TO. KALKULUS II

1A.

$$\boxed{\begin{aligned}&\frac{dy}{dx} + \frac{y}{x} = e^x\\&\frac{dy}{dx} + P(x).y = Q(x)\end{aligned}} \qquad \begin{aligned}P(x) &= \frac{1}{x}\\ Q(x) &= e^x\end{aligned}$$

PD. LINEAR

$$S(x) = e^{\int P(x)\,dx} = e^{\int \frac{1}{x}dx} = e^{\int \frac{dx}{x}} = e^{\ln x} = x$$

SOLUSI UMUM PD

$$y = \frac{1}{S(x)}\left[\int S(x).Q(x)\,dx + c\right] \rightarrow y = \frac{1}{x}\left[\int x.e^x dx + c\right]$$

$$\rightarrow y = \frac{1}{x}\left[\int \underset{u}{x}.\,d(\underset{v}{e^x}) + c\right] \rightarrow y = \frac{1}{x}\left[x.e^x - \int e^x d(x) + c\right]$$

$$\rightarrow y = \frac{1}{x}\left[x.e^x - e^x + c\right]$$

$$\boxed{y = e^x - \frac{e^x}{x} + \frac{c}{x}}$$

1B

$$(\sin^2 x - y)\,dx - x\,dy = 0$$

$$-x.\frac{dy}{dx} + (\sin^2 x - y) = 0$$

$$\frac{dy}{dx} + \frac{(\sin^2 x - y)}{-x} = 0$$

$$\frac{dy}{dx} - \frac{\sin^2 x}{x} + \frac{y}{x} = 0$$

$$\boxed{\begin{aligned}&\frac{dy}{dx} + \frac{y}{x} = \frac{\sin^2 x}{x}\\&\frac{dy}{dx} + P(x).y = Q(x)\end{aligned}} \qquad \begin{aligned}P(x) &= \frac{1}{x}\\ Q(x) &= \frac{\sin^2 x}{x}\end{aligned}$$

PD. LINEAR

$$S(x) = e^{\int P(x)\,dx} = e^{\int \frac{1}{x}dx} = e^{\int \frac{dx}{x}}$$

$$= e^{\ln x} = x$$

SOLUSI UMUM PD

$$y = \frac{1}{s(x)}\left[\int s(x)\cdot Q(x)\,dx + C\right]$$

$$y = \frac{1}{x}\left[\int \cancel{x}\,\frac{\sin^2 x}{\cancel{x}}\,dx + C\right]$$

$$y = \frac{1}{x}\left[\int \sin^2 x\,dx + C\right]$$

$$\boxed{\begin{aligned}\cos 2x &= 1 - 2\sin^2 x\\ 2\sin^2 x &= 1-\cos 2x\\ \sin^2 x &= \tfrac{1}{2} - \tfrac{1}{2}\cos 2x\end{aligned}}$$

$$y = \frac{1}{x}\left[\int\left(\tfrac{1}{2} - \tfrac{1}{2}\cos 2x\right)dx + C\right]$$

$$y = \frac{1}{x}\left[\tfrac{1}{2}\int dx - \tfrac{1}{2}\int \cos 2x\,dx + C\right]$$

$$y = \frac{1}{x}\left[\tfrac{1}{2}x - \tfrac{1}{2}\cdot\left(\tfrac{1}{2}\right)\int \cos 2x\cdot\frac{2\,dx}{d(2x)} + C\right]$$

$$y = \frac{1}{x}\left[\tfrac{1}{2}x - \tfrac{1}{4}\sin 2x + C\right]$$

$$y = \frac{1}{2} - \frac{1}{4}\cdot\frac{\sin 2x}{x} + \frac{C}{x}$$

$$\boxed{y = \frac{1}{2} - \frac{\sin 2x}{4x} + \frac{C}{x}}$$

2A) $\underbrace{\left(y\cdot\sin x + \tfrac{1}{8}y^2 + x^3\right)}_{M(x,y)}dx + \underbrace{\left(\tfrac{1}{2}y - \cos x + \tfrac{1}{4}xy\right)}_{N(x,y)}dy = 0$

