UTS

KALKULUS I (K0424 / K0314 (DUMMY))

JUMAT, 25 APRIL 2014
pk 08.00 - 09.40 (100 MENIT)

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① a) $\lim_{x\to 0} \frac{3^x - 2^x}{x} = \lim_{x\to 0} \frac{3^x \ln 3 - 2^x \ln 2}{1} = 3^0 \ln 3 - 2^0 \ln 2$

$= \ln 3 - \ln 2$

$= \ln \frac{3}{2}$.

$\left[\frac{0}{0}\right]$ I

b) $\lim_{x\to 0} (\sec x)^{\frac{1}{x}} = \lim_{x\to 0} \left[1 + (\sec x - 1)\right]^{\frac{1}{x}}$ V

$\left[1^{\infty}\right]$

$= e^{\lim_{x\to 0} (\sec x - 1)(\frac{1}{x})} = e^{\lim_{x\to 0} \frac{\sec x - 1}{x}}$ $\frac{0}{0}$ I

$= e^{\lim_{x\to 0} \frac{\sec x \cdot \tan x - 0}{1}} = e^{\lim_{x\to 0} \frac{\tan x}{x} \cdot \frac{\sec x}{1}}$

$= e^{\lim_{x\to 0} x \cdot \sec x} = e^{\lim_{x\to 0} 0 \cdot \frac{\sec 0}{1}} = e^0 = \boxed{1}$

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② TENTUKAN TURUNAN PERTAMA $\left(\frac{dy}{dx}\right)$ DARI FUNGSI BERIKUT :

a) $\boxed{y = \arctan(3x+5)}$

$y' = \frac{dy}{dx} = \frac{1}{1+(3x+5)^2} \cdot (3x+5)' = \frac{3}{1+9x^2+30x+25}$

$= \frac{3}{9x^2+30x+26}$

(2) b) $x^2 + y^2 - 4x + 8xy - 6y + 5 = 0$

$\rightarrow 2x + 2y \cdot y' - 4 + (8x)'y + (8x) \cdot y' - 6y' + 0 = 0$

$2x + 2y \cdot y' - 4 + 8y + 8x \cdot y' - 6y' = 0$

$8x \cdot y' + 2y \cdot y' - 6y' = -2x - 8y + 4$

(:2)

$4x \cdot y' + y \cdot y' - 3y' = -x - 4y + 2$

$y'(4x + y - 3) = -x - 4y + 2$

$$y' = \frac{-x - 4y + 2}{4x + y - 3}$$

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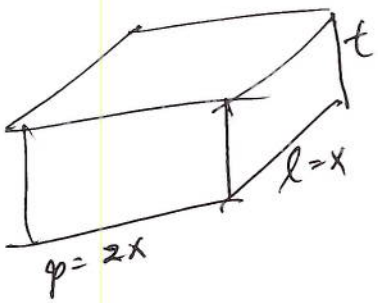
(3) SUATU KALENG (TANPA TUTUP BAGIAN ATASNYA) BERBENTUK BALOK DENGAN VOLUME $\frac{500}{3}$ cm^3, DIMANA panjang KALENG SAMA DENGAN DUA KALI LEBARNYA. TENTUKAN UKURAN KALENG TERSEBUT, AGAR BAHAN PEMBUATNYA SEHEMAT MUNGKIN!



$V = p \cdot l \cdot t = \frac{500}{3}$

$V = (2x) \cdot (x) \cdot t = \frac{500}{3}$

$2x^2 \cdot t = \frac{500}{3}$

$t = \frac{500}{6x^2} = \frac{250}{3x^2}$

$L = p \cdot l + 2 \cdot (p + l) \cdot t$

$L = (2x) \cdot (x) + 2(2x + x) \cdot t$

$L = 2x^2 + 6x \cdot \frac{250}{3x^2}$

$$L = 2x^2 + \frac{500}{x}$$

$$L = 2x^2 + 500 \cdot x^{-1}$$

$$\rightarrow L' = 4x + 500 \cdot (-1)\, x^{-2} = 4x - \frac{500}{x^2}$$

SYARAT L MIN : $L' = 0$

$$\rightarrow 4x - \frac{500}{x^2} = 0$$

$$4x = \frac{500}{x^2}$$

$$x^3 = \frac{500}{4} = 125$$

$$x = \sqrt[3]{125} = \sqrt[3]{5^3} = 5$$

$$p = 2x = 2(5) = 10$$

$$l = x = 5$$

$$t = \frac{250}{3x^2} = \frac{250}{3(5)^2} = \frac{\overset{10}{\cancel{250}}}{3\overset{1}{\cancel{(25)}}} = \frac{10}{3} = 3\frac{1}{3}$$

JADI UKURAN KALENG TERSEBUT :

10 cm X 5 cm X $3\frac{1}{3}$ cm

(4) DIKETAHUI $z = x^2 - xy + 3y^2 + 11x - 6$

TENTUKAN z EKSTRIM DAN JENIS EKSTRIM TERSEBUT!

SOLUSI :

$z = x^2 - xy + 3y^2 + 11x - 6$

$z_x = \frac{\partial z}{\partial x} = 2x - y + 11$

$z_{xx} = r = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) = \frac{\partial^2 z}{\partial x^2} = 2$

$z = x^2 - xy + 3y^2 + 11x - 6$

$z_y = \frac{\partial z}{\partial y} = -x + 6y$

$z_{yy} = t = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) = \frac{\partial^2 z}{\partial y^2} = 6$

$\frac{\partial z}{\partial x} = 2x - y + 11$

$z_{xy} = s = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) = -1$

DISKRIMINAN :

$\triangle = r.t - s^2$

$\triangle = z_{xx}.z_{yy} - (z_{xy})^2$

$\triangle = (2).(6) - (-1)^2$

$\triangle = 12 - 1$

$\triangle = 11 > 0 \rightarrow$ ADA HARGA EKSTRIM

SYARAT TITIK KRITIS

$\frac{\partial z}{\partial x} = 0 \rightarrow 2x - y + 11 = 0 \rightarrow 2x - y = -11 \quad | \times 1 |$

$\frac{\partial z}{\partial y} = 0 \rightarrow -x + 6y = 0 \rightarrow -x + 6y = 0 \quad | \times 2 |$

$2x - y = -11$

$-2x + 12y = 0 \; +$

$11y = -11$

$y = \frac{-11}{11} = -1$

$-x + 6y = 0$

$6y = x$

$\rightarrow x = 6y$

$x = 6(-1)$

$x = -6 \rightarrow$ TITIK KRITIS (x, y)

TITIK KRITIS $(-6, -1)$

$\Delta = 11 > 0$
$r = z_{xx} = 2 > 0$
} MINIMUM DI TITIK $(-6, -1)$

HARGA/NILAI MINIMUM $= z = x^2 - xy + 3y^2 + 11x - 6$

$= (-6)^2 - (-6)(-1) + 3(-1)^2 + 11(-6) - 6$

$= 36 - 6 + 3 - 66 - 6$

$= \underline{-39}$