UTS "MATEMATIKA TEKNIK I"

JUMAT, 14 NOV. 2014

pk. 15.00 – 16.40 (100 MENIT)

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① $\underbrace{(9x+8y)}_{M(x,y)}\,dx + \underbrace{(8x+9y)}_{N(x,y)}\,dy = 0$

$M(x,y) = 9x + 8y$ | $N(x,y) = 8x + 9y$

$\frac{\partial M}{\partial y} = 0 + 8(1) = 8$ | $\frac{\partial N}{\partial x} = 8(1) + 0 = 8$

$\longrightarrow \boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}} \Longrightarrow$ PD. EKSAK

$F(x,y) = \int M(x,y)\,dx + c(y) \longrightarrow \boxed{\frac{\partial F}{\partial y} = N(x,y)}$

$F(x,y) = \int (9x+8y)\,dx + c(y)$

$\boxed{F(x,y) = \frac{9}{2}x^2 + 8y\,x + c(y)}$

$\frac{\partial F}{\partial y} = 0 + 8x(1) + c'(y) = N(x,y)$

$8x + c'(y) = 8x + 9y$

$c'(y) = 9y$

$c(y) = \int c'(y)\,dy = \int 9y\,dy = \frac{9}{2}y^2 + C$

Solusi umum PD

$F(x,y) = 0$

$\frac{9}{2}x^2 + 8xy + c(y) = 0$

$\boxed{\frac{9}{2}x^2 + 8xy + \frac{9}{2}y^2 + C = 0}$

atau

$9x^2 + 16xy + 9y^2 + 2C = 0$

$\boxed{9x^2 + 16xy + 9y^2 + C = 0}$

② $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = e^{4x}$

$D^2y - 6Dy + 8y = e^{4x}$

$\boxed{(D^2 - 6D + 8).y = e^{4x}}$

(I) $(D^2 - 6D + 8).y = 0$

PERS. EIGEN

$\alpha^2 - 6\alpha + 8 = 0$

$(\alpha - 2)(\alpha - 4) = 0$

$\alpha_1 = 2$, $\alpha_2 = 4$

$y_h = c_1 . e^{\alpha_1 x} + c_2 . e^{\alpha_2 x}$

$y_h = c_1 e^{2x} + c_2 e^{4x}$

(II) $(D^2 - 6D + 8).y = e^{4x}$ → $k = 4$, $D = k = 4$

$y_K = \frac{e^{4x}}{D^2 - 6D + 8} = \frac{e^{4x}}{(D-2)(D-4)}$

$= \frac{1}{D-2} \cdot (D-4)^{-1} . e^{4x}(1)$

$= \frac{1}{4-2} \cdot e^{4x} . D^{-1}(1)$

$= \frac{1}{2} . e^{4x} \int (1)\, dx$

$= \frac{1}{2} e^{4x} . x$

$y_K = \frac{1}{2} x . e^{4x}$

Solusi umum PD

$y = y_h + y_K$

$\boxed{y = c_1 e^{2x} + c_2 e^{4x} + \frac{1}{2} x . e^{4x}}$

③ $\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 10y = \cos 3x$

$D^2y - 7Dy + 10y = \cos 3x$

$\boxed{(D^2 - 7D + 10).y = \cos 3x}$

Ⓘ $(D^2 - 7D + 10).y = 0$

PERS. EIGEN

$\alpha^2 - 7\alpha + 10 = 0$

$(\alpha - 2)(\alpha - 5) = 0$

$\alpha_1 = 2, \alpha_2 = 5$

$y_h = c_1 \cdot e^{\alpha_1 x} + c_2 \cdot e^{\alpha_2 x}$

$y_h = c_1 e^{2x} + c_2 e^{5x}$

Ⓘ $(D^2 - 7D + 10).y = \cos 3x$ → $a = 3$

$D^2 = -a^2$

$D^2 = -(3)^2 = -9$

$y_k = \frac{\cos 3x}{D^2 - 7D + 10}$

$= \frac{\cos 3x}{-9 - 7D + 10} = \frac{\cos 3x}{1 - 7D} \cdot \frac{1 + 7D}{1 + 7D}$

$= \frac{\cos 3x\,(1 + 7D)}{(1)^2 - (7D)^2} = \frac{\cos 3x + 7D(\cos 3x)}{1 - 49D^2}$

$= \frac{\cos 3x + 7(-\sin 3x).(3)}{1 - 49(-9)}$

$= \frac{\cos 3x - 21 \cdot \sin 3x}{1 + 441}$

$y_k = \frac{\cos 3x - 21 \sin 3x}{442} = \frac{1}{442}\cos 3x - \frac{21}{442}\sin 3x$

SOLUSI UMUM PD

$y = y_h + y_k$

$\boxed{y = c_1 e^{2x} + c_2 e^{5x} + \frac{1}{442}\cos 3x - \frac{21}{442}\sin 3x}$

④ JUMLAH PENDUDUK SEKARANG = $P(0)$

JUMLAH PENDUDUK SETELAH t TAHUN = $P(t)$

$P(5) = 4\,P(0)$

$P(t) = 8\,P(0) \longrightarrow t = ?$

LAJU PERTUMBUHAN PENDUDUK

$$\frac{\partial P}{\partial t} = k \cdot P$$

$$\frac{\partial P}{P} = k \cdot dt \longrightarrow \frac{\partial P}{P} - k \cdot dt = 0$$

$$\int \frac{\partial P}{P} - k \cdot \int dt = \int 0$$

$$\ln P - k \cdot t = c$$

$$\ln P - \ln e^{kt} = \ln c$$

$$\ln \frac{P(t)}{e^{kt}} = \ln c$$

$$\frac{P(t)}{e^{kt}} = c$$

$$P(t) = c \cdot e^{kt}$$

$$P(0) = c \cdot e^{k(0)}$$

$$P(0) = c \cdot \frac{e^0}{1}$$

$$P(0) = c$$

$$P(t) = \textcircled{c} \cdot e^{kt}$$

$$P(t) = P(0) \cdot e^{kt}$$

$$P(5) = P(0) \cdot e^{k(5)} = 4\,P(0)$$

$$e^{5k} = 4 \longrightarrow \ln e^{5k} = \ln 4$$

$$5k = \ln 4$$

$$k = \frac{\ln 4}{5}$$

$$P(t) = P(0) \cdot e^{\left(\frac{\ln 4}{5}\right)t}$$

$$P(t) = P(0) \cdot e^{k \cdot t} \longrightarrow P(t) = 8\,P(0) \longrightarrow 8\,P(0) = P(0) \cdot e^{\left(\frac{\ln 4}{5}\right) \cdot t}$$

$$8 = e^{\left(\frac{\ln 4}{5}\right) \cdot t}$$

$$\ln 8 = \ln e^{\left(\frac{\ln 4}{5}\right)t}$$

$$\ln 8 = \left(\frac{\ln 4}{5}\right) \cdot t \longrightarrow t = \frac{5 \ln 8}{\ln 4} = \frac{5 \log_2 8}{\log_2 4} = \frac{(3)(5)\log 2}{2 \log 2} = \frac{15}{2} = 7\tfrac{1}{2}$$

JUMLAH PENDUDUK MENJADI DELAPAN KALI LIPAT SETELAH $7\frac{1}{2}$ TAHUN