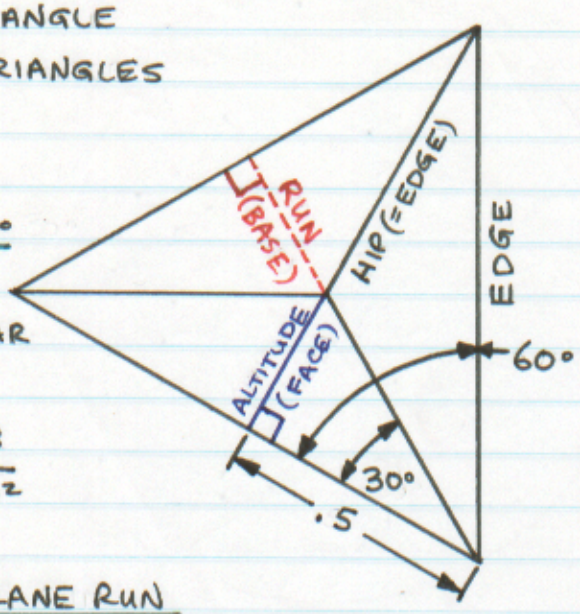
TETRAHEDRON:

BASE: EQUILATERAL TRIANGLE

FACES: EQUILATERAL TRIANGLES

SIDES: 3

EDGES (= HIPS): 1



PLANE RUN = .5 × tan 30°

= .2886751346

ALTITUDE OF TRIANGULAR FACE = .5 × tan 60°

= .8660254038

RISE = $\sqrt{\text{ALTITUDE}^2 - \text{RUN}^2}$

= .8164965809

PLANE PITCH = RISE / PLANE RUN

= 2.828427125 (= $2\times\sqrt{2}$)

LENGTH UNDER HIP (HIP RUN)

= $\sqrt{\text{HIP}^2 - \text{RISE}^2}$

= .5773502692

HIP ANGLE TO PLANE = arc sin (RISE/HIP) = arcos (HIP RUN/HIP)

= 54.73561032°

PROGRAM CALCULATION:

ENTER: SIDES = 3

PLANE PITCH = $2\sqrt{2}$

RUN = .2886751346

EDGE = 1

DISPLAY: RISE = .8164965809

ALTITUDE = .8660254038

HIP = 1

HIP ∠° = 54.73561032°

PLANE ∠° = 35.26438968°

INCLUDED ANGLE BETWEEN FACES $\cong$ 35.26° × 2 = 70.52°

OUTSIDE ANGLE BETWEEN ADJACENT FACES

= 180° − 70.52° = 109.48°

CUBE:

BASE: EQUILATERAL TRIANGLE

FACES: ISOSCELES TRIANGLES

SIDES: 3

EDGES = 1.414213562 (= $\sqrt{2}$)

PLANE RUN = .5 x $\sqrt{2}$ x tan 30°

= .4082482905

ALTITUDE OF TRIANGULAR FACE = .5 X EDGE

= .7071067812

RISE = $\sqrt{\text{ALTITUDE}^2 - \text{RUN}^2}$

= .5773502692

PLANE PITCH = RISE / PLANE RUN

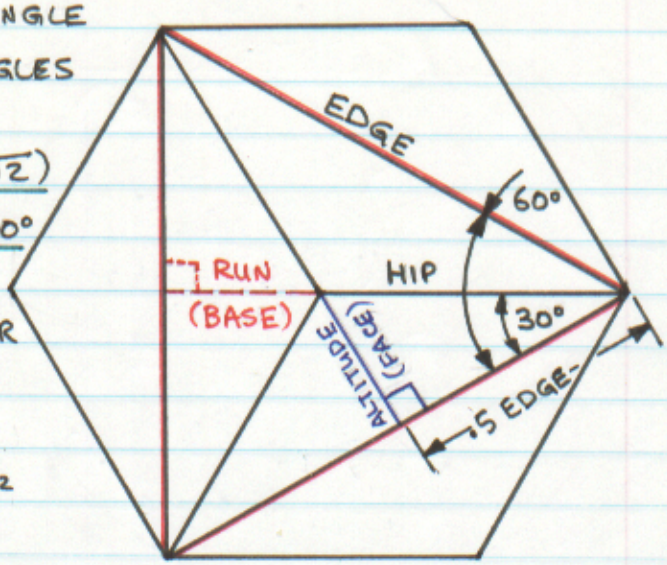
= 1.414213562

LENGTH UNDER HIP (HIP RUN) = $\sqrt{\text{HIP}^2 - \text{RISE}^2}$

= .8164965809

HIP ANGLE TO PLANE = arcsin (RISE/HIP) = arccos (HIP RUN/HIP)

= 35.26438968°



EDGES FORMING BASE OF TETRAHEDRON = $\sqrt{2}$

PROGRAM CALCULATION:

ENTER: SIDES = 3
PLANE PITCH = $\sqrt{2}$
RUN = .4082482905
EDGE = $\sqrt{2}$

DISPLAY: RISE = .5773502692
ALTITUDE = .7071067812
HIP = 1
HIP ∠° = 35.26438969°
PLANE ∠° = 45°

INCLUDED ANGLE BETWEEN FACES = 45° x 2 = 90°

OUTSIDE ANGLE BETWEEN ADJACENT FACES = 180° − 90° = 90°

OCTAHEDRON:

BASE: SQUARE

FACES: EQUILATERAL TRIANGLES

SIDES: 4

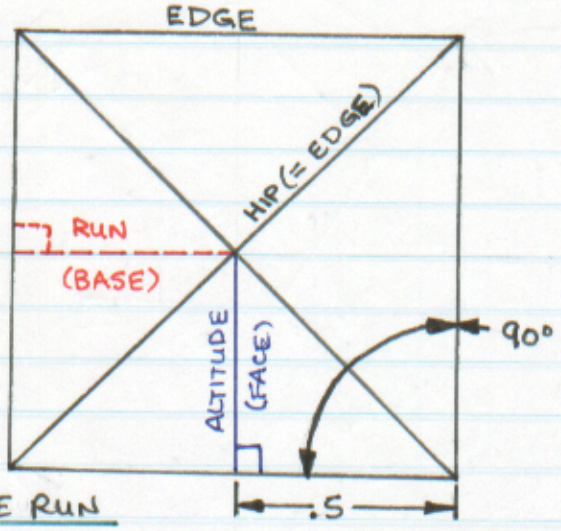
EDGES(=HIPS): 1

PLANE RUN = .5 x EDGE = .5

ALTITUDE OF TRIANGULAR FACE = .5 x tan 60°
= .8660254038

RISE = $\sqrt{\text{ALTITUDE}^2 - \text{RUN}^2}$
= .7071067812

PLANE PITCH = RISE / PLANE RUN
= 1.414213562



LENGTH UNDER HIP (HIP RUN)
= $\sqrt{\text{HIP}^2 - \text{RISE}^2}$
= .7071067814

HIP ANGLE TO PLANE = arcsin(RISE/HIP) = arcos(HIP RUN / HIP)
= 45°

PROGRAM CALCULATION:

