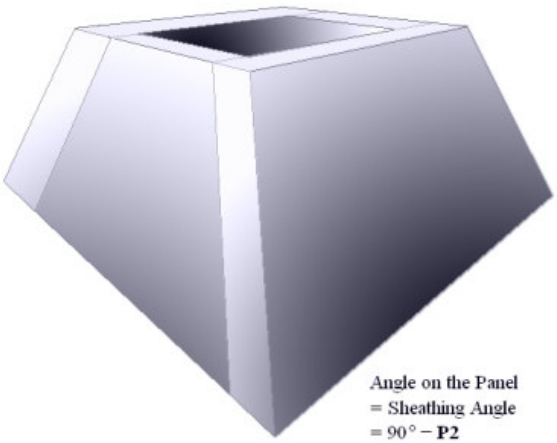
Compound Angle for Hopper

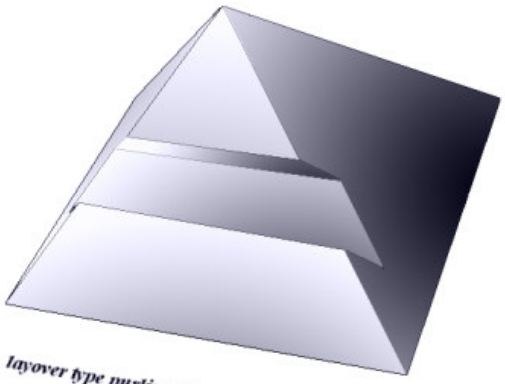
Hopper Compound Angles compared to Layover Hip Roof

... the panels composing the hopper follow the purlins in a Hip roof



Angle on the Panel
= Sheathing Angle
= 90° − **P2**

Saw Blade Bevel Angle along edge of Panel
= 90° − Sum of Backing Angles
= 90° − (C5m + C5a)

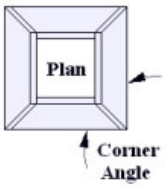


... layover type purlin extracted from Hip roof

Angle on the Stick
= Sheathing Angle
= 90° − **P2**

Saw Blade Bevel Angle
= 90° − Sum of Backing Angles
= 90° − (C5m + C5a)

Wireframe sketches of Hopper Angles



Slope Angle = $\arctan\left[\frac{\textbf{Rise}}{\textbf{Run}}\right]$

Angle Formulas ... General Case

unequal panel slope angles , irregular corner angle in plan view

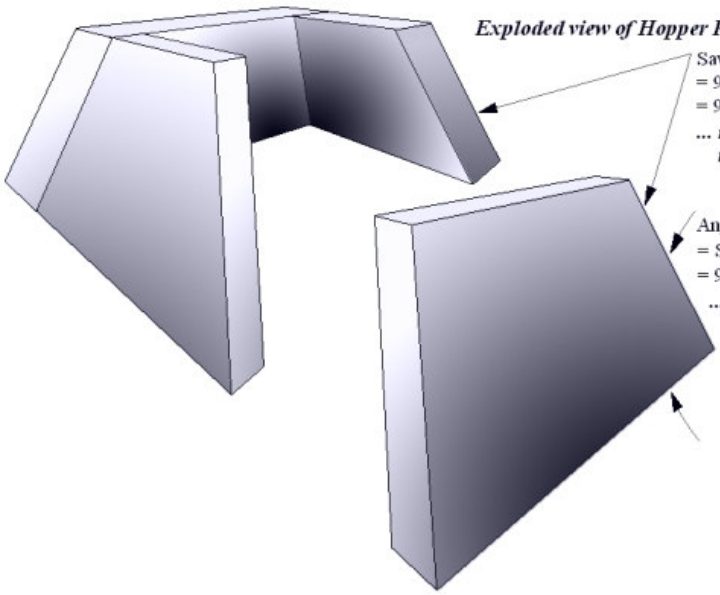
Corner Angle in Plan View = **W**
Main Panel Slope Angle = **SS**
Adjoining Panel Slope Angle = **S**

Main Sheathing Angle = $\arctan\left[\frac{\tan S \sin W}{\sin SS + \tan S \cos SS \cos W}\right]$

Adjoining Sheathing Angle = $\arctan\left[\frac{\tan SS \sin W}{\sin S + \tan SS \cos S \cos W}\right]$