$\frac{1}{8}$

$$M(x,y) = y\sin x + \tfrac{1}{8}y^2 + x^3$$

$$\frac{\partial M}{\partial y} = (1)\cdot\sin x + \tfrac{1}{8}(2y) + 0 = \sin x + \tfrac{1}{4}y$$

$$N(x,y) = \tfrac{1}{2}y - \cos x + \tfrac{1}{4}xy$$

$$\frac{\partial N}{\partial x} = 0 - (-\sin x) + \tfrac{1}{4}y(1) = \sin x + \tfrac{1}{4}y$$

$$\boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}} \Longrightarrow \text{PD. EKSAK}$$

$$F(x,y) = \int N(x,y)\,dy + C(x) \longrightarrow \boxed{\frac{\partial F}{\partial x} = M(x,y)}$$

$$F(x,y) = \int\left(\tfrac{1}{2}y - \cos x + \tfrac{1}{4}xy\right)dy + C(x)$$

$$F(x,y) = \tfrac{1}{2}\int y\,dy - \cos x\cdot\int dy + \tfrac{1}{4}x\int y\,dy + C(x)$$

$$F(x,y) = \tfrac{1}{2}\cdot\left(\tfrac{1}{2}y^2\right) - (\cos x)(y) + \tfrac{1}{4}x\left(\tfrac{1}{2}y^2\right) + C(x)$$

$$\boxed{F(x,y) = \tfrac{1}{4}y^2 - y.\cos x + \tfrac{1}{8}xy^2 + c(x)}$$

$$\frac{\partial F}{\partial x} = 0 - y.(-\sin x) + \tfrac{1}{8}y^2(1) + c'(x) = M(x,y)$$

$$y.\sin x + \cancel{\tfrac{1}{8}y^2} + c'(x) = y.\sin x + \cancel{\tfrac{1}{8}y^2} + x^3$$

$$\underline{c'(x) = x^3}$$

$$c(x) = \int c'(x)\,dx = \int x^3\,dx = \underline{\tfrac{1}{4}x^4 + c}$$

SOLUSI UMUM PD

$$F(x,y) = 0$$

$$\tfrac{1}{4}y^2 - y.\cos x + \tfrac{1}{8}xy^2 + c(x) = 0$$

$$\boxed{\tfrac{1}{4}y^2 - y.\cos x + \tfrac{1}{8}xy^2 + \tfrac{1}{4}x^4 + c = 0} \quad \times 8$$

atau : $\boxed{2y^2 - 8y.\cos x + xy^2 + 2x^4 + \underset{c}{\cancel{8c}} = 0}$

2B) $\underbrace{(5x^4y - xy^2)}_{M(x,y)}dx + \underbrace{(x^5 - x^2y + 2y)}_{N(x,y)}dy = 0$

$$M(x,y) = 5x^4y - xy^2$$

$$\frac{\partial M}{\partial y} = 5x^4(1) - x(2y) = \underline{5x^4 - 2xy}$$

$$N(x,y) = x^5 - x^2y + 2y$$

$$\frac{\partial N}{\partial x} = 5x^4 - (2x).y + 0 = \underline{5x^4 - 2xy}$$

$$\longrightarrow \boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}} \Longrightarrow \text{PD. EKSAK}$$

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$$F(x,y) = \int M(x,y)\,dx + C(y) \longrightarrow \boxed{\frac{\partial F}{\partial y} = N(x,y)}$$

$$F(x,y) = \int (5x^4 y - x y^2)\,dx + C(y)$$

$$F(x,y) = 5y \int x^4\,dx - y^2 \int x\,dx + C(y)$$

$$F(x,y) = 5y\left(\tfrac{1}{5}x^5\right) - y^2\left(\tfrac{1}{2}x^2\right) + C(y)$$

$$\boxed{F(x,y) = x^5 y - \tfrac{1}{2}x^2 y^2 + C(y)}$$

$$\frac{\partial F}{\partial y} = x^5(1) - \tfrac{1}{2}x^2(2y) + C'(y) = N(x,y)$$

$$\cancel{x^5} - \cancel{x^2 y} + C'(y) = \cancel{x^5} - \cancel{x^2 y} + 2y$$

$$\boxed{C'(y) = 2y}$$

$$C(y) = \int C'(y)\,dy = \int 2y\,dy = 2\left(\tfrac{1}{2}y^2\right) + C$$
$$= y^2 + C$$

Solusi umum PD

$$F(x,y) = 0$$

$$x^5 y - \tfrac{1}{2}x^2 y^2 + C(y) = 0$$

$$\boxed{x^5 y - \tfrac{1}{2}x^2 y^2 + y^2 + C = 0}$$

ATAU

$$x^5 y - \tfrac{1}{2}x^2 y^2 + y^2 = -C$$
$$= C$$

3A) $\frac{d^2y}{dt^2} - 3 + 2t = e^t$ DI MANA $\underline{y(0)=0}$, $\underline{y'(0)=0}$, $\cancel{y''(0)=6}$

$\frac{d^2y}{dt^2} = e^t + (2t-3)$

$\boxed{D^2y = e^t + (2t-3)}$

$D^2y = 0$

PERS. EIGEN

$\alpha^2 = 0$

$\alpha_{1,2} = 0$

$\alpha_1 = \alpha_2 = 0$

$y_h = (c_1 + c_2.t)e^{\alpha t}$

$y_h = (c_1 + c_2.t)\underbrace{e^{0t}}_{1}$

$\boxed{y_h = c_1 + c_2.t}$

$\boxed{D^2y = \underbrace{e^t}_{\mathrm{I}} + \underbrace{(2t-3)}_{\mathrm{II}}}$

I $D^2y = e^t \longrightarrow K = 1$, $D = K = 1$

$y_{K1} = \frac{e^t}{D^2}$

$y_{K1} = \frac{e^t}{1^2} = \underline{e^t}$

II $D^2y = 2t - 3$

$y_{K2} = \frac{1}{D^2}(2t-3) = D^{-2}(2t-3)$

$= D^{-1}.D^{-1}(2t-3)$

$= \int\int (2t-3).dt.dt$

$= \int (\frac{2}{2}t^2 - 3t)\,dt$

$= \int (t^2 - 3t)\,dt = \underline{\frac{1}{3}t^3 - \frac{3}{2}t^2}$

SOLUSI KHUSUS : $y_K = y_{K1} + y_{K2}$

$\boxed{y_K = e^t + \frac{1}{3}t^3 - \frac{3}{2}t^2}$

SOLUSI UMUM PD : $y = y_h + y_K$

$\boxed{y(t) = c_1 + c_2.t + e^t + \frac{1}{3}t^3 - \frac{3}{2}t^2}$

$y'(t) = 0 + c_2(1) + e^t + \frac{1}{3}(3t^2) - \frac{3}{2}(2t)$

$\boxed{y'(t) = c_2 + e^t + t^2 - 3t}$

⑥

$$\cancel{y''(t) = 0 + e^t + 2t - 3(1)}$$

$$\cancel{\boxed{y''(t) = e^t + 2t - 3}}$$

$y(0) = 0 \longrightarrow y(t) = c_1 + c_2 t + e^t + \frac{1}{3}t^3 - \frac{3}{2}t^2$

$$y(0) = c_1 + \underline{c_2(0)} + e^0 + \underline{\tfrac{1}{3}(0)^3} - \underline{\tfrac{3}{2}(0)^2} = 0$$