ENTER: SIDES = 4
PLANE PITCH = 1.414213562
RUN = .5
EDGE = 1

DISPLAY: RISE = .707106781
ALTITUDE = .866025403 6
HIP = .9999999999
HIP ∠° = 44.99999999°
PLANE ∠° = 54.73561032

INCLUDED ANGLE BETWEEN FACES ≅ 54.74° x 2 = 109.48°

OUTSIDE ANGLE BETWEEN ADJACENT FACES
= 180° − 109.48° = 70.52°

DODECAHEDRON:

BASE: EQUILATERAL TRIANGLE

FACES: ISOSCELES TRIANGLES

SIDES: 3

EDGES = 1.618033988

PLANE RUN = $\frac{.5 \times \text{EDGE} \times \tan 60°}{3}$

$= .4670861797$

ALTITUDE OF TRIANGULAR FACE $= \sqrt{1^2 - (.5\ \text{EDGE})^2}$

$= .5877852521$

RISE $= \sqrt{\text{ALTITUDE}^2 - \text{RUN}^2}$

$= .3568220892$

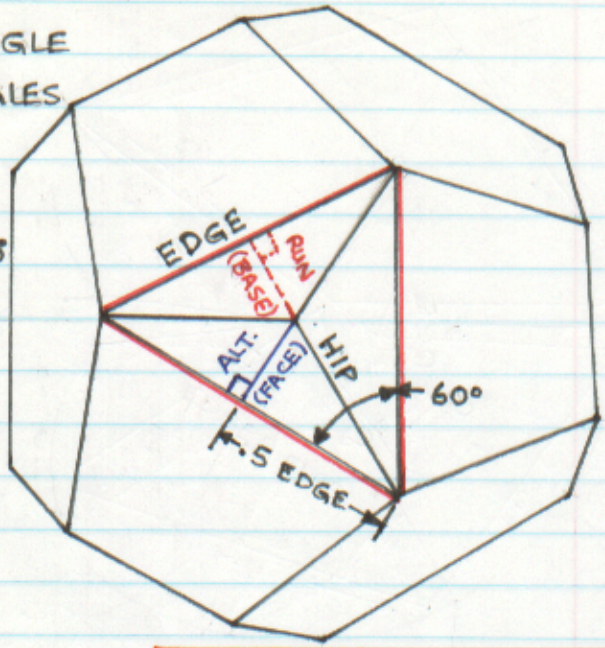
PLANE PITCH = RISE/PLANE RUN

$= .7639320209$

LENGTH UNDER HIP (HIP RUN)

$= \sqrt{\text{HIP}^2 - \text{RISE}^2}$

$= .9341723592$



IF THE EDGE OF A REGULAR PENTAGON = 1, THE DIAGONAL = 1.618033988 (GOLDEN RATIO). THIS WILL FORM THE EDGE OF THE TETRAHEDRON BASE.

HIP ANGLE TO PLANE = arcsin(RISE/HIP) = arcos(HIP RUN/HIP)

$= 20.90515741°$

PROGRAM CALCULATION

ENTER: SIDES = 3

PLANE PITCH = .7639320209

RUN = .4670861797

EDGE = 1.618033988

DISPLAY: RISE = .3568220892

ALTITUDE = .5877852521

HIP = .9999999996

HIP ∠° = 20.90515742°

PLANE ∠° = 58.28252562°

INCLUDED ANGLE BETWEEN FACES $\cong 58.28° \times 2 = 116.56°$

OUTSIDE ANGLE BETWEEN ADJACENT FACES

$= 180° - 116.56° = 63.44°$

ICOSAHEDRON:

BASE: EQUILATERAL PENTAGON

FACES: EQUILATERAL TRIANGLES

SIDES: 5

EDGES (=HIPS): 1

PLANE RUN = .5 x tan 54°
= .6881909602

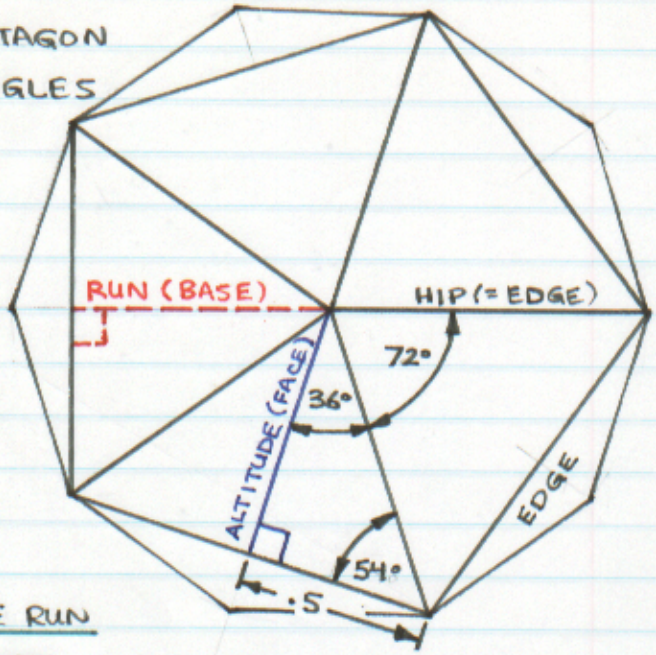
ALTITUDE OF TRIANGULAR FACE = .5 x tan 60°
= .8660254038

RISE = $\sqrt{\text{ALTITUDE}^2 - \text{RUN}^2}$
= .5257311122

PLANE PITCH = RISE / PLANE RUN
= .7639320227

LENGTH UNDER HIP (HIP RUN)
= $\sqrt{\text{HIP}^2 - \text{RISE}^2}$
= .8506508083

HIP ANGLE TO PLANE = arcsin(RISE/HIP) = arcos(HIP RUN / HIP)
= 31.71747442°



PROGRAM CALCULATION:

