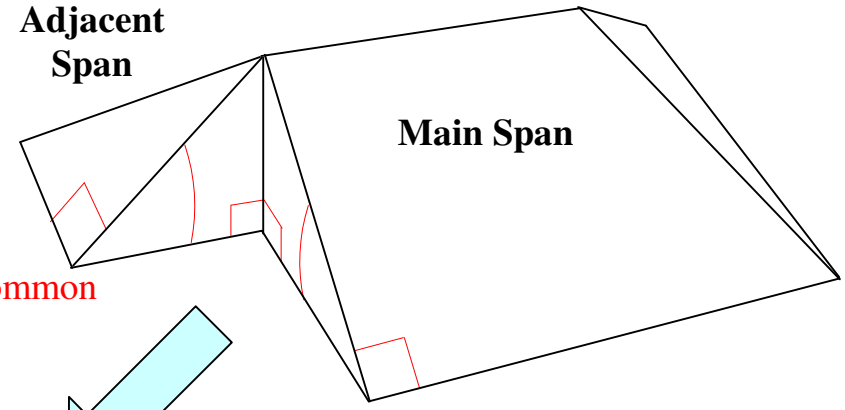
GENERAL DECK ANGLE EQUATIONS

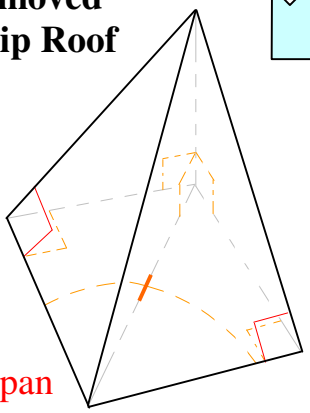
Hip Roof Model and Definition of Angles



S : Adjacent Common Pitch Angle

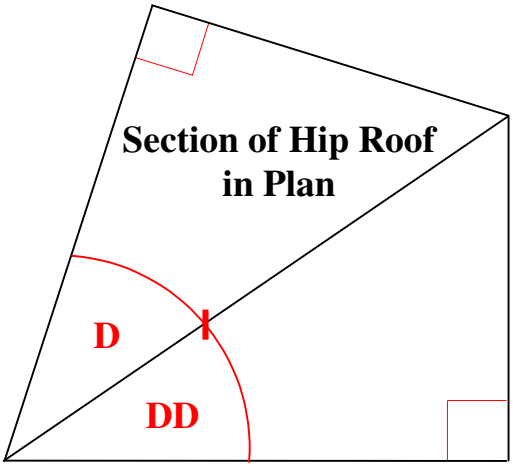
SS : Main Common Pitch Angle

Section removed from the Hip Roof



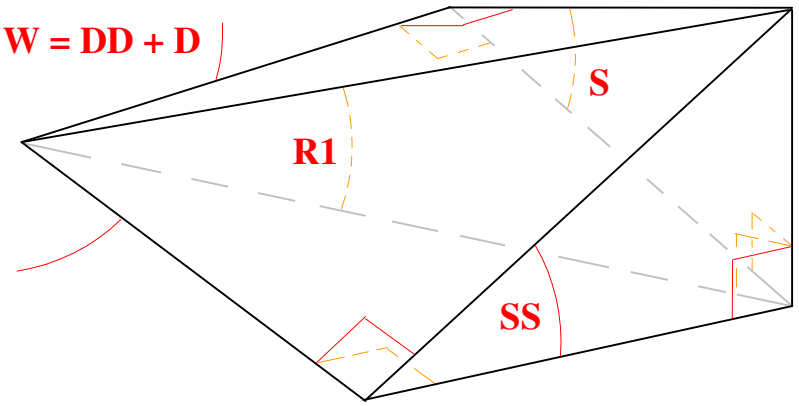
D : Adjacent Span Deck Angle

DD : Main Span Deck Angle



W : Total Deck Angle between Eaves

W = DD + D



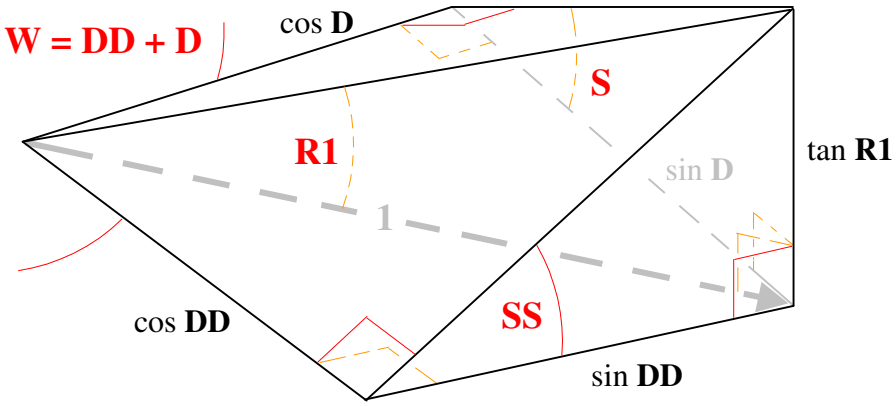
Trigonometric Model of Hip Roof

R1 : Hip Pitch Angle

The Hip Pitch Angle links the Main and Adjacent spans of the Hip Roof

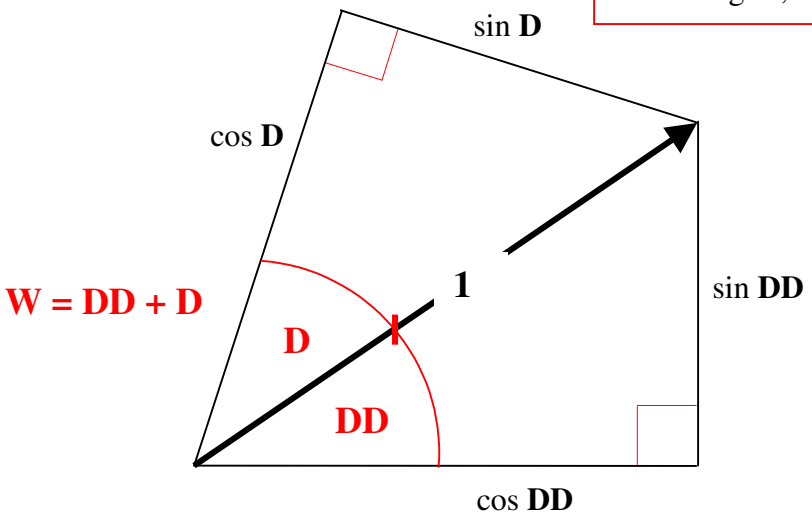
GENERAL DECK ANGLE EQUATIONS

Unit radius vector method



Isometric Projection
The unit radius vector returns a value for the rise equal to the tangent of Hip Pitch Angle **R1**.

Plan
The unit radius vector yields trigonometric functions for the Deck Angles, **DD** and **D**.



GENERAL DECK ANGLE EQUATIONS

Unit radius vector method

tan **SS** = tan **R1** ÷ sin **DD**, therefore tan **R1** = tan **SS** sin **DD**