$$c_1 + 0 + 1 + 0 - 0 = 0$$

$$\boxed{c_1 = -1}$$

$y'(0) = 0 \longrightarrow y'(t) = c_2 + e^t + t^2 - 3t$

$$y'(0) = c_2 + e^0 + 0^2 - 3(0) = 0$$

$$c_2 + 1 + 0 - 0 = 0$$

$$\boxed{c_2 = -1}$$

SOLUSI ~~——~~ PD

$$y(t) = c_1 + c_2 \cdot t + e^t + \frac{1}{3}t^3 - \frac{3}{2}t^2$$

$$\boxed{y(t) = -1 - t + e^t + \frac{1}{3}t^3 - \frac{3}{2}t^2}$$

3 B) $\frac{d^2y}{dx^2} - \frac{dy}{dx} + \frac{5}{4}y = 5 + 18x + 12x^2$

$D^2y - Dy + \frac{5}{4}y = 12x^2 + 18x + 5$

$$\boxed{\left(D^2 - D + \frac{5}{4}\right) \cdot y = 12x^2 + 18x + 5}$$

I $\left(D^2 - D + \frac{5}{4}\right) \cdot y = 0$

PERS. EIGEN

$\alpha^2 - \alpha + \frac{5}{4} = 0 \quad \times 4$

~~$a = 1,\ b = 1,\ c = \frac{5}{4}$~~

$4\alpha^2 - 4\alpha + 5 = 0$

$a = 4,\ b = -4,\ c = 5$

RUMUS abc

$\alpha_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\alpha_{1,2} = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(4)(5)}}{2(4)}$

$= \frac{4 \pm \sqrt{16 - 80}}{8} = \frac{4 \pm \sqrt{-64}}{8}$

$= \frac{4 \pm \sqrt{64} \cdot \sqrt{-1}}{8} = \frac{4 \pm 8i}{8}$

$\alpha_{1,2} = \frac{4}{8} \pm \frac{8}{8}i$

$$\boxed{\alpha_{1,2} = \frac{1}{2} \pm i}$$

$a = \frac{1}{2}$ $\quad b = 1$

$y_h = e^{ax}(c_1 \cdot \cos bt + c_2 \cdot \sin bt)$

$$\boxed{y_h = e^{\frac{1}{2}x}(c_1 \cdot \cos t + c_2 \cdot \sin t)}$$

⑧

II $(D^2 - D + \frac{5}{4})y = 12x^2 + 18x + 5$

$D(12x^2+18x+5) = 24x + 18$
$D^2(12x^2+18x+5) = 24$
$D^3(12x^2+18x+5) = 0$ ×

$$y_k = \frac{1}{\frac{5}{4} - D + D^2} \cdot (12x^2+18x+5)$$

$$\frac{5}{4} - D + D^2 \ / \ 1 \ \backslash \ \frac{4}{5} + \frac{16}{25}D - \frac{16}{125}D^2$$

$$1 - \frac{4}{5}D + \frac{4}{5}D^2 \ -$$

$$\frac{4}{5}D - \frac{4}{5}D^2$$

$$\frac{4}{5}D - \frac{16}{25}D^2 + \frac{16}{25}D^3 \ -$$

$$-\frac{4}{25}D^2 - \frac{16}{25}D^3$$

$$y_k = \left(\frac{4}{5} + \frac{16}{25}D - \frac{16}{125}D^2\right)(12x^2+18x+5)$$

$$y_k = \frac{4}{5}(12x^2+18x+5) + \frac{16}{25}D(12x^2+18x+5) - \frac{16}{125}D^2(12x^2+18x+5)$$

$$y_k = \frac{48}{5}x^2 + \frac{72}{5}x + 4 + \frac{16}{25}(24x+18) - \frac{16}{125}(24)$$

$$y_k = \frac{48}{5}x^2 + \frac{72}{5}x + 4 + \frac{384}{25}x + \frac{288}{25} - \frac{384}{125}$$

$$y_k = \frac{48}{5}x^2 + \left(\frac{360}{25}x + \frac{384}{25}x\right) + \left(\frac{500}{125} + \frac{1440}{125} - \frac{384}{125}\right)$$

$$\boxed{y_k = \frac{48}{5}x^2 + \frac{744}{25}x + \frac{1556}{125}}$$

SOLUSI UMUM PD

$$y = y_h + y_k$$

$$\boxed{y = e^{\frac{1}{2}x}(c_1 . \cos t + c_2 . \sin t) + \frac{48}{5}x^2 + \frac{744}{25}x + \frac{1556}{125}}$$