ENTER: SIDES = 5
PLANE PITCH = .7639320227
RUN = .6881909602
EDGE = 1

DISPLAY: RISE = .5257311122
ALTITUDE = .8660254038
HIP = 1
HIP ∠° = 31.71747442°
PLANE ∠° = 69.09484255°

INCLUDED ANGLE BETWEEN FACES ≅ 69.09° x 2 = 138.18°

OUTSIDE ANGLE BETWEEN ADJACENT FACES
= 180° − 138.18° = 41.82°

REGULAR POLYHEDRA DATA

	ICOSAHEDRON	DODECAHEDRON	OCTAHEDRON	CUBE	TETRAHEDRON
BASE	EQUILATERAL PENTAGON	EQUILATERAL TRIANGLE	SQUARE	EQUILATERAL TRIANGLE	EQUILATERAL TRIANGLE
FACES	EQUILATERAL TRIANGLES	ISOSCELES TRIANGLES	EQUILATERAL TRIANGLES	ISOSCELES TRIANGLES	EQUILATERAL TRIANGLES
NUMBER OF FACES	5	3	4	3	3
EDGES	1	1.6180	1	1.4142	1
HIPS	1	1	1	1	1
PLANE RUN	.6882	.4671	.5000	.4028	.2887
ALTITUDE OF FACES	.8660	.5878	.8660	.7071	.8660
RISE	.5257	.3568	.7071	.5774	.8615
PLANE PITCH	.7639	.7639	1.4142	1.4142	2.8284
HIP RUN	.8507	.9342	.7071	.8165	.5774
HIP PITCH	.6180	.3819	1	.7071	1.4142
HIP ANGLE FROM BASE	31.72°	20.91	45°	35.26°	54.74°
ANGLE BETWEEN ADJACENT FACES	138.18°	116.56	109.48°	90°	70.52°
VERTEX ANGLES FACE TO OPPOSITE EDGE	110.91°	121.72°		90°	54.74°
OPPOSITE FACES			70.53°		
OPPOSITE EDGES			90°		

PLATONIC SOLIDS

ICOSAHEDRON

Deck: Regular Pentagon	Deck Angle = 108°
Faces: 5 × Equilateral Triangle	Pitch = .763932
SS = 37.37737°	**DD** = 54.00000°
R1 = 31.71747°	90° – **P2** = 60.00000°

20 × Equilateral Triangle, Saw Blade Bevel = C5 = 20.90516°

DODECAHEDRON

Deck: Equilateral Triangle	Deck Angle = 60°
Faces: 3 × Isosceles Triangle	Pitch = .763932
SS = 37.37737°	**DD** = 30.00000°
R1 = 20.90516°	90° – **P2** = 36.00000°

12 × Regular Pentagon, Saw Blade Bevel = C5 = 31.71747°

OCTAHEDRON

Deck: Square	Deck Angle = 90°
Faces: 4 × Equilateral Triangle	Pitch = 1.414214
SS = 54.73562°	**DD** = 45.00000°
R1 = 45.00000°	90° – **P2** = 60.00000°

8 × Equilateral Triangle, Saw Blade Bevel = C5 = 35.26439°

CUBE

Deck: Equilateral Triangle	Deck Angle = 60°
Faces: 3 × Isosceles Triangle	Pitch = 1.414214
SS = 54.73562°	**DD** = 30.00000°
R1 = 35.26439°	90° – **P2** = 45.00000°

6 × Square, Saw Blade Bevel = C5 = 45.00000°

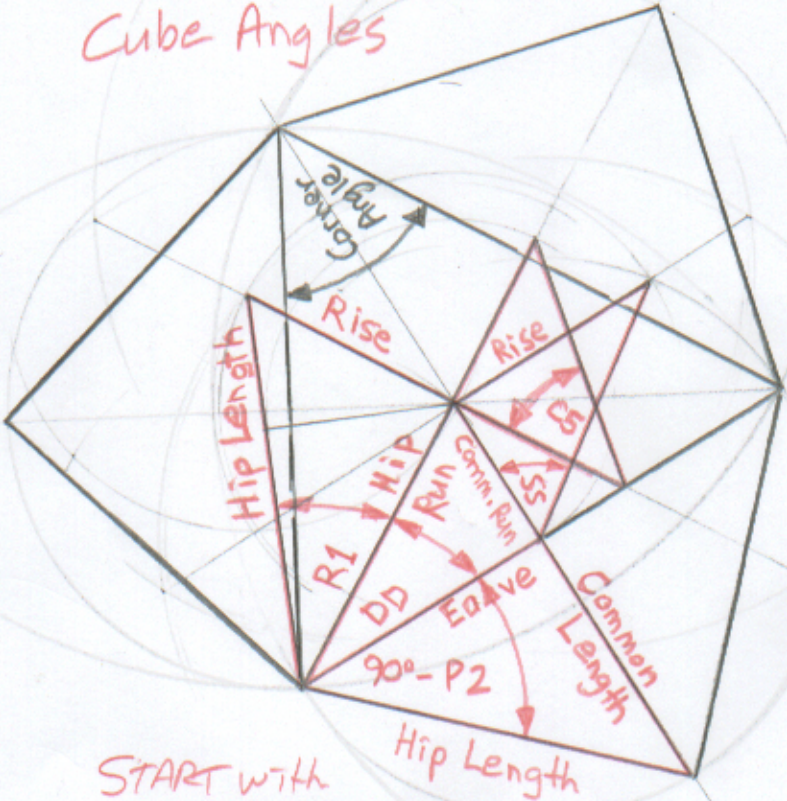
TETRAHEDRON

Deck: Equilateral Triangle	Deck Angle = 60°
Faces: 3 × Equilateral Triangle	Pitch = 2.828427
SS = 70.52878°	**DD** = 30.00000°
R1 = 54.73561°	90° – **P2** = 60.00000°