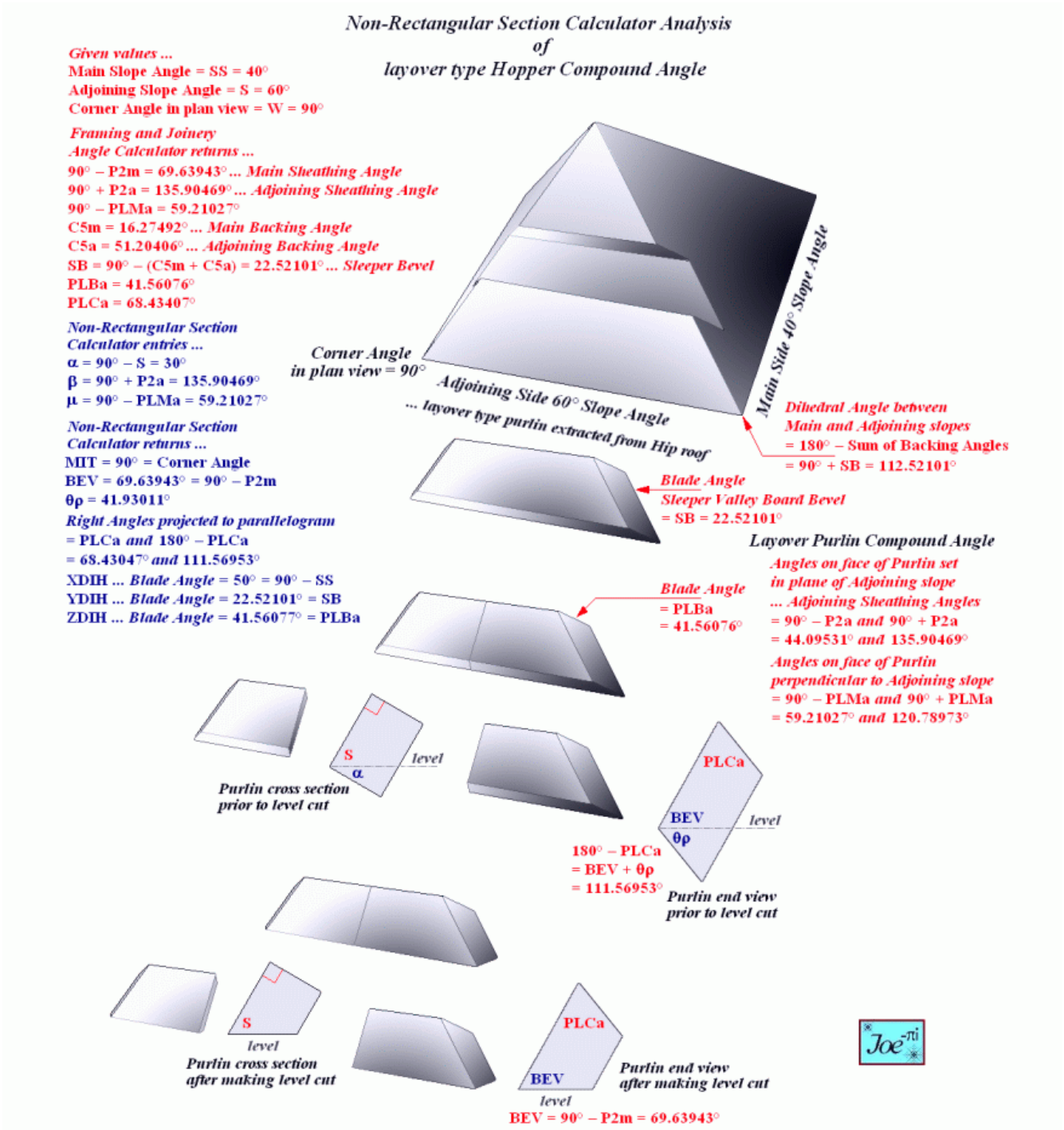
Saw Blade Bevel Angle = $\arcsin(\cos SS \cos S - \sin SS \sin S \cos W)$

Exploded view of Hopper Panel

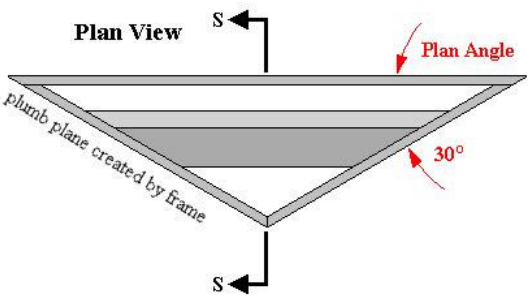


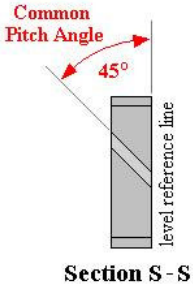
Saw Blade Bevel Angle along edge of Panel
= 90° − Sum of Backing Angles
= 90° − (C5m + C5a)
... note! the Saw Blade Bevel is equal along the edges of both of the abutting panels

Angle on the Panel
= Sheathing Angle
= 90° − **P2**
... this angle may be unequal on each panel for different combinations of panel slope angles and irregular corner angles in plan view


