JIKA $f(x) = X^n$, BUKTIKAN BAHWA $f'(x) = n \cdot X^{n-1}$!

BUKTI : $f(x) = X^n \longrightarrow f(x+h) = (x+h)^n$

$$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

, DI MANA DERET BINOMIUM NEWTON : $\boxed{(a+b)^n = \sum_{r=0}^{n} C(n,r)\, a^{n-r} b^r}$

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^n - x^n}{h} = \lim_{h \to 0} \frac{\sum_{r=0}^{n} C(n,r)\, X^{n-r} h^r - X^n}{h}$$

$$= \lim_{h \to 0} \frac{C(n,0) \cdot X^{n-0} \cdot h^0 + C(n,1)\, X^{n-1} \cdot h + C(n,2) X^{n-2} h^2 + \dots + C(n,n)\, X^{n-n} h^n - X^n}{h}$$

$$= \lim_{h \to 0} \frac{\frac{n!}{0!(n-0)!} X^n \cdot (1) + \frac{n!}{1!(n-1)!} X^{n-1} h + \frac{n!}{2!(n-2)!} X^{n-2} h^2 + \dots + \frac{n!}{n!(n-n)!} X^0 h^n - X^n}{h}$$

$$= \lim_{h \to 0} \frac{\frac{\cancel{n!}}{(1)\cancel{n!}} X^n + \frac{n\cancel{(n-1)!}}{1 \cdot \cancel{(n-1)!}} X^{n-1} h + \frac{n!}{2(n-2)!} X^{n-2} h^2 + \dots + \frac{\cancel{n!}}{\cancel{n!}\, 0!} (1) h^n - X^n}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{X^n} + n \cdot X^{n-1} \cdot h + \frac{n!}{2(n-2)!} X^{n-2} h^2 + \dots + h^n - \cancel{X^n}}{h}$$

$$= \lim_{h \to 0} \left(\frac{n \cdot X^{n-1} \cdot \cancel{h}}{\cancel{h}} + \frac{n!}{2(n-2)!} X^{n-2} h + \dots + h^{n-1} \right)$$

$$= n \cdot X^{n-1} + \underbrace{\frac{n!}{2(n-2)!} X^{n-2} (0)}_{0} + \dots + \underbrace{(0)^{n-1}}_{0}$$

$$\boxed{f'(x) = n \cdot X^{n-1}}$$