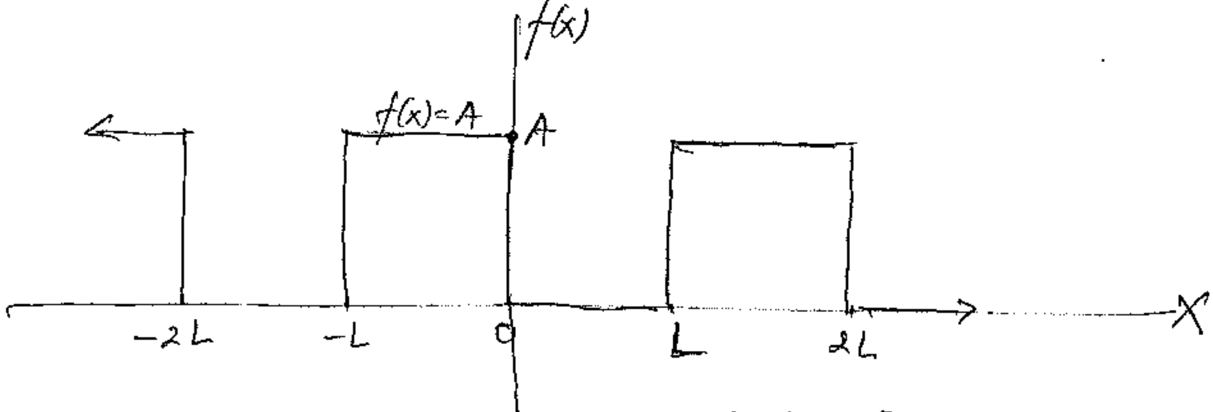
# KALKULUS II (TI-MATH / TD)

(1)




a) NYATAKAN BENTUK PERS. PIECE-WISE-NYA.
b) CARILAH DERET FOURIER DARI $f(x)$.

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SOLUSI :

a) $$f(x) = \begin{cases} 0, & 0 < x < L \\ A, & L < x < 2L \end{cases}$$

b) $$a_0 = \frac{1}{L}\int_0^{2L} f(x)\,dx = \frac{1}{L}\left[\cancel{\int_0^L (0).dx} + \int_L^{2L} (A).dx\right]$$
$$= \frac{1}{L} \cdot A \int_L^{2L} dx$$
$$= \frac{1}{L} \cdot A \cdot \left[ x \Big|_L^{2L} \right]$$
$$= \frac{1}{L} \cdot A \cdot (2L - L) = \frac{1}{\cancel{L}} \cdot A \cdot \cancel{L} = A$$

$$a_n = \frac{1}{L}\int_0^{2L} f(x) \cdot \cos\left(\frac{n\pi x}{L}\right) dx = \frac{1}{L}\left[\cancel{\int_0^L (0).\cos\left(\frac{n\pi x}{L}\right) dx} + \int_L^{2L} (A) \cos\left(\frac{n\pi x}{L}\right) dx\right]$$
$$= \frac{1}{\cancel{L}} \cdot A \cdot \left(\frac{\cancel{L}}{n\pi}\right) \int_L^{2L} \cos\left(\frac{n\pi x}{L}\right) d\left(\frac{n\pi x}{L}\right)$$
$$= \frac{A}{n\pi}\left[\sin \frac{n\pi x}{L} \Big|_L^{2L}\right]$$
$$= \frac{A}{n\pi}\left[\sin \frac{n\pi(2\cancel{L})}{\cancel{L}} - \sin \frac{n\pi \cancel{L}}{\cancel{L}}\right]$$
$$= \frac{A}{n\pi}\left[\underbrace{\sin n(2\pi)}_{0} - \underbrace{\sin n\pi}_{0}\right] = 0$$

$$b_n = \frac{1}{L}\int_0^{2L} f(x) \cdot \sin\left(\frac{n\pi x}{L}\right) dx$$
$$= \frac{1}{L}\left[\cancel{\int_0^L (0) \cdot \sin\left(\frac{n\pi x}{L}\right) dx} + \int_L^{2L} (A) \cdot \sin\left(\frac{n\pi x}{L}\right) dx\right]$$
$$= \frac{1}{\cancel{L}} \cdot A \cdot \left(\frac{\cancel{L}}{n\pi}\right) \int_L^{2L} \sin\left(\frac{n\pi x}{L}\right) \cdot d\left(\frac{n\pi x}{L}\right)$$
$$= \frac{A}{n\pi}\left[-\cos \frac{n\pi x}{L} \Big|_L^{2L}\right]$$
$$= -\frac{A}{n\pi}\left[\cos \frac{n\pi(2\cancel{L})}{\cancel{L}} - \cos \frac{n\pi(\cancel{L})}{\cancel{L}}\right]$$
$$= -\frac{A}{n\pi}\left(\cos n(2\pi) - \cos n\pi\right)$$

