$\boxed{\text{TAHAP I}}$ UAS "$\underline{\text{KALKULUS II}}$" (K0434)

KAMIS, 13 FEBRUARI 2014

pk 10.00 – 11.40 (100 MENIT)

$$L\{t^n . e^{at}\} = L\{e^{at} . t^n\} = F(s-a)$$

① $$L\{t^5 . e^{7t}\} = L\{e^{7t} . \underbrace{t^5}_{f(t)}\} = F(s-7) = \boxed{\frac{120}{(s-7)^6}}$$

$\underline{f(t) = t^5}$

$F(s) = L\{f(t)\}$

$F(s) = L\{t^5\} = \frac{5!}{s^{5+1}}$

$\boxed{F(s) = \frac{120}{s^6}}$

$\rightarrow F(s-7) = \frac{120}{(s-7)^6}$

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② $$\int_{-3}^{7}\int_{0}^{2x}\int_{y}^{x-1} dz . dy . dx = \int_{x=-3}^{7} dx \int_{y=0}^{2x} dy \int_{z=y}^{x-1} dz$$

$$= \int_{x=-3}^{7} dx \int_{y=0}^{2x} dy \left[z \Big|_{y}^{x-1} \right] = \int_{x=-3}^{7} dx \int_{y=0}^{2x} dy \left[(x-1) - y \right]$$

$$= \int_{x=-3}^{7} dx \int_{y=0}^{2x} \left[(x-1) - y \right] dy = \int_{x=-3}^{7} dx \left[(x-1) . y - \tfrac{1}{2} y^2 \Big|_{0}^{2x} \right]$$

$$= \int_{x=-3}^{7} dx \left[(x-1)(2x-0) - \tfrac{1}{2}\{(2x)^2 - 0^2\} \right]$$

$$= \int_{x=-3}^{7} \left[2x^2 - 2x - \tfrac{1}{2}(4x^2) \right] dx = \int_{-3}^{7} \left(\cancel{2x^2} - 2x - \cancel{2x^2} \right) dx$$

$$= -2\int_{-3}^{7} x\, dx = -\cancel{2}\left[\tfrac{1}{\cancel{2}} x^2 \Big|_{-3}^{7} \right]$$

$$= -\left[x^2 \Big|_{-3}^{7} \right] = -\left[(7)^2 - (-3)^2 \right] = -(49 - 9) = \boxed{-40}$$

③ a) $\varphi(x,y,z) = x^2 + y^2 + z^2$

$$\frac{\partial \varphi}{\partial x} = 2x + 0 + 0 = 2x$$

$\varphi(x,y,z) = x^2 + y^2 + z^2$

$$\frac{\partial \varphi}{\partial y} = 0 + 2y + 0 = 2y$$

$\varphi(x,y,z) = x^2 + y^2 + z^2$

$$\frac{\partial \varphi}{\partial z} = 0 + 0 + 2z = 2z$$

$$\text{GRAD}\,\varphi = \nabla\varphi = \left(\frac{\partial \varphi}{\partial x}, \frac{\partial \varphi}{\partial y}, \frac{\partial \varphi}{\partial z}\right) = \underline{\underline{(2x, 2y, 2z)}}$$

b) $\operatorname{div} \vec{A} = \nabla \cdot \vec{A}$, $\vec{A} = 3x\vec{i} + y\vec{j} + 2z\vec{k} = (3x, y, 2z)$

$$= \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \cdot (3x, y, 2z)$$

$$= \frac{\partial}{\partial x}(3x) + \frac{\partial}{\partial y}(y) + \frac{\partial}{\partial z}(2z)$$

$$= 3 + 1 + 2 = \underline{\underline{6}}.$$

c) $\operatorname{curl} \vec{A} = \nabla \times \vec{A} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \times (3x, y, 2z)$

$$= \left|\begin{array}{ccc|cc} \vec{i} & \vec{j} & \vec{k} & \vec{i} & \vec{j} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} & \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ 3x & y & 2z & 3x & y \end{array}\right.$$

$-\quad -\quad -\quad +\quad +\quad +$

$$= \left[\vec{i}\frac{\partial}{\partial y}(2z) + \vec{j}\frac{\partial}{\partial z}(3x) + \vec{k}\frac{\partial}{\partial x}(y)\right] - \left[\vec{k}\frac{\partial}{\partial y}(3x) + \vec{i}\frac{\partial}{\partial z}(y) + \vec{j}\frac{\partial}{\partial x}(2z)\right]$$

$$= (\vec{i}\cdot 0 + \vec{j}\cdot 0 + \vec{k}\cdot 0) - (\vec{k}\cdot 0 + \vec{i}\cdot 0 + \vec{j}\cdot 0)$$

$$= 0\vec{i} + 0\vec{j} + 0\vec{k} = \underline{\underline{(0, 0, 0)}}$$

④ $\vec{V} = (2, 3, 1)$ DAN $\vec{r} = 3t\,\vec{i} + t\,\vec{j} + 2t\,\vec{k}$

$$\vec{r} = (3t, t, 2t)$$

$$\vec{V} \times \vec{r} = \left|\begin{array}{ccc|cc} \vec{i} & \vec{j} & \vec{k} & \vec{i} & \vec{j} \\ 2 & 3 & 1 & 2 & 3 \\ 3t & t & 2t & 3t & t \end{array}\right|$$