tan **S** = tan **R1** ÷ sin **D**, therefore tan **R1** = tan **S** sin **D**

tan **SS** sin **DD** = tan **S** sin **D**

tan **SS** ÷ tan **S** = sin **D** ÷ sin **DD**

D = **W** – **DD** ,

therefore sin **D** = sin (**W** – **DD**)

= sin **W** cos **DD** – cos **W** sin **DD**

(Sine of Difference of Angles)

$$\tan \mathbf{SS} \div \tan \mathbf{S} = \frac{\sin \mathbf{W} \cos \mathbf{DD} - \cos \mathbf{W} \sin \mathbf{DD}}{\sin \mathbf{DD}}$$

$$= (\sin \mathbf{W} \div \tan \mathbf{DD}) - \cos \mathbf{W}$$

$$\text{Therefore, } \cot \mathbf{DD} = \frac{(\tan \mathbf{SS} \div \tan \mathbf{S}) + \cos \mathbf{W}}{\sin \mathbf{W}}$$

$$\text{and } \tan \mathbf{DD} = \frac{\sin \mathbf{W}}{(\tan \mathbf{SS} \div \tan \mathbf{S}) + \cos \mathbf{W}}$$

Note : If **W** = 90°, then sin **D** = cos **DD**, and tan **DD** = tan **S** ÷ tan **SS**

Also, if **W** = 90°, cos **W** = 0, sin **W** = 1,

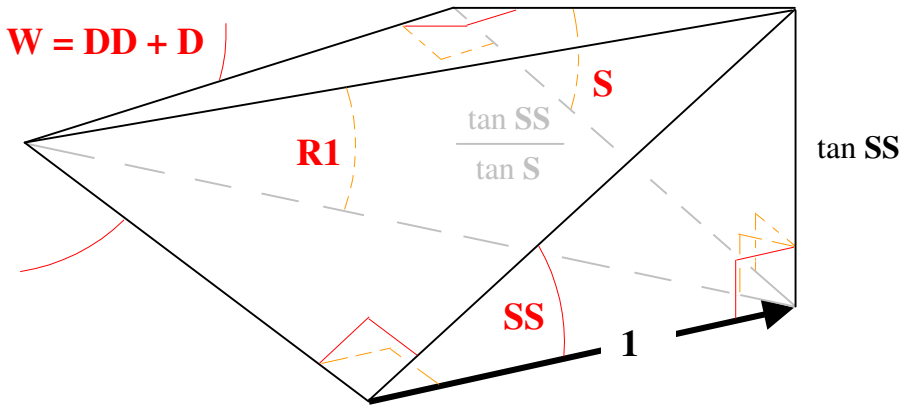
and tan **DD** = 1 ÷ (tan **SS** ÷ tan **S** + 0) = tan **S** ÷ tan **SS**