$$b_n = -\frac{A}{n\pi}\left(1 - \cos n\pi\right)$$

$$\boxed{b_n = \frac{A}{n\pi}\left(\cos n\pi - 1\right)}$$

DERET FOURIER

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[\cancel{\underset{0}{a_n} \cdot \cos\left(\frac{n\pi x}{L}\right)} + b_n \cdot \sin\left(\frac{n\pi x}{L}\right)\right]$$

$$f(x) = \frac{A}{2} + \sum_{n=1}^{\infty}\frac{A}{n\pi}\left(\cos n\pi - 1\right) \cdot \sin\left(\frac{n\pi x}{L}\right)$$

$$\boxed{f(x) = \frac{A}{2} + \frac{A}{\pi}\sum_{n=1}^{\infty}\frac{\cos n\pi - 1}{n} \cdot \sin\left(\frac{n\pi x}{L}\right)}$$

2. DIKETAHUI FUNGSI $f(x) = e^{-Ax}$

a) TENTUKAN TRANSFORMASI SINUS FOURIER DARI $f(x)$!

b) TENTUKAN TRANSFORMASI COSINUS FOURIER DARI $f(x)$!

a)

$$F_s(\alpha) = \sqrt{\frac{2}{\pi}}\int_0^{\infty} f(x) \cdot \sin(\alpha x)\,dx$$

$$= \sqrt{\frac{2}{\pi}}\int_0^{\infty} e^{-\overset{s}{A}x}\sin(\alpha x)\,dx$$

$$= \sqrt{\frac{2}{\pi}} \cdot L\{\sin(\alpha x)\}$$

$$= \sqrt{\frac{2}{\pi}} \cdot \left(\frac{\alpha}{A^2+\alpha^2}\right)$$

$$\int_0^{\infty} e^{-st} \cdot \sin(\alpha t)\,dt = L\{\sin \alpha t\} = \boxed{\frac{\alpha}{s^2+\alpha^2}}$$

b)

$$F_c(\alpha) = \sqrt{\frac{2}{\pi}}\int_0^{\infty} f(x) \cdot \cos(\alpha x)\,dx$$

$$= \sqrt{\frac{2}{\pi}}\int_0^{\infty} e^{-\overset{s}{A}x}\cos(\alpha x)\,dx$$

$$= \sqrt{\frac{2}{\pi}} \cdot L\{\cos \alpha x\}$$

$$= \sqrt{\frac{2}{\pi}} \cdot \left(\frac{A}{A^2+\alpha^2}\right)$$

$$\int_0^{\infty} e^{-st} \cdot \cos(\alpha t)\,dt = L\{\cos(\alpha t)\} = \boxed{\frac{s}{s^2+\alpha^2}}$$

3a) CARILAH TRANSFORMASI – Z

DARI $f[n] = An^2 + Bn + C$

SOLUSI :

TRANSFORMASI – Z :

- $Z\{f[n]\} = \sum_{n=1}^{\infty} f(n).\, z^{-n}$

$$= \sum_{n=1}^{\infty} (An^2 + Bn + C)\, z^{-n}$$

$$= \sum_{n=1}^{\infty} A n^2 z^{-n} + \sum_{n=1}^{\infty} B n\, z^{-n} + \sum_{n=1}^{\infty} C.\, z^{-n}$$

$$= A \sum_{n=1}^{\infty} n^2 . z^{-n} + B \sum_{n=1}^{\infty} n .\, z^{-n} + C \sum_{n=1}^{\infty} 1 .\, z^{-n}$$

$$= A \,.\, Z\{n^2\} + B \,.\, Z\{n\} + C \,.\, Z\{1\}$$

$$= A \cdot \frac{z(z+1)}{(z-1)^3} + B \cdot \frac{z}{(z-1)^2} + C \cdot \frac{z}{z-1}$$

DARI TABEL

$$\boxed{\begin{aligned} Z\{1\} &= \frac{z}{z-1} \\ Z\{n\} &= \frac{z}{(z-1)^2} \\ Z\{n^2\} &= \frac{z(z+1)}{(z-1)^3} \end{aligned}}$$

3b) TENTUKAN INVERS TRANSFORMASI - Z

DARI $F(z) = \dfrac{A}{(1-Bz^{-1})(1-Cz^{-1})}$

SOLUSI

$$F(z) = \frac{A}{(1-B.z^{-1}).(1-Cz^{-1})} \cdot \frac{z^2}{z^2}$$

$$F(z) = \frac{Az^2}{z(1-B.z^{-1}).z(1-Cz^{-1})}$$

$$F(z) = \frac{Az^2}{(z-B)(z-C)}$$

$$\frac{F(z)}{z} = \frac{Az}{(z-B)(z-C)} = \frac{p}{(z-B)} + \frac{q}{(z-C)} \qquad \text{KPK } (z-B)(z-C)$$

$$\boxed{A(z) = p(z-C) + q(z-B)}$$

$\boxed{z=B} \rightarrow AB = p(B-C) + q\underbrace{\cancel{(B-B)}}_{0}$

$AB = p(B-C) \rightarrow \boxed{p = \dfrac{AB}{B-C}}$

$\boxed{z=C} \rightarrow AC = p\underbrace{\cancel{(C-C)}}_{0} + q(C-B)$

$AC = q(C-B)$

$AC = -q(B-C)$

$q(B-C) = -AC$

$$\boxed{q = \frac{-AC}{B-C} = -\frac{AC}{B-C}}$$

$$\boxed{\begin{aligned} &Z\{a^n\} = \frac{z}{z-a} \\ &\rightarrow Z^{-1}\left\{\frac{z}{z-a}\right\} = a^n \end{aligned}}$$

$$\frac{F(z)}{z} = \frac{p}{z-B} + \frac{q}{z-C} \quad (z)$$

$$F(z) = p\cdot\left(\frac{z}{z-B}\right) + q\cdot\left(\frac{z}{z-C}\right)$$

$$F(z) = \frac{AB}{B-C}\left(\frac{z}{z-B}\right) - \frac{AC}{B-C}\left(\frac{z}{z-C}\right)$$

$$\rightarrow Z^{-1}\{F(z)\} = \frac{AB}{B-C}\cdot Z^{-1}\left\{\frac{z}{z-B}\right\} - \frac{AC}{B-C}\cdot Z^{-1}\left\{\frac{z}{z-C}\right\}$$

$$\boxed{f(n) = \frac{AB}{B-C}\cdot B^n - \frac{AC}{B-C}\cdot C^n}$$

4. JIKA SEBUAH PERS. BEDA YANG MENYATAKAN HUBUNGAN ANTARA INPUT DAN OUTPUT DARI SUATU SYSTEM DISCRETE TIME SEBAGAI BERIKUT

$$Y(n) + Y(n-1) - 2Y(n-2) = x(n) + x(n-1)$$

a) TENTUKAN FUNGSI TRANSFER $H(z)$ !

b) TENTUKAN DISCRETE TIME IMPULSE RESPONSE $h[n]$ !

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SOLUSI :

a) $Y(n) + Y(n-1) - 2Y(n-2) = x(n) + x(n-1)$

$$y(z).z^{-0} + y(z).z^{-1} - 2y(z).z^{-2} = X(z).z^{-0} + X(z).z^{-1}$$

$$y(z).[1 + z^{-1} - 2z^{-2}] = X(z)[1 + z^{-1}]$$

$$H(z) = \boxed{\frac{y(z)}{X(z)} = \frac{1+z^{-1}}{1+z^{-1}-2z^{-2}}} \cdot \frac{z^2}{z^2}$$

$$\boxed{H(z) = \frac{z^2+z}{z^2+z-2}}$$

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b) $h[n] = Z^{-1}\{H(z)\}$

$$H(z) = \frac{z^2+z}{z^2+z-2} \longrightarrow H(z) = \frac{z(z+1)}{z^2+z-2}$$

$$\frac{H(z)}{z} = \frac{z+1}{z^2+z-2}$$

$$\frac{H(z)}{z} = \frac{z+1}{(z-1)(z+2)} = \frac{p}{(z-1)} + \frac{q}{(z+2)} \qquad \frac{KPK}{(z-1)(z+2)}$$

$$z+1 = p(z+2) + q(z-1)$$

$\boxed{z=1} \rightarrow 1+1 = p(1+2) + q\underbrace{(1-1)}_{0}$

$2 = 3p \longrightarrow \boxed{p = \frac{2}{3}}$

$\boxed{z=-2} \rightarrow -2+1 = p\underbrace{(-2+2)}_{0} + q(-2-1)$

$-1 = -3q \longrightarrow \boxed{q = \frac{-1}{-3} = \frac{1}{3}}$

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$$\frac{H(z)}{z} = \frac{p}{z-1} + \frac{q}{z+2} \quad \cdot (z)$$

$$H(z) = p\cdot\left(\frac{z}{z-1}\right) + q\cdot\left(\frac{z}{z+2}\right)$$

$$H(z) = \frac{2}{3}\left(\frac{z}{z-1}\right) + \frac{1}{3}\left(\frac{z}{z+2}\right)$$

$$Z^{-1}\{H(z)\} = \frac{2}{3}\, Z^{-1}\left\{\frac{z}{z-1}\right\} + \frac{1}{3}\, Z^{-1}\left\{\frac{z}{z+2}\right\} \rightarrow z-(-2)$$

$$h[n] = \frac{2}{3}\cdot(1)^n + \frac{1}{3}\cdot(-2)^n$$

$$\boxed{h[n] = \frac{2}{3} + \frac{1}{3}(-2)^n}$$