$$- \quad - \quad - \qquad + \quad + \quad +$$

$$= \left(6t \cdot \vec{i} + 3t \cdot \vec{j} + 2t\,\vec{k}\right) - \left(9t \cdot \vec{k} + t\,\vec{i} + 4t\,\vec{j}\right)$$

$$= 5t \cdot \vec{i} - t \cdot \vec{j} - 7t \cdot \vec{k}$$

$$\vec{V} \times \vec{r} = \underline{(5t, -t, -7t)}$$

$$\frac{d}{dt}\left(\vec{V} \times \vec{r}\right) = \frac{d}{dt}(5t, -t, -7t) = \underline{\underline{(5, -1, -7)}}$$

TAHAP II

UAS "KALKULUS II" (K0434)

KAMIS, 13 FEBRUARI 2014

pk 13.00 – 14.40 (100 MENIT)

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① $\frac{dx}{dt} - 4x = e^{7t}$, SYARAT AWAL : $x(0) = 0$

$$L\{\frac{dx}{dt} - 4x\} = L\{e^{7t}\}$$

$$\underline{L\{\frac{dx}{dt}\}} - L\{4x\} = \frac{1}{s-7}$$

$$s \cdot L\{x\} - \underbrace{x(0)}_{=0} - 4 \cdot L\{x\} = \frac{1}{s-7}$$

$$s \cdot L\{x\} - 4 \cdot L\{x\} = \frac{1}{s-7}$$

$$L\{x\} \cdot (s-4) = \frac{1}{s-7}$$

$$L\{x\} = \boxed{\frac{1}{(s-4)(s-7)} = \frac{A}{(s-4)} + \frac{B}{(s-7)}}$$

$$A = \cancel{(s-4)} \cdot \frac{1}{\cancel{(s-4)}(s-7)} = \frac{1}{4-7} = \frac{1}{-3} = \boxed{-\frac{1}{3}}$$

$s-4=0$

$s=4$

$$B = \cancel{(s-7)} \cdot \frac{1}{(s-4)\cancel{(s-7)}} = \frac{1}{7-4} = \boxed{\frac{1}{3}}$$

$s-7=0$

$s=7$

$$L\{x\} = \frac{A}{s-4} + \frac{B}{s-7}$$

$$L\{x\} = \frac{-\frac{1}{3}}{s-4} + \frac{\frac{1}{3}}{s-7}$$

$$x(t) = L^{-1}\{\frac{-\frac{1}{3}}{s-4} + \frac{\frac{1}{3}}{s-7}\}$$

$$x(t) = -\frac{1}{3} L^{-1}\{\frac{1}{s-4}\} + \frac{1}{3} L^{-1}\{\frac{1}{s-7}\}$$

$$x(t) = -\frac{1}{3} e^{4t} + \frac{1}{3} e^{7t}$$

$$\boxed{x(t) = \frac{1}{3} e^{7t} - \frac{1}{3} e^{4t}}$$

ATAU (CARA LAIN)

(1) $\frac{dx}{dt} - 4x = e^{7t}$, SYARAT AWAL : $x(0) = 0$

$L\{\frac{dx}{dt} - 4x\} = L\{e^{7t}\}$

$L\{\frac{dx}{dt}\} - L\{4x\} = \frac{1}{s-7}$

$s \cdot L\{x\} - \underbrace{x(0)}_{0} - 4 \cdot L\{x\} = \frac{1}{s-7}$

$s \cdot L\{x\} - 4 \cdot L\{x\} = \frac{1}{s-7}$

$L\{x\} \cdot (s-4) = \frac{1}{s-7}$

$$\boxed{L\{x\} = \frac{1}{(s-7)(s-4)}}$$

$L\{x\} = \frac{1}{s^2 - 11s + 28}$

$x(t) = L^{-1}\{\frac{1}{s^2 - 11s + 28}\}$

$x(t) = L^{-1}\{\frac{1}{(s-\frac{11}{2})^2 - \frac{9}{4}}\}$

$x(t) = e^{\frac{11}{2}t} \cdot L^{-1}\{\frac{1}{s^2 - \frac{9}{4}}\}$

$x(t) = e^{\frac{11}{2}t} \cdot L^{-1}\{\frac{1}{s^2 - (\frac{3}{2})^2}\}$

$x(t) = \frac{2}{3} \cdot e^{\frac{11}{2}t} \cdot L^{-1}\{\frac{\frac{3}{2}}{s^2 - (\frac{3}{2})^2}\}$

$$\boxed{x(t) = \frac{2}{3} e^{\frac{11}{2}t} \cdot \sinh \frac{3}{2}t}$$

$s^2 - 11s + 28$

$= (s^2 - 11s + \frac{121}{4}) - \frac{121}{4} + 28$

$= (s - \frac{11}{2})^2 - \frac{121}{4} + \frac{112}{4}$

$= (s - \frac{11}{2})^2 - \frac{9}{4}$

$\left(\frac{\text{Koef. } s}{2}\right)^2 = \left(\frac{-11}{2}\right)^2 = \frac{121}{4}$

$L^{-1}\{F(s-a)\} = e^{at} \cