SOLUSI UAS KO424 "KALKULUS I" HAL 1

SEMESTER GENAP 2013/2014
JUMAT, 20 JUNI 2014
pk 15.30 - 17.10 (100 MENIT)

1 a) $\int x^2 (x^3+5)^4 dx = \int (x^3+5)^4 \cdot \frac{x^2 dx}{3x^2 dx}$

$d(x^3+5) = 3\,\underline{x^2 dx}$

$$= \frac{1}{3}\int (x^3+5)^4 \cdot d(x^3+5)$$

$$= \frac{1}{3} \cdot \frac{1}{4+1} (x^3+5)^{4+1} + C$$

$$= \underline{\underline{\frac{1}{15} (x^3+5)^5 + C}}$$

b) $\int e^x \sin 2x\, dx = \int \underbrace{\sin 2x}_{u} \cdot \underbrace{e^x dx}_{dv}$

$\boxed{dv = e^x dx}$

$\int dv = \int e^x dx$

$\boxed{v = e^x}$

$$= \int \underbrace{\sin 2x}_{u}\ d(\underbrace{e^x}_{v})$$

$$= u.v - \int v \cdot d(u)$$

$$= (\sin 2x)(e^x) - \int e^x\, d(\sin 2x)$$

$$= e^x . \sin 2x - \int e^x . (\cos 2x) . (2) dx$$

$$= e^x . \sin 2x - 2 \int \underbrace{\cos 2x}_{u} . \underbrace{e^x dx}_{dv}$$

$$= e^x . \sin 2x - 2 \int \underbrace{\cos 2x}_{u} . d(\underbrace{e^x}_{v})$$

$$= e^x . \sin 2x - 2[u.v - \int v . d(u)]$$

$$= e^x . \sin 2x - 2[(\cos 2x).(e^x) - \int e^x\, d(\cos 2x)]$$

$$= e^x . \sin 2x - 2 e^x \cos 2x + 2 \int e^x (-\sin 2x)(2) dx$$

$$\underline{\int e^x . \sin 2x\, dx} = e^x . \sin 2x - 2 e^x \cos 2x - 4 \underline{\int e^x \sin 2x\, dx}$$

$$\int e^x . \sin 2x\, dx + 4 \int e^x \sin 2x\, dx = e^x . \sin 2x - 2 e^x \cos 2x + C$$

$$5 \int e^x . \sin 2x\, dx = e^x (\sin 2x - 2 \cos 2x) + C \quad \times \frac{1}{5}$$

$$\underline{\underline{\int e^x \sin 2x\, dx = \frac{1}{5} e^x (\sin 2x - 2 \cos 2x) + C}}$$

↓

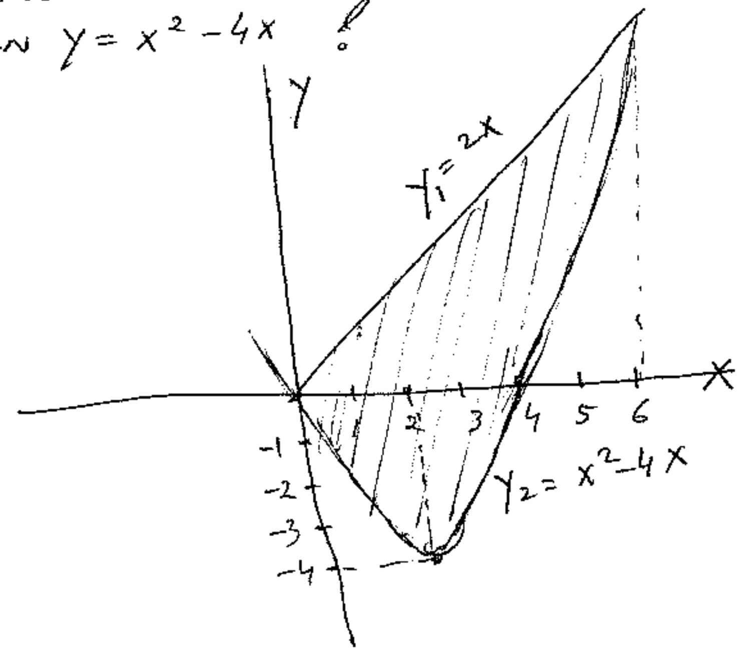
ATAU

$$\int e^{ax} \cdot \sin bx \, dx = \frac{e^{ax}}{a^2+b^2}\left(a \cdot \sin bx - b \cdot \cos bx\right) + C$$

$$\int e^{\overset{a=1}{x}} \cdot \sin \overset{b=2}{2}x \, dx = \frac{e^{x}}{1^2+2^2}\left(1 \cdot \sin 2x - 2 \cdot \cos 2x\right) + C$$

$$= \frac{1}{5} e^{x}\left(\sin 2x - 2 \cdot \cos 2x\right) + C$$

② TENTUKAN LUAS DAERAH YANG DIBATASI ANTARA DUA KURVA YANG BERPOTONGAN $y = 2x$ DAN $y = x^2 - 4x$!