4 × Equilateral Triangle, Saw Blade Bevel = C5 = 54.73561°

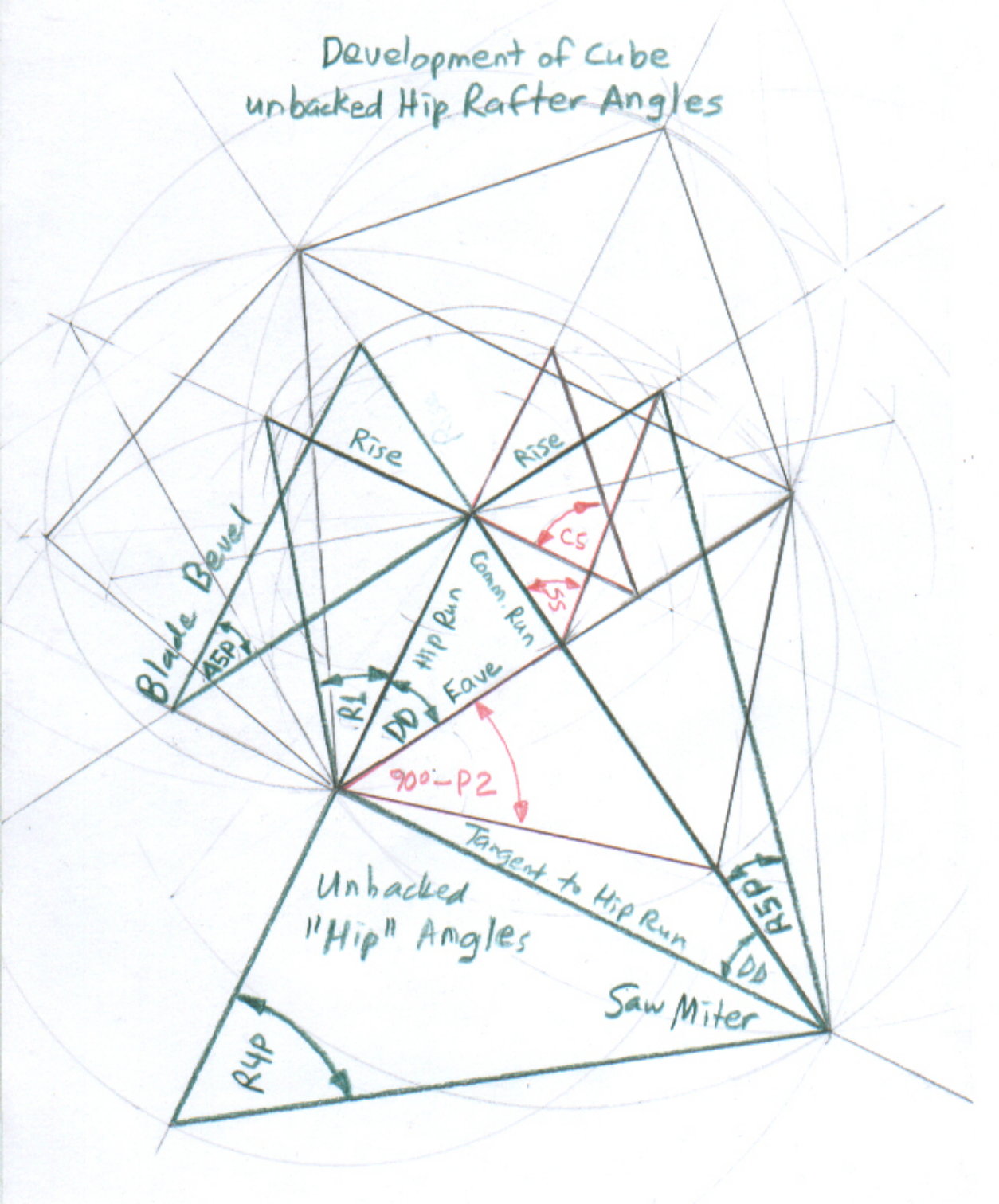
(Roof Tetrahedron)

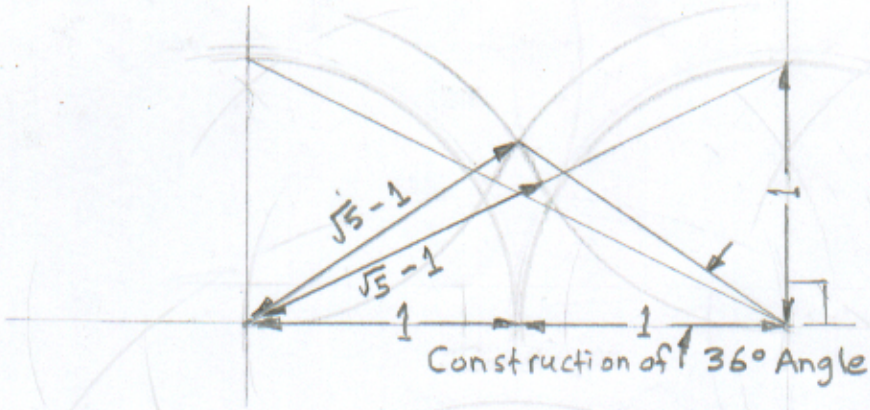
Development of Cube Angles



START with
DD and 90°-P2

Development of Cube
unbacked Hip Rafter Angles

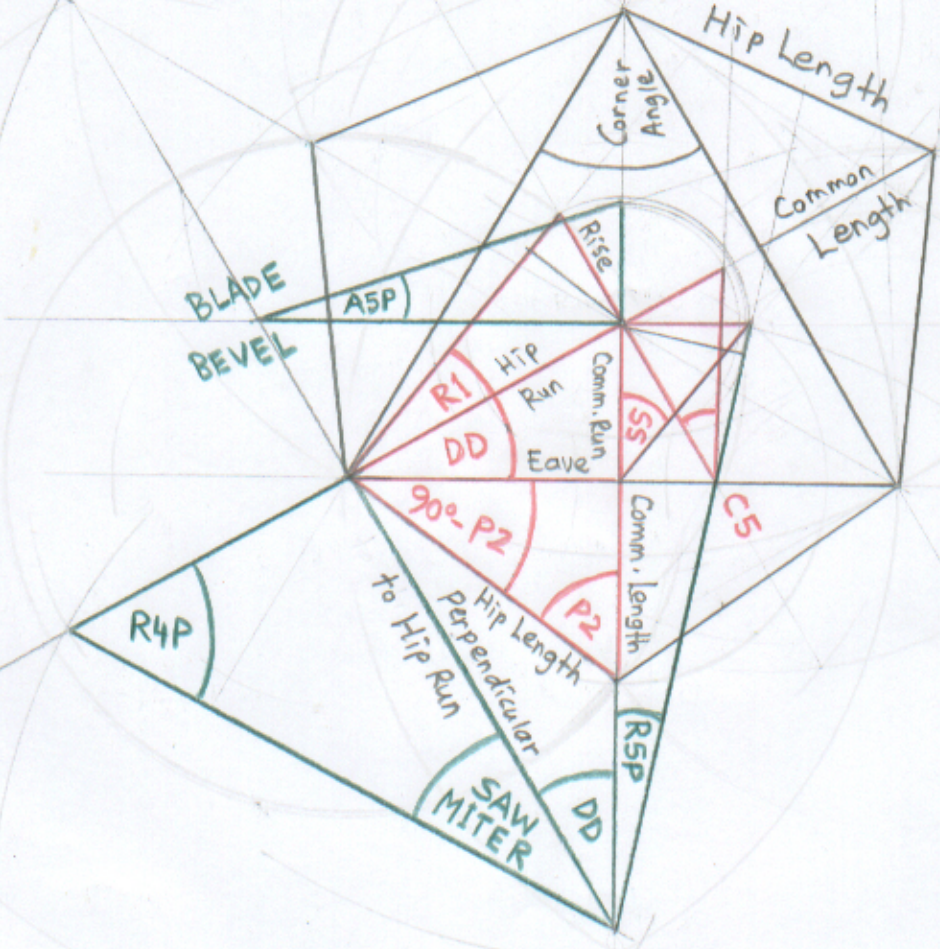




Development of Dodecahedron Compound Angles

Roof Tetrahedron Angles

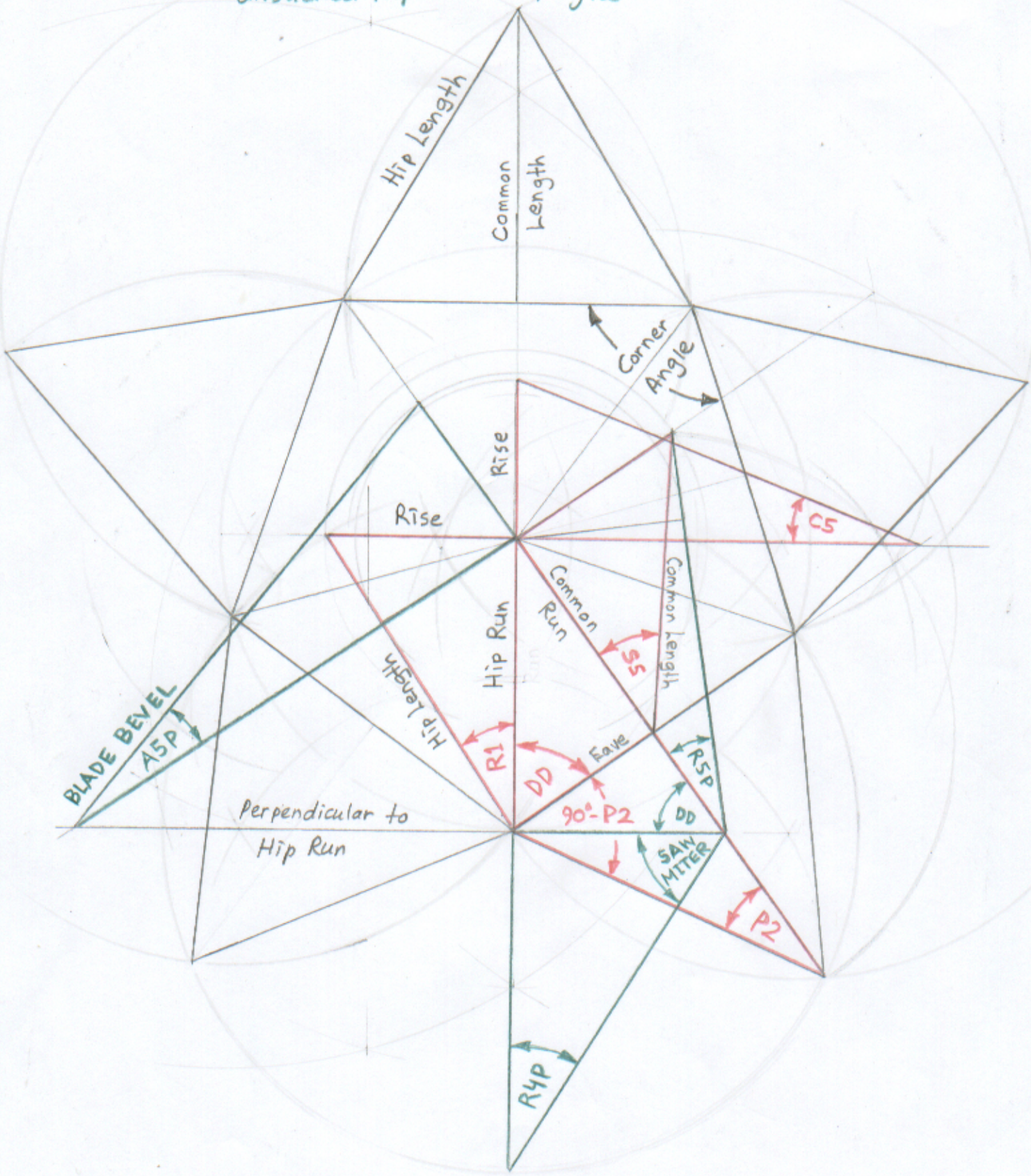
Unbacked Hip Rafter Angles



Development of Icosahedron Compound Angles

Roof Tetrahedron Angles

Unbacked Hip Rafter Angles



Development of Regular Tetrahedron Angles

Real Roof Footprint and Surfaces
= 60° Equilateral Triangles

Given: DD = 30° and 90° - P2 = 60°

Construct R1 = 54.73561°

Construct SS = 70.52878°

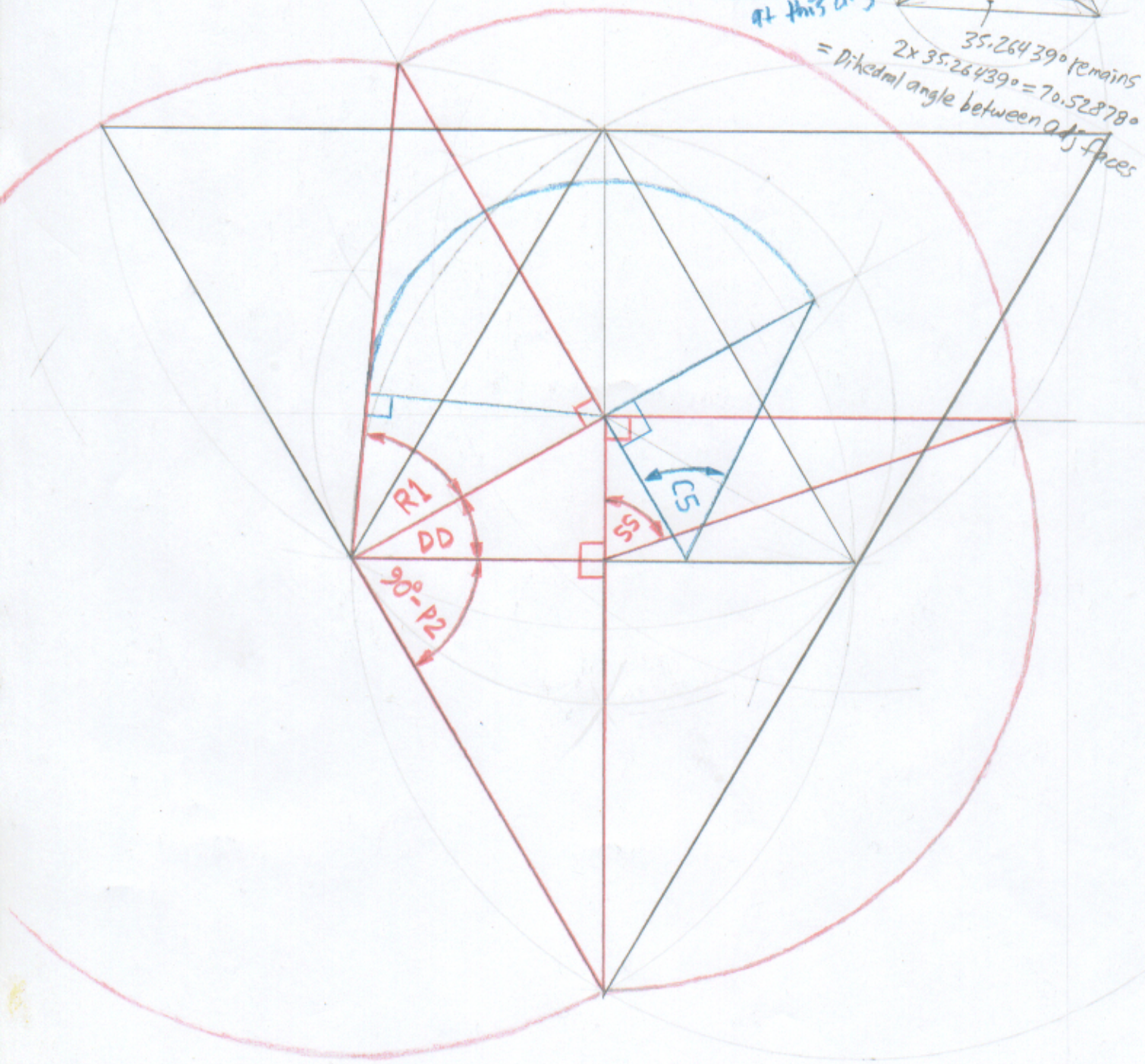
Construct C5 = 54.73561°

Bevel all edges at this angle

Plywood faces

35.26439° remains

2x 35.26439° = 70.52878°
= Dihedral angle between adj faces



Development of Regular Tetrahedron Angles

This development is a continuation of, or built upon, the triangles of DD, SS, R1, et al...

*note! ... RSP = 35.26439° (rise = rise of ΔSS)

R1 = 54.73561° (given)

DD = 30° (given)

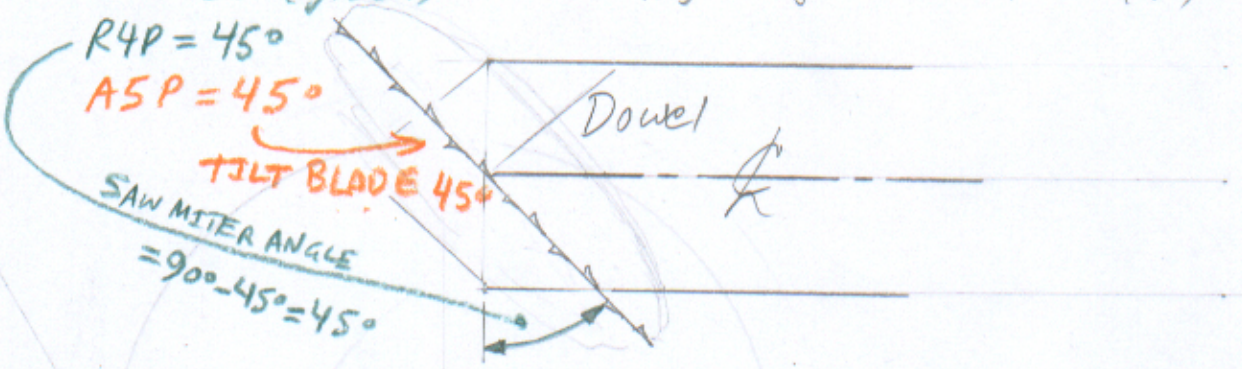
R4P = 45°

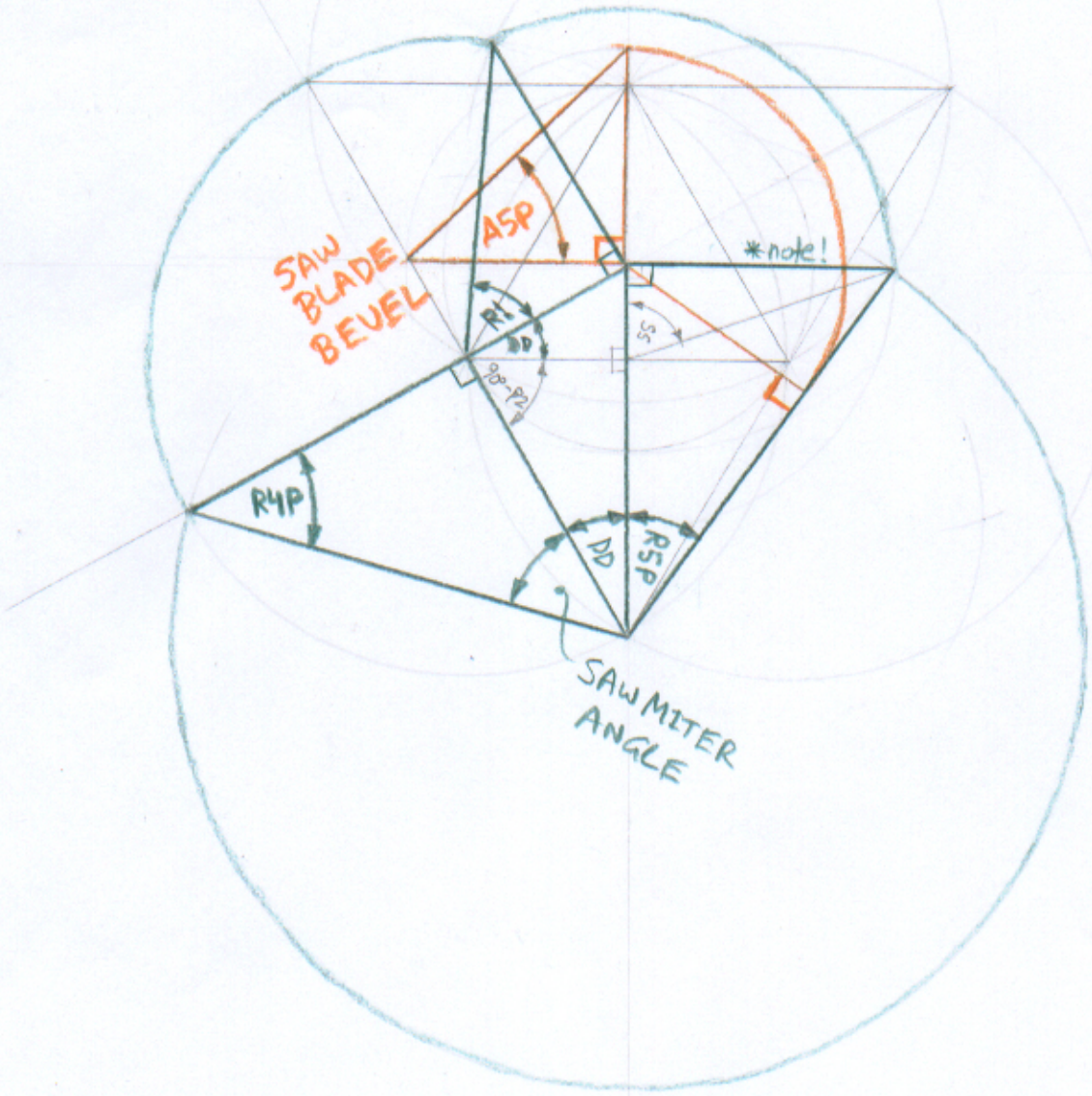
ASP = 45°

If using miter box...