(4)

$$\boxed{\begin{aligned}\frac{dx}{dt} &= 4x+2y-8t \\ \frac{dy}{dt} &= 3x-y+2t\end{aligned}} \longrightarrow \boxed{\frac{dx}{dt}-4x+8t = 2y}$$

$$\frac{d^2x}{dt^2} = 4\frac{dx}{dt} + 2\frac{dy}{dt} - 8$$

$$\frac{d^2x}{dt^2} = 4\frac{dx}{dt} + 2(3x-y+2t) - 8$$

$$\frac{d^2x}{dt^2} = 4\frac{dx}{dt} + 6x - 2y + 4t - 8$$

$$\frac{d^2x}{dt^2} = 4\frac{dx}{dt} + 6x - \left(\frac{dx}{dt} - 4x + 8t\right) + 4t - 8$$

$$\frac{d^2x}{dt^2} = 4\frac{dx}{dt} + 6x - \frac{dx}{dt} + 4x - 8t + 4t - 8$$

$$\frac{d^2x}{dt^2} = 3\frac{dx}{dt} + 10x - 4t - 8$$

$$\boxed{\frac{d^2x}{dt^2} - 3\frac{dx}{dt} - 10x = -4t-8}$$

$$D^2x - 3Dx - 10x = -4t-8$$

$$\boxed{(D^2-3D-10)x = -4t-8}$$

(I) $(D^2-3D-10)x = 0$

PERS. EIGEN

$$\alpha^2 - 3\alpha - 10 = 0$$

$$(\alpha-5)(\alpha+2) = 0$$

$$\alpha_1 = 5, \quad \alpha_2 = -2$$

$$x_h = c_1 \cdot e^{\alpha_1 t} + c_2 \cdot e^{\alpha_2 t}$$

$$\boxed{x_h = c_1 \cdot e^{5t} + c_2 \cdot e^{-2t}}$$

(II) $(D^2-3D-10)x = -4t-8$

$$\boxed{D(-4t-8) = -4}$$

$$D^2(-4t-8) = 0$$

$$x_K = \frac{1}{-10-3D+D^2} \cdot (-4t-8)$$

$$\begin{array}{r|l} -10-3D+D^2 & 1 \\ \hline & -\frac{1}{10}+\frac{3}{100}D \end{array}$$

$$1$$

$$1 + \frac{3}{10}D - \frac{1}{10}D^2$$

$$-\frac{3}{10}D + \frac{1}{10}D^2$$

$$-\frac{3}{10}D - \frac{9}{100}D^2 + \frac{3}{100}D^3$$

$$x_K = \left(-\frac{1}{10} + \frac{3}{100}D\right)(-4t-8)$$

$$x_K = -\frac{1}{10}(-4t-8) + \frac{3}{100}D(-4t-8)$$

$$x_K = \frac{2}{5}t + \frac{4}{5} + \frac{3}{100}(-4)$$

$$x_K = \frac{2}{5}t + \frac{4}{5} - \frac{3}{25}$$

$$x_K = \frac{2}{5}t + \frac{20}{25} - \frac{3}{25}$$

$$\boxed{x_K = \frac{2}{5}t + \frac{17}{25}}$$

$$x(t) = x_h + x_k$$

$$\boxed{x(t) = c_1 \cdot e^{5t} + c_2 \cdot e^{-2t} + \frac{2}{5}t + \frac{17}{25}}$$

$$\boxed{\frac{dx}{dt} = 5c_1 \cdot e^{5t} - 2c_2 \cdot e^{-2t} + \frac{2}{5}}$$

$$\frac{dx}{dt} = 5c_1 \cdot e^{5t} - 2c_2\, e^{-2t} + \frac{10}{25}$$

$$-4x = -4c_1 \cdot e^{5t} - 4c_2 \cdot e^{-2t} - \frac{8}{5}t - \frac{68}{25}$$

$$8t = 8t \quad +$$

$$2y(t) = c_1 \cdot e^{5t} - 6 \cdot c_2 \cdot e^{-2t} + \frac{32}{5}t - \frac{58}{25} \qquad \times\frac{1}{2}$$

$$\boxed{y(t) = \frac{1}{2}c_1 \cdot e^{5t} - 3c_2\, e^{-2t} + \frac{16}{5}t - \frac{29}{25}}$$