For any **W**, if **SS** = **S**, then $\tan \mathbf{DD} = \tan \mathbf{D} = \frac{\sin \mathbf{W}}{1 + \cos \mathbf{W}}$

GENERAL DECK ANGLE EQUATIONS

Arcos and arctan fomulas from trig functions of W

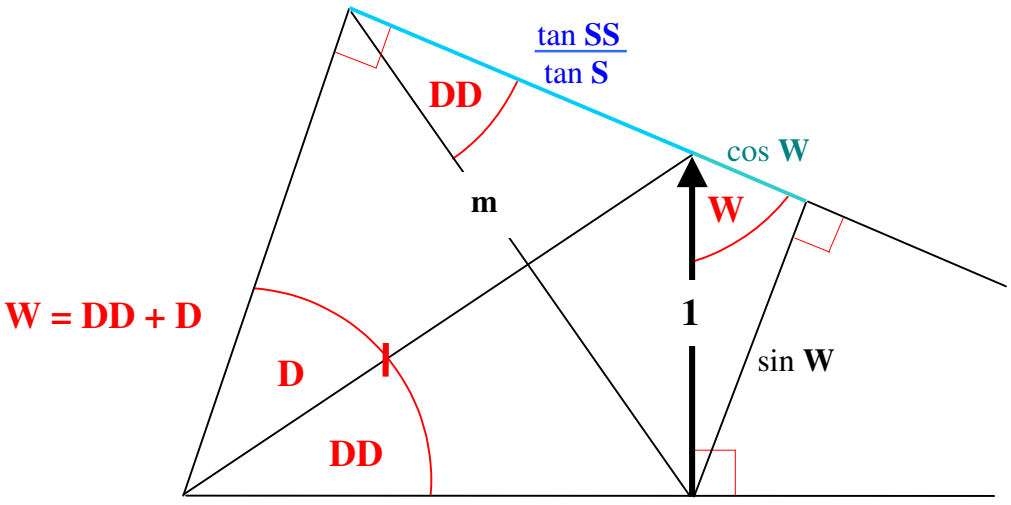
Isometric Projection
The unit length returns a rise of tan **SS**.



$$\mathbf{m} = \sqrt{(\tan \mathbf{SS} \div \tan \mathbf{S})^2 + 2\,(\tan \mathbf{SS} \div \tan \mathbf{S}) \cos \mathbf{W} + 1} \quad \text{(Law of Cosines)}$$

$$\tan \mathbf{DD} = \frac{\sin \mathbf{W}}{(\tan \mathbf{SS} \div \tan \mathbf{S}) + \cos \mathbf{W}}$$

$$\cos \mathbf{DD} = \frac{(\tan \mathbf{SS} \div \tan \mathbf{S}) + \cos \mathbf{W}}{\sqrt{(\tan \mathbf{SS} \div \tan \mathbf{S})^2 + 2\,(\tan \mathbf{SS} \div \tan \mathbf{S}) \cos \mathbf{W} + 1}}$$



Plan
Trigonometric functions of **W** are a consequence of the chosen unit length.

GENERAL DECK ANGLE EQUATIONS

Arcos and arctan fomulas from trig functions of W

The quadrilateral formed by the deck angles in plan has supplementary opposite angles, and can therefore be circumscribed. The inscribed angles labeled **DD** must be equal, since they subtend the same arc.

For any **W**, if **SS** = **S**, then $\tan \mathbf{DD} = \dfrac{\sin \mathbf{W}}{1 + \cos \mathbf{W}}$

If **DD** = **D**, then:
$\sin \mathbf{W} = \sin 2\,\mathbf{DD} = 2 \sin \mathbf{DD} \cos \mathbf{DD}$
$1 + \cos \mathbf{W} = 1 + \cos 2\,\mathbf{DD} = 2 \cos^2 \mathbf{DD}$
Therefore, $2 \sin \mathbf{DD} \cos \mathbf{DD} \div 2 \cos^2 \mathbf{DD} = \tan \mathbf{DD}$
(Double angle formulas)
The longer and less elegant cos **DD** formula on the previous page always returns a positive value if **DD** or **D** exceed 90 degrees.