Non-Rectangular Section Calculator Analysis
of
layover type Hopper Compound Angle
Given values ...
Main Slope Angle = SS = 40°
Adjoining Slope Angle = S = 60°
Corner Angle in plan view = W = 90°
Framing and Joinery
Angle Calculator returns ...
90° – P2m = 69.63943° ... Main Sheathing Angle
90° + P2a = 135.90469° ... Adjoining Sheathing Angle
90° – PLMa = 59.21027°
C5m = 16.27492° ... Main Backing Angle
C5a = 51.20406° ... Adjoining Backing Angle
SB = 90° – (C5m + C5a) = 22.52101° ... Sleeper Bevel
PLBa = 41.56076°
PLCa = 68.43407°
Non-Rectangular Section
Calculator entries ...
α = 90° – S = 30°
β = 90° + P2a = 135.90469°
μ = 90° – PLMa = 59.21027°
Non-Rectangular Section
Calculator returns ...
MIT = 90° = Corner Angle
BEV = 69.63943° = 90° – P2m
θρ = 41.93011°
Right Angles projected to parallelogram
= PLCa and 180° – PLCa
= 68.43047° and 111.56953°
XDIH ... Blade Angle = 50° = 90° – SS
YDIH ... Blade Angle = 22.52101° = SB
ZDIH ... Blade Angle = 41.56077° = PLBa
Corner Angle
in plan view = 90°
Adjoining Side 60° Slope Angle
Main Side 40° Slope Angle
... layover type purlin extracted from Hip roof
Dihedral Angle between
Main and Adjoining slopes
= 180° – Sum of Backing Angles
= 90° + SB = 112.52101°
Blade Angle
Sleeper Valley Board Bevel
= SB = 22.52101°
Layover Purlin Compound Angle
Blade Angle
= PLBa
= 41.56076°
Angles on face of Purlin set
in plane of Adjoining slope
... Adjoining Sheathing Angles
= 90° – P2a and 90° + P2a
= 44.09531° and 135.90469°
Angles on face of Purlin
perpendicular to Adjoining slope
= 90° – PLMa and 90° + PLMa
= 59.21027° and 120.78973°
S
α
level
Purlin cross section
prior to level cut
PLCa
BEV
θρ
level
180° – PLCa
= BEV + θρ
= 111.56953°
Purlin end view
prior to level cut
S
level
Purlin cross section
after making level cut
PLCa
BEV
level
Purlin end view
after making level cut
BEV = 90° – P2m = 69.63943°
Joe-πi

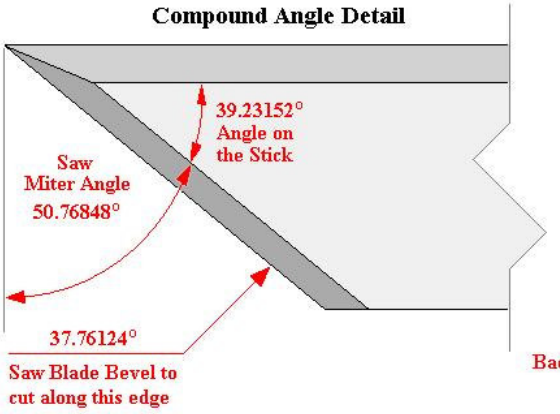
Vent Louver Compound Angle





Calculation of the Compound Angle on the Vent

Comparison of the angles on the components of a vent to the related parts of a Valley roof



Angle on the Stick

$$\text{Sheathing Angle} = \arctan\left[\frac{\tan \text{Plan Angle}}{\cos \text{Common Pitch Angle}}\right]$$

$$= \arctan\left[\frac{\tan 30^\circ}{\cos 45^\circ}\right] = 39.23152^\circ$$

Saw Miter Angle

$$\text{Jack Rafter Side Cut Angle} = 90^\circ - \text{Sheathing Angle}$$

$$= 90^\circ - 39.23152^\circ$$

$$= 50.76848^\circ$$

Saw Blade Bevel Angle

$$\text{Backing Angle} = \arcsin\left[\sin \text{Common Pitch Angle} \times \cos \text{Plan Angle}\right]$$

$$= \arcsin\left[\sin 45^\circ \times \cos 30^\circ\right] = 37.76124^\circ$$

Main Roof Pitch Angle = 30° Dormer Pitch Angle = 45°

Corner Angle between Eaves in Plan View = 90°

... for this combination of intersecting roof slopes

Dormer Sheathing Angle = Angle on the Stick = 39.23152°

Dormer Jack Rafter Side Cut Angle = Saw Miter Angle = 50.76848°

... since this is a layover type intersection

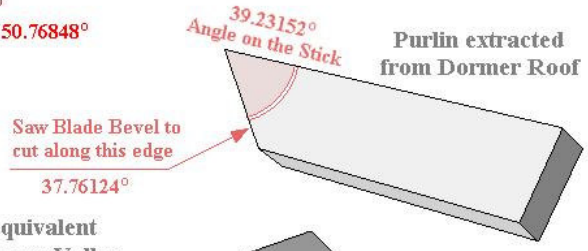
Main Backing Angle = 14.47751°

Dormer Backing Angle = 37.76124°

Valley Board Bevel Angle = Saw Blade Bevel Angle

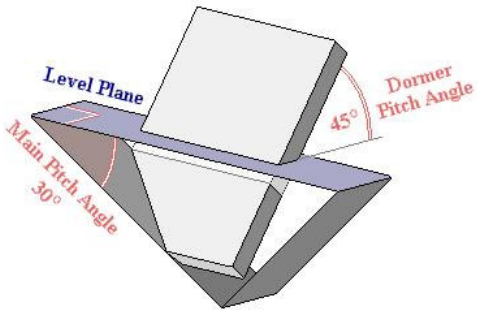
= 90° – (14.47751° + 37.76124°) = 37.76125°