SOLUSI :

$$\left.\begin{aligned} y &= x^2 - 4x \\ y &= 2x \end{aligned}\right\}$$

$\longrightarrow \quad y = y$

$$x^2 - 4x = 2x$$
$$x^2 - 4x - 2x = 0$$
$$x^2 - 6x = 0$$
$$x(x-6) = 0$$
$$x = 0, \quad x = 6$$

TERHADAP SUMBU X

$$L = \int_0^6 (y_1 - y_2)\, dx$$

$$L = \int_0^6 \left[2x - (x^2 - 4x)\right] dx$$

$$L = \int_0^6 \left(2x - x^2 + 4x\right) dx$$

$$L = \int_0^6 \left(6x - x^2\right) dx$$

$$L = \frac{6}{2}x^2 - \frac{1}{3}x^3 \Big|_0^6$$

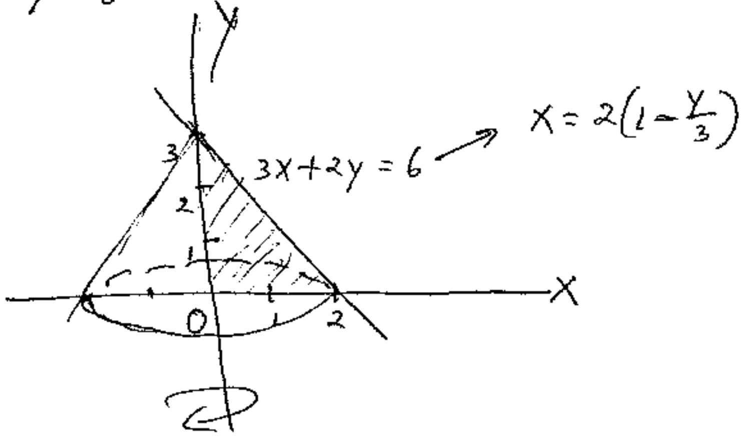
$$= 3\left(6^2 - 0^2\right) - \frac{1}{3}\left(6^3 - 0^3\right)$$

$$= 3(36) - \frac{216}{3}$$

$$= 108 - 72$$

$$= 36$$

3) TENTUKAN VOLUME BENDA PUTAR YG DIBATASI OLEH SUMBU-X, SUMBU-Y DAN GARIS LURUS $3x + 2y = 6$ YANG DIPUTAR TERHADAP SUMBU Y!



SOLUSI :

$$3x + 2y = 6$$

$$\frac{3x}{6} + \frac{2y}{6} = \frac{6}{6}$$

$$\boxed{\frac{x}{2} + \frac{y}{3} = 1}$$

$$\frac{x}{2} = 1 - \frac{y}{3}$$

$$x = 2\left(1 - \frac{y}{3}\right) \longrightarrow x^2 = 4\left(1 - \frac{y}{3}\right)^2$$

DIPUTAR TERHADAP SUMBU Y

$$V = \pi \int_0^3 x^2 . dy$$

$$V = \pi \int_0^3 4\left(1 - \frac{y}{3}\right)^2 dy$$

$$-\frac{1}{3}\,dy$$

$$V = 4\pi\left(-\frac{3}{1}\right)\int_0^3 \left(1 - \frac{y}{3}\right)^2 . \, d\left(1 - \frac{y}{3}\right)$$

$$V = \cancel{-12}\;4\,\pi \cdot \frac{1}{\cancel{3}\;1}\left(1 - \frac{y}{3}\right)^3 \Big|_0^3$$

$$V = -4\pi\left[\left(1 - \frac{3}{3}\right)^3 - \left(1 - \frac{0}{3}\right)^3\right]$$

$$V = -4\pi\left[(1-1)^3 - (1-0)^3\right]$$

$$V = -4\pi\left[0 - 1\right]$$

$$V = 4\pi$$

4 UJI KONVERGENSI DERET : $\sum_{n=1}^{\infty} \frac{5^n}{n!}$

(APAKAH DERET TERSEBUT KONVERGEN ATAU DIVERGEN ?)

SOLUSI :

$$\sum_{n=1}^{\infty} \frac{5^n}{n!} \longrightarrow U_n = \frac{5^n}{n!}$$

$$U_{n+1} = \frac{5^{n+1}}{(n+1)!} = \frac{5^n \cdot 5}{(n+1) \cdot n!}$$

TES RATIO

$$\lim_{n \to \infty} \frac{U_{n+1}}{U_n} = \lim_{n \to \infty} \frac{\frac{5^n \cdot 5}{(n+1) \cdot n!}}{\frac{5^n}{n!}}$$

$$= \lim_{n \to \infty} \frac{\cancel{5^n} \cdot 5}{(n+1) \cdot \cancel{n!}} \cdot \frac{\cancel{n!}}{\cancel{5^n}}$$

$$= \lim_{n \to \infty} \frac{5}{n+1} = \lim_{n \to \infty} \frac{5}{\infty + 1}$$

$$= \lim_{n \to \infty} \frac{5}{\infty} = \underline{0 < 1}$$

$\longrightarrow$ DERET KONVERGEN