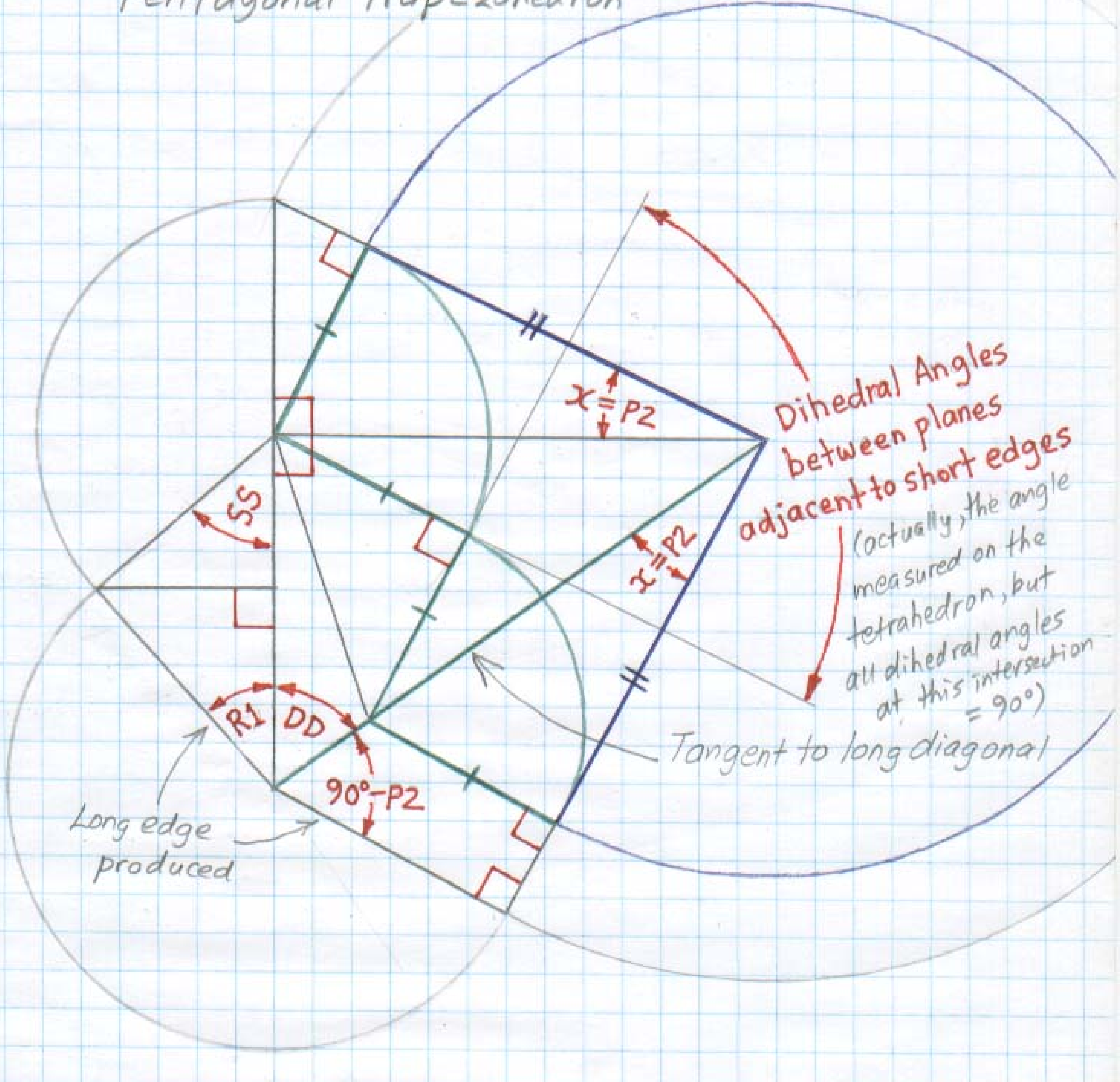


Dihedral Angle between planes adjacent to the long edge:

Since $C5 = P2$: $180° - (2 \times 25.91365°) = 128.17271°$

Dihedral Angle between planes adjacent to Short edge = 90°

Irregular Tetrahedron extracted from Pentagonal Trapezohedron



SS = 48.03008°

DD = 54°

R1 = 41.96992°

P2 = X = 25.91365°

CS = 25.91365°

Dihedral Angle, planes adjacent to long edge:

180° − (2×CS) = 128.17271°