Layover Valley method of solving the Compound Angle on the Vent

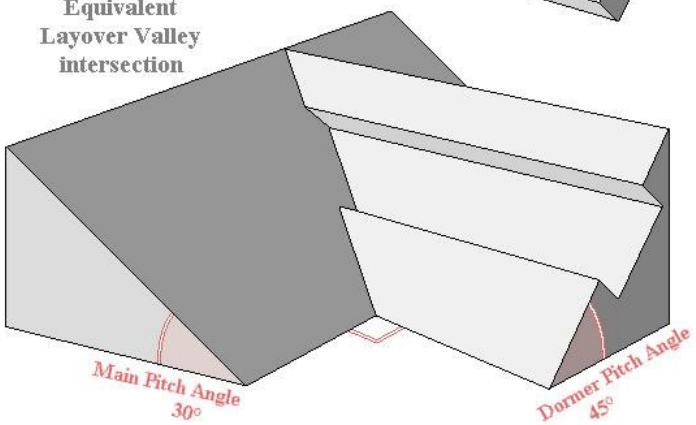


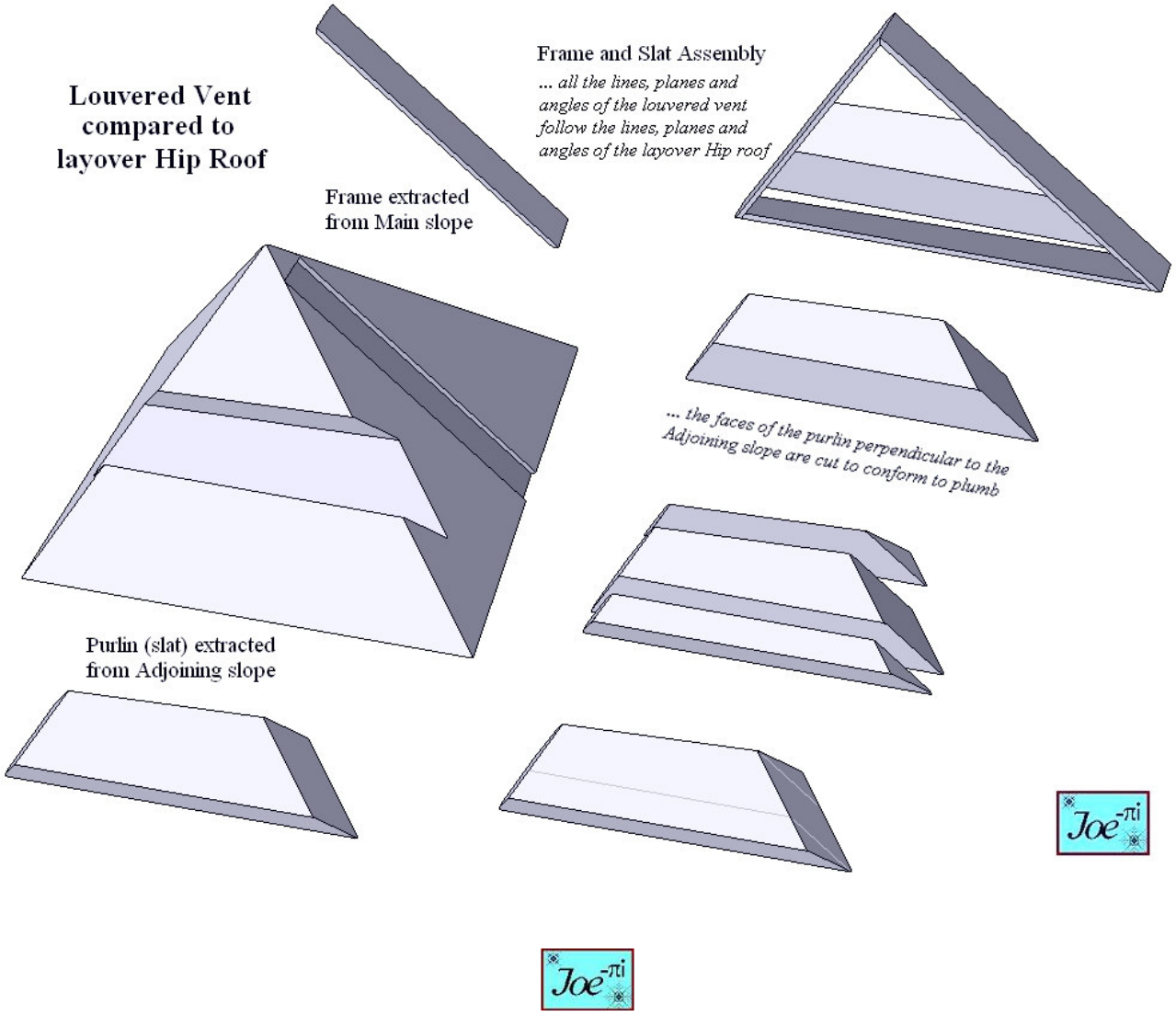
Simplified sketch of Vent Components and Angle Relationships