Angle on upper (open) face = 45°

Angle on adjacent face = 54.73561° (=R1)



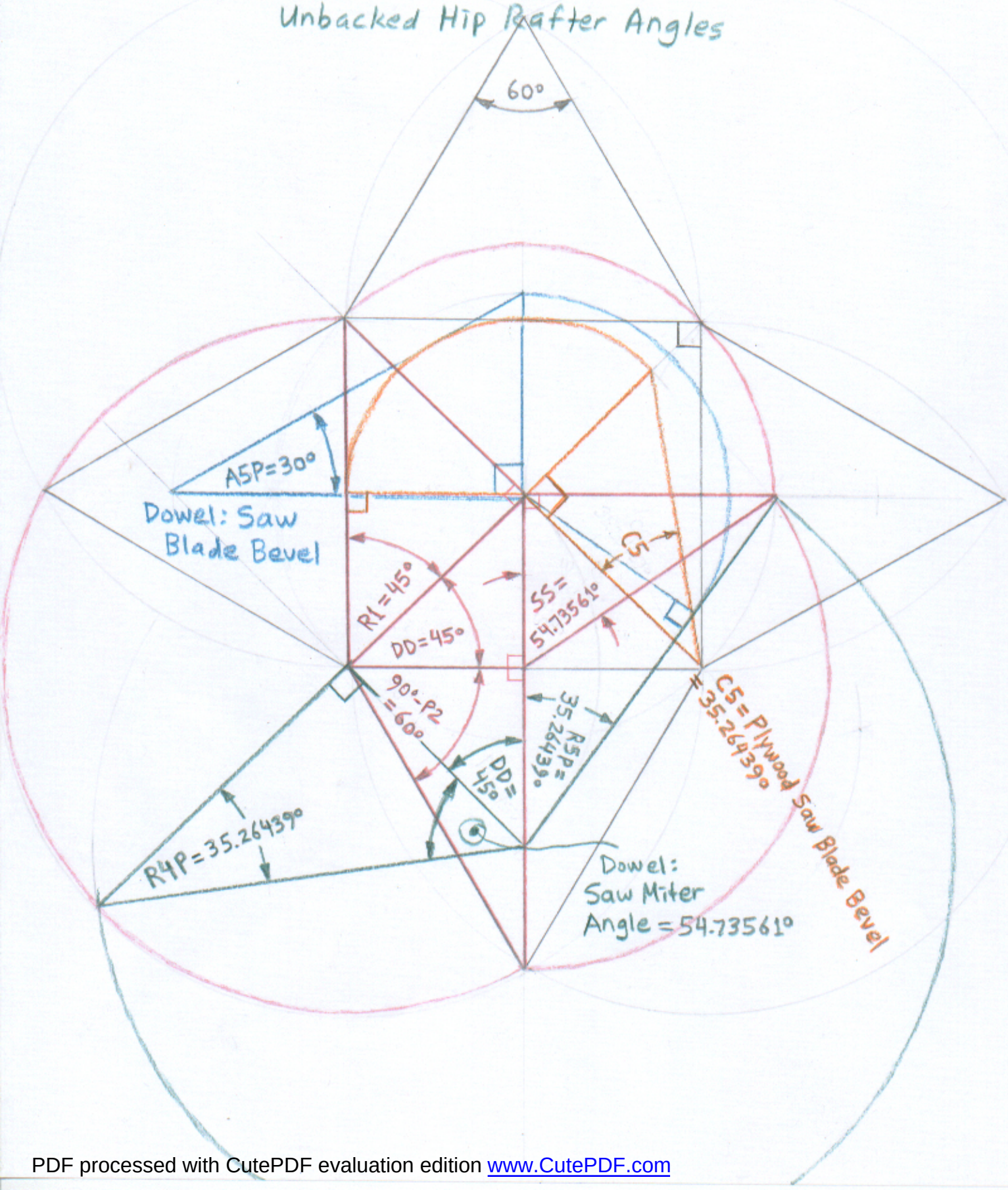


Developments of Regular Octahedron Compound Angles

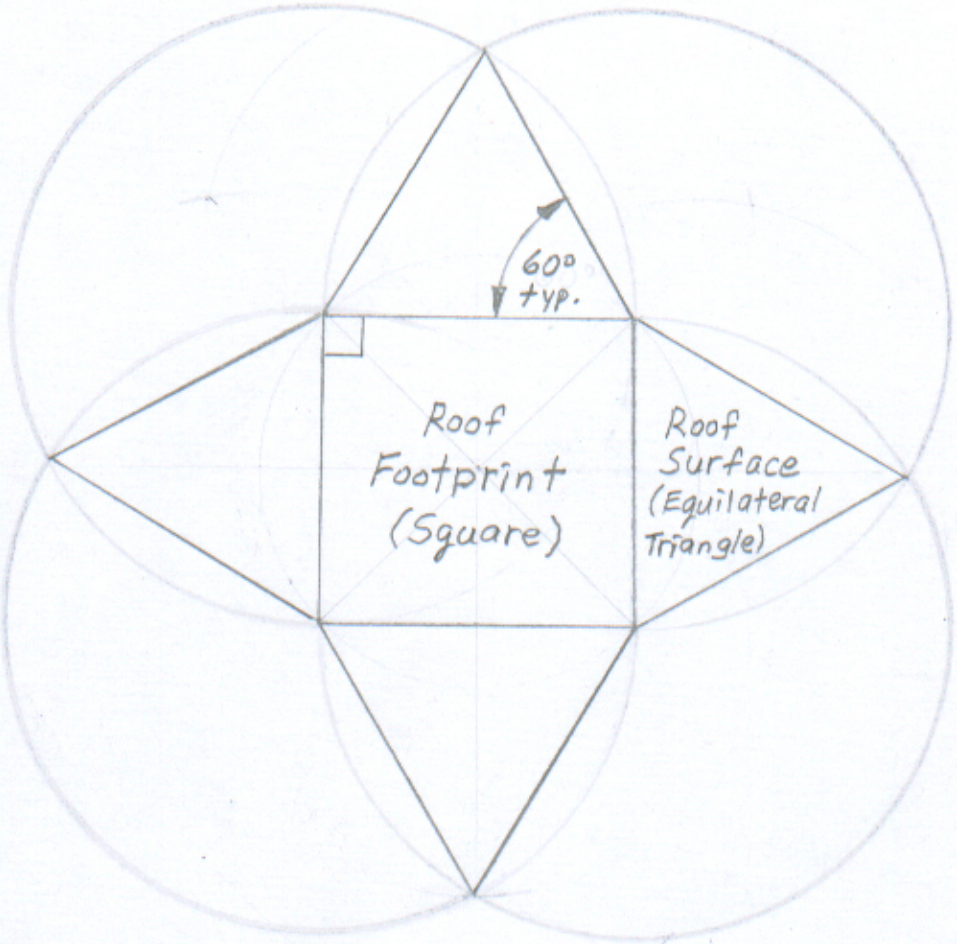
Real Roof Footprint (Square) and Surfaces (Equilateral Triangles)