Solusi umum PD :

$$x(t) = c_1 \cdot e^{5t} + c_2 \cdot e^{-2t} + \frac{2}{5}t + \frac{17}{25}$$

$$y(t) = \frac{1}{2}c_1 \cdot e^{5t} - 3c_2 \cdot e^{-2t} + \frac{16}{5}t - \frac{29}{25}$$

⑤ $L\{e^{-2t}(3\cos 4t - 2\sin 7t) + t.\cos t + 5 + t^2.e^t\}$

$= L\{e^{-2t}(3\cos 4t - 2\sin 7t)\} + L\{t.\cos t\} + L\{5\} + L\{t^2 e^t\}$

$L\{e^{-2t}\underbrace{(3\cos 4t - 2\sin 7t)}_{F(t)}\} = f[s-(-2)]$

$= f(s+2)$

$= \dfrac{3(s+2)}{(s+2)^2+16} - \dfrac{14}{(s+2)^2+49}$

$F(t) = 3\cos 4t - 2\sin 7t$

$f(s) = L\{F(t)\}$

$f(s) = L\{3\cos 4t - 2\sin 7t\}$

$= 3L\{\cos 4t\} - 2L\{\sin 7t\}$

$= 3\left(\dfrac{s}{s^2+4^2}\right) - 2\left(\dfrac{7}{s^2+7^2}\right)$

$\boxed{f(s) = \dfrac{3s}{s^2+16} - \dfrac{14}{s^2+49}}$

$L\{t.\underbrace{\cos t}_{F(t)}\} = (-1)^1 . f^{(1)}(s)$

$= (-1) . \dfrac{-s^2+1}{(s^2+1)^2} = \boxed{\dfrac{s^2-1}{(s^2+1)^2}}$

$F(t) = \cos t$

$f(s) = L\{F(t)\}$

$f(s) = L\{\cos t\}$

$f(s) = \dfrac{s}{s^2+1^2}$

$\boxed{f(s) = \dfrac{s}{s^2+1}}$

$f^{(1)}(s) = \dfrac{(s)'(s^2+1) - s.(s^2+1)'}{(s^2+1)^2}$

$= \dfrac{(1)(s^2+1) - s(2s)}{(s^2+1)^2}$

$= \dfrac{s^2+1-2s^2}{(s^2+1)^2}$

$= \dfrac{-s^2+1}{(s^2+1)^2}$

$L\{5\} = 5\,L\{1\} = 5\left(\frac{1}{s}\right) = \boxed{\frac{5}{s}}$

$L\{t^2 . e^t\} = L\{e^{\overset{a=1}{t}} . \underbrace{t^2}_{F(t)}\} = f(s-a)$

$= f(s-1)$

$F(t) = t^2$

$f(s) = L\{F(t)\}$

$f(s) = L\{t^2\}.$

$f(s) = \frac{2!}{s^{2+1}} = \frac{2.1}{s^3} = \boxed{\frac{2}{s^3}}$

$= \boxed{\frac{2}{(s-1)^3}}$

$L\{e^{-2t}(3\cos 4t - 2\sin 7t) + t.\cos t + 5 + t^2 . e^t\}$

$= \underline{L\{e^{-2t}(3\cos 4t - 2\sin 7t)\}} + \underline{L\{t.\cos t\}} + \underline{L\{5\}} + \underline{L\{t^2 . e^t\}}$

$= \frac{3(s+2)}{(s+2)^2+16} - \frac{14}{(s+2)^2+49} + \frac{s^2-1}{(s^2+1)^2} + \frac{5}{s} + \frac{2}{(s-1)^3}$