... pitch angles are measured with respect to the level reference plane

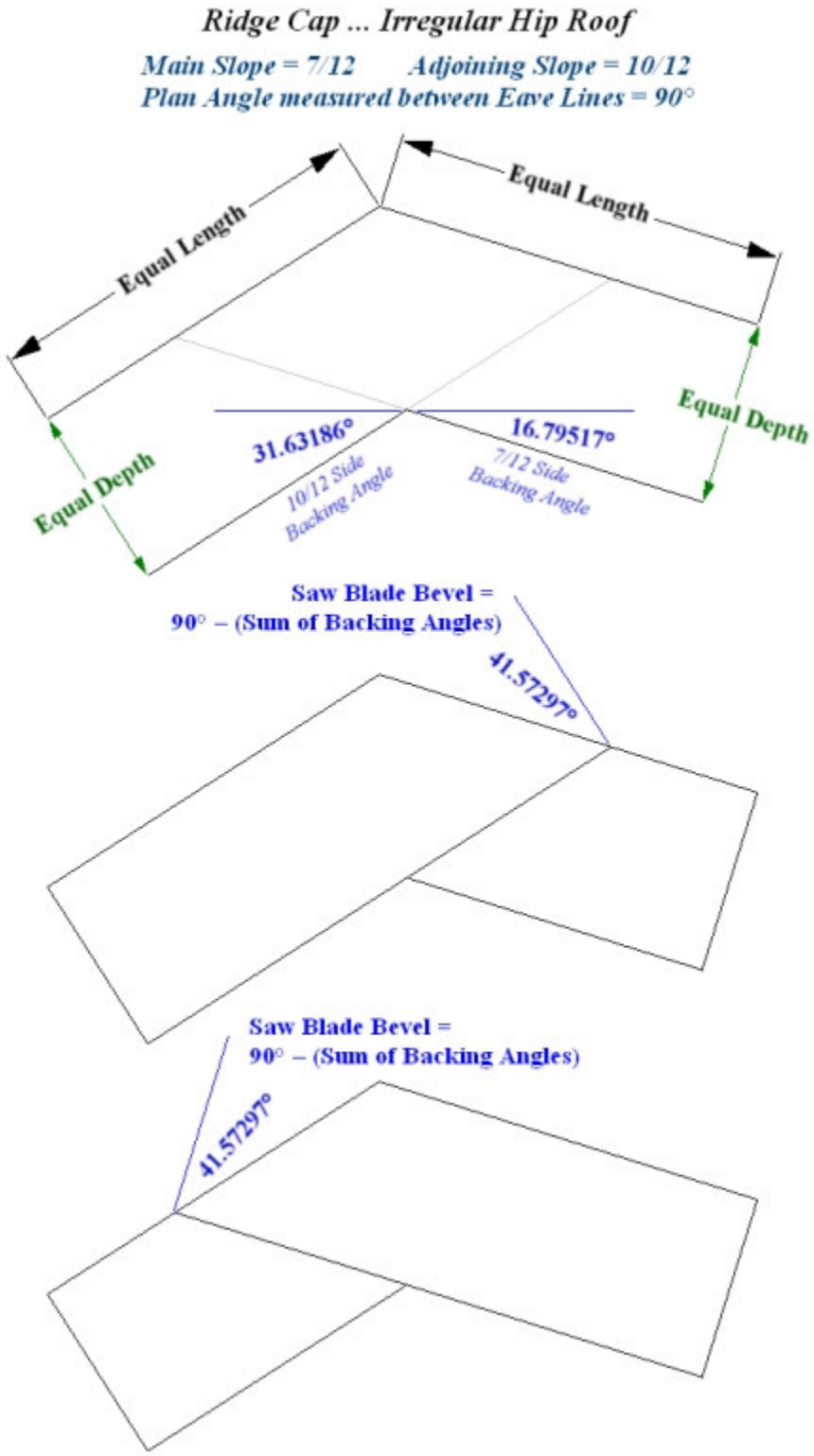


Equivalent Layover Valley intersection




Louvered Vent compared to layover Hip Roof
Frame extracted from Main slope
Frame and Slat Assembly
... all the lines, planes and angles of the louvered vent follow the lines, planes and angles of the layover Hip roof
... the faces of the purlin perpendicular to the Adjoining slope are cut to conform to plumb
Purlin (slat) extracted from Adjoining slope

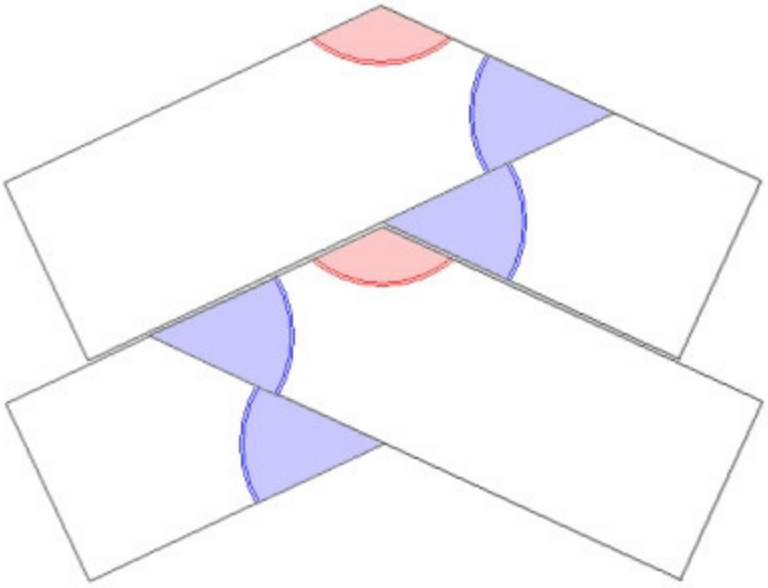
Irregular Hip Roof Ridge Cap



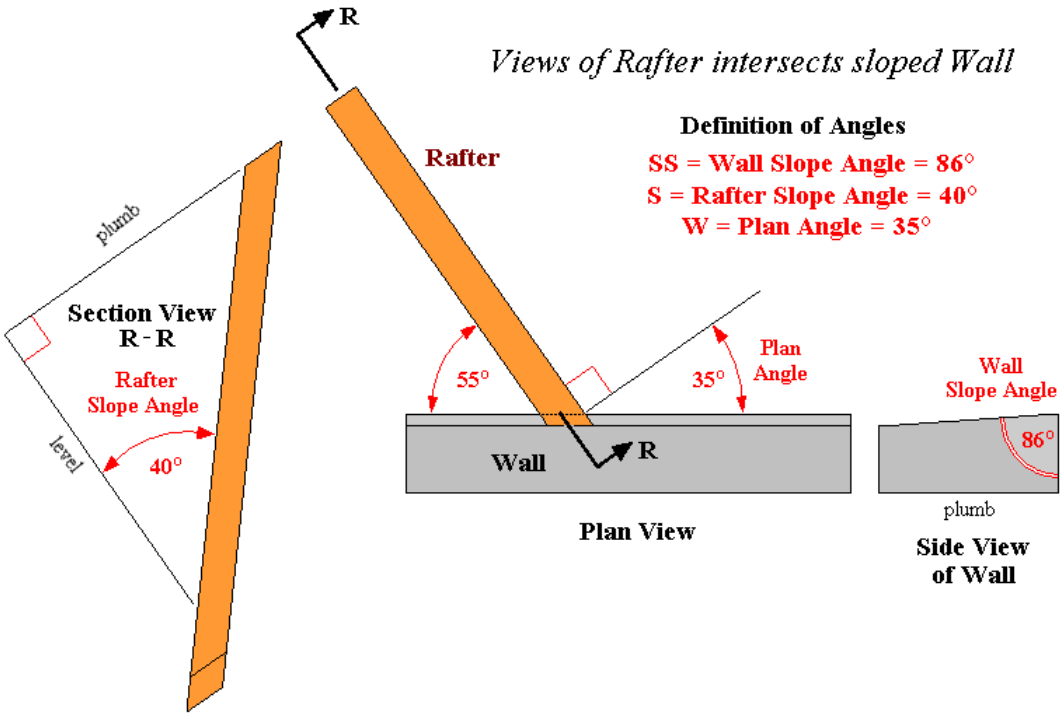
Ridge Cap ... Irregular Hip Roof

Main Slope = 7/12 Adjoining Slope = 10/12
Plan Angle measured between Eave Lines = 90°

Sum of Backing Angles = 48.42703°
180° – (Sum of Backing Angles) = 131.57297°



Compound Angle: Rafter intersects sloped Wall



Angle Formulas for
Rafter intersection with sloped Wall

Angles on the sloped surface of the Wall

Angle of Rafter mortise line from Level

$$= \arctan\left[\frac{\tan S \sin W}{\sin SS + \tan S \cos SS \cos W}\right]$$

Mortise or Housing Angle to accommodate Rafter Foot in sloped face of Wall

$$= \arctan\left[\frac{\tan S + \tan SS \cos W}{\tan SS \sin W\,(\cos SS - \sin SS \tan S \cos W)}\right]$$