Roof Tetrahedron Angles

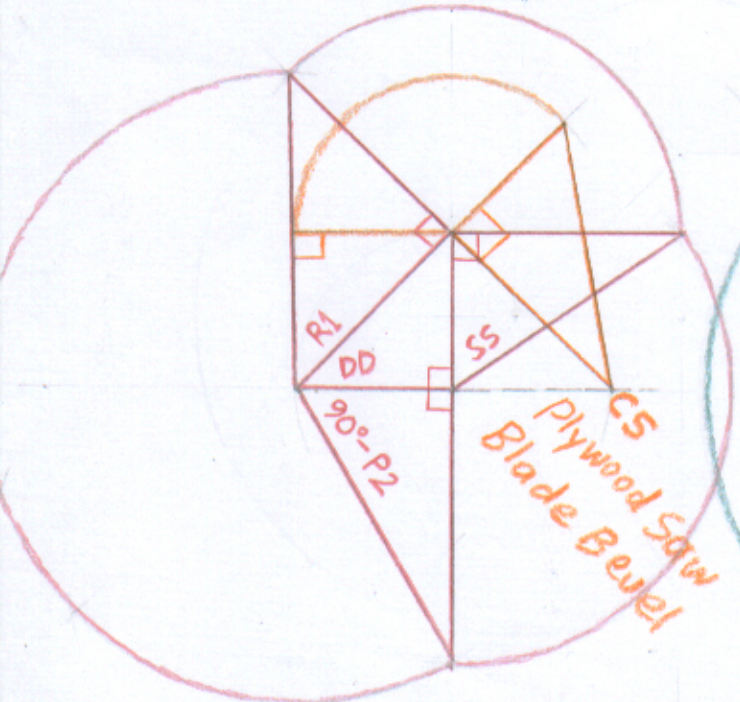
Unbacked Hip Rafter Angles



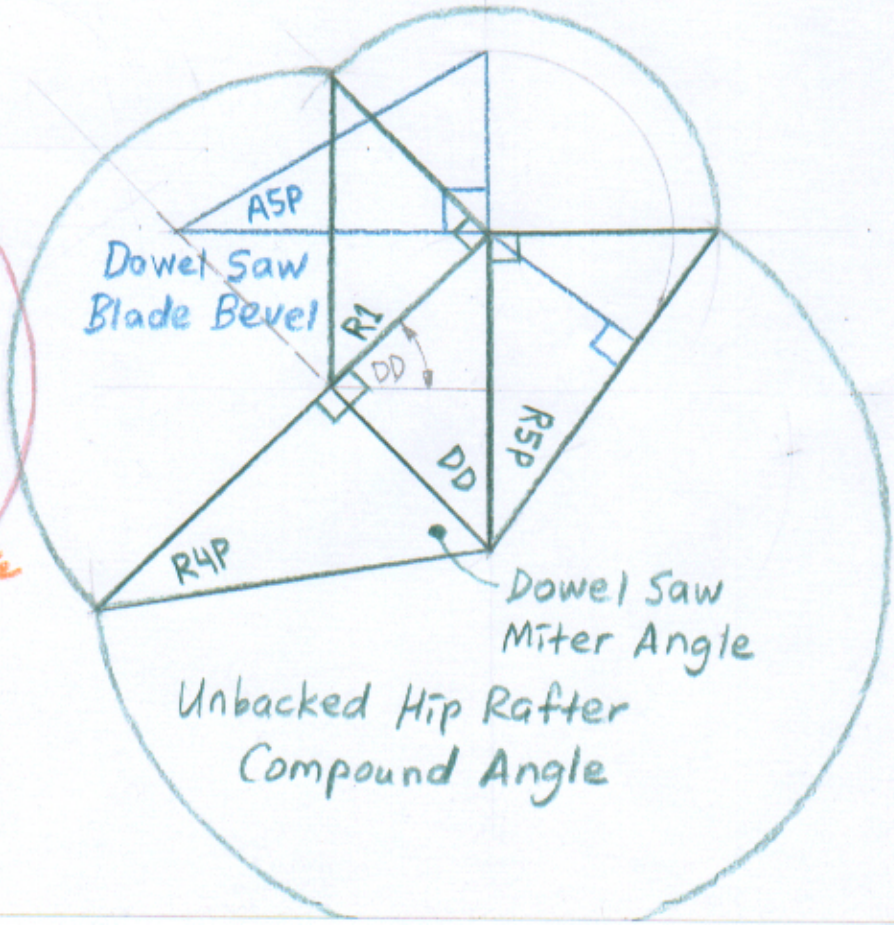
Developments of Regular Octahedron Compound Angles



Real Roof Footprint and Surfaces



Tetrahedron extracted from Real Roof

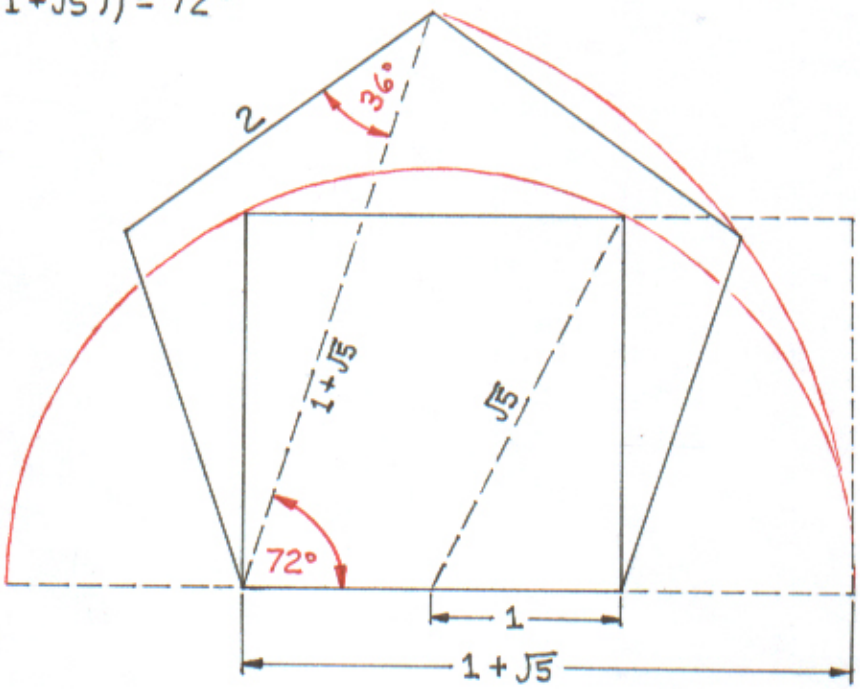


Unbacked Hip Rafter Compound Angle

GOLDEN SECTION

$$\arccos\left(\frac{(1+\sqrt{5})/2}{2}\right) = 36°$$

$$\arccos\left(1/(1+\sqrt{5})\right) = 72°$$



$$\frac{a+b}{a} = \frac{a}{b}$$

$$\therefore ab + b^2 = a^2$$

Let $a = 1$

$$\therefore b^2 + b - 1 = 0$$

$$b = \frac{-1 \pm \sqrt{5}}{2}$$