Angles on the Rafter

Angle measured on the upper shoulder of the Rafter

$$= \arctan\left[\frac{\sin S + \tan SS \cos S \cos W}{\tan SS \sin W}\right]$$

Saw Blade Bevel for upper shoulder of the Rafter

$$= \arcsin(\cos SS \cos S - \sin SS \sin S \cos W)$$

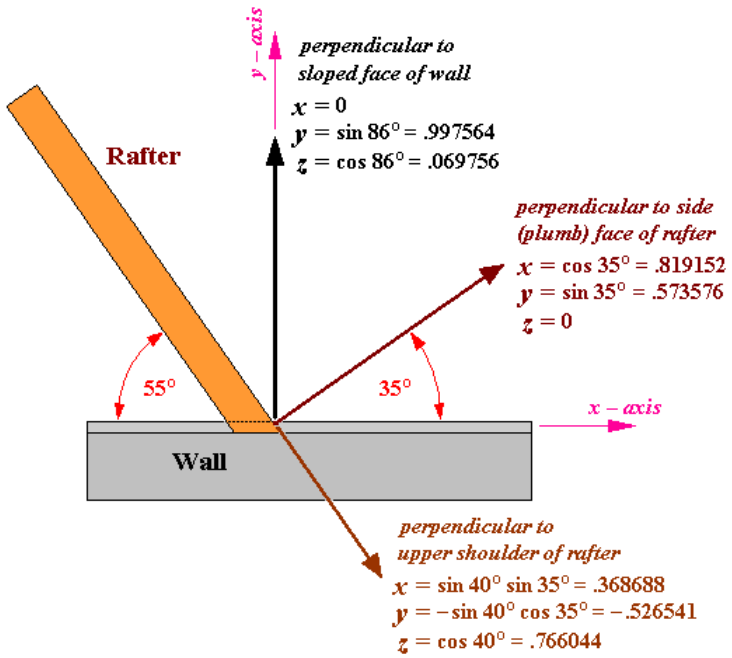
Angle measured on the plumb face of the Rafter

$$= \arctan\left[\frac{\tan S + \tan SS \cos W}{1 - \tan SS \tan S \cos W}\right]$$

Saw Blade Bevel for plumb face of the Rafter

$$= \arctan\left[\frac{\tan SS \sin W}{\sqrt{1 + \tan^2 SS \cos^2 W}}\right]$$

Plan View of Unit Vectors
perpendicular to Rafter and Wall



Dihedral Angles and Saw Blade Bevels

Dihedral Angle between side (plumb) face of rafter and sloped face of wall = 55.09769°
Saw Blade Bevel for plumb face of rafter = 90° – 55.09767° = 34.90231°

Dihedral Angle between upper shoulder of rafter and sloped face of wall = 118.15265°
Saw Blade Bevel for upper shoulder of rafter = 118.15265° – 90° = 28.15265°

Supplementary Angles on footprint created on the wall by projected cross section of the stick = 68.07793° and 111.92207°

Angles measured on the Stick

Angle on the side (plumb) face of the rafter = 54.87917°
Angle on the upper shoulder of the rafter = 49.53719°

Solution of the Compound Angle on the Rafter

Angle between vectors a and b
(Dihedral Angle between Planes)

$$= \arccos\left[\frac{x_a x_b + y_a y_b + z_a z_b}{\sqrt{x_a^2 + y_a^2 + z_a^2}\ \sqrt{x_b^2 + y_b^2 + z_b^2}}\right]$$

*... since **a** and **b** are unit vectors the denominator equals one*

*... since the **complementary** **Saw Blade Bevel***
*is required we take the **arcsin** of the expression*

... substituting the trig functions of the terminal points of the unit vectors

Saw Blade Bevel for plumb face of the Rafter

$$= \arcsin(\sin SS \sin W)$$

Saw Blade Bevel for upper shoulder of the Rafter

$$= \arcsin(\cos SS \cos S - \sin SS \sin S \cos W)$$

*... applying the **Compound Angle Formulas***

Mortise or Housing Angle

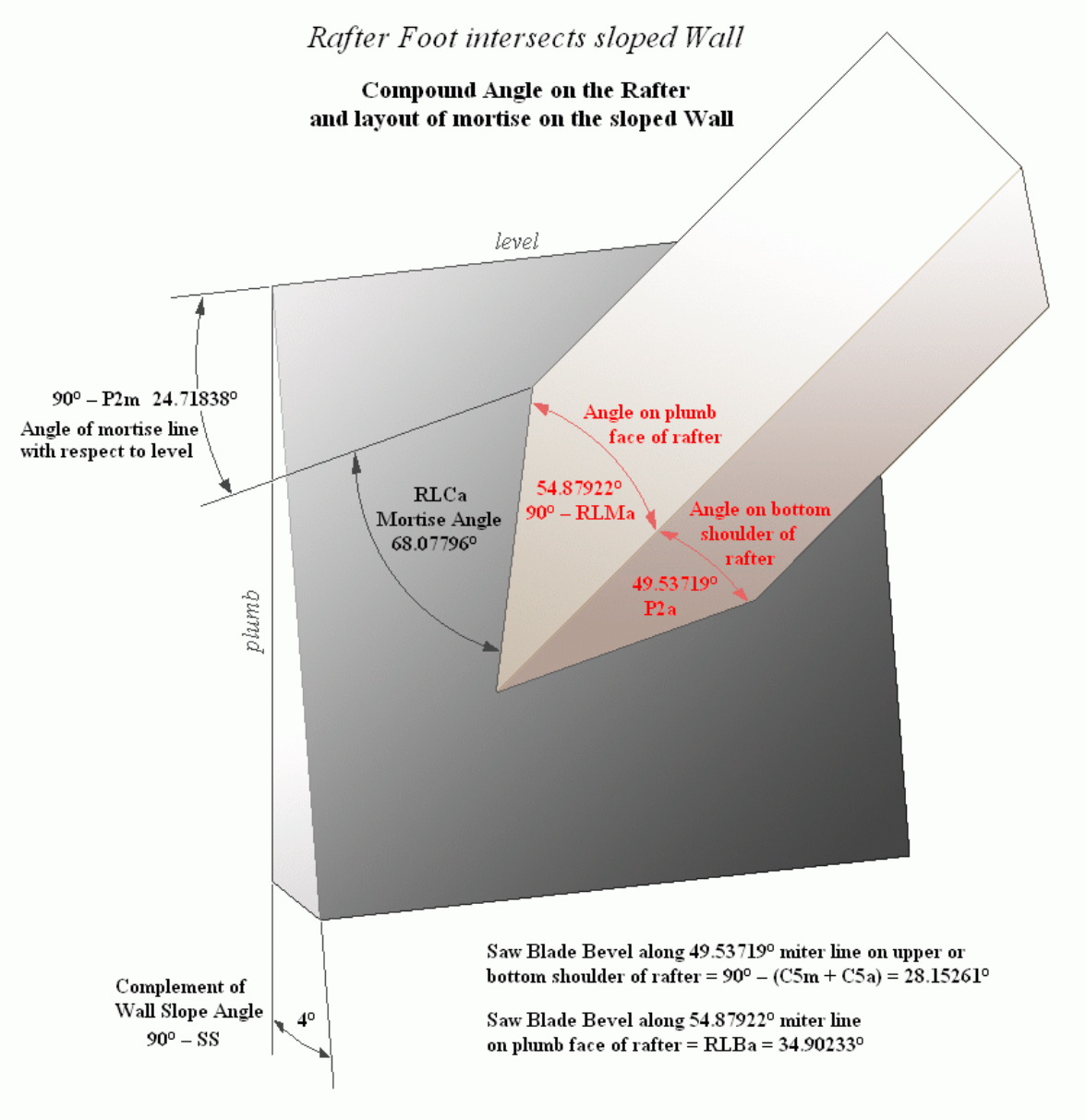
$$= \arccos(\tan \textbf{Saw Blade Bevel for plumb face of the Rafter} \times \tan \textbf{Saw Blade Bevel for upper shoulder of the Rafter})$$

Angle measured on the plumb face of the Rafter

$$= \arcsin(\sin \textbf{Mortise Angle} \times \cos \textbf{Saw Blade Bevel for upper shoulder of the Rafter})$$

Angle measured on the upper shoulder of the Rafter

$$= \arcsin(\sin \textbf{Mortise Angle} \times \cos \textbf{Saw Blade Bevel for plumb face of the Rafter})$$


Rafter Foot intersects sloped Wall
Compound Angle on the Rafter
and layout of mortise on the sloped Wall
level
90° – P2m 24.71838°
Angle of mortise line
with respect to level
RLCa
Mortise Angle
68.07796°
plumb
Angle on plumb
face of rafter
54.87922°
90° – RLMa
Angle on bottom
shoulder of
rafter
49.53719°
P2a
Complement of
Wall Slope Angle
90° – SS
4°
Saw Blade Bevel along 49.53719° miter line on upper or
bottom shoulder of rafter = 90° – (C5m + C5a) = 28.15261°
Saw Blade Bevel along 54.87922° miter line
on plumb face of rafter = RLBa = 34.90233°

Rafter intersects sloped Wall
Angle Calculations

Given values ...

Main Roof Slope Angle = 86°
Adjoining Roof Slope Angle = 40°
Angle in Plan View between Ridge Lines = 35°

Layover Valley angle calculations ...
(apply standard Framing Angle Formulas)

Main Plan Angle = 1.83922°
Adjoining Plan Angle = 33.16078°

Valley Pitch Angle = 24.65415°

Main Backing Angle = 85.59804°
Adjoining Backing Angle = 32.55456°

Sleeper Bevel Angle = – 28.15260°
... saw blade bevel for upper shoulder of rafter
Adjoining Jack Rafter Side Cut Angle = 49.53719°
... angle measured on upper shoulder of rafter
Main Sheathing Angle = 24.71838°
... angle of mortise line with respect to level

Compound Angle calculations ...

Angle measured on plumb face of Rafter
= arctan (sin 49.53719° ÷ tan 28.15260°) = 54.87922°

Saw Blade Bevel for plumb face of Rafter
= arctan (sin 54.87922° ÷ tan 49.53719°) = 34.90233°

Mortise or Housing Angle
= arccos (cos 54.87922° × cos 49.53719°) = 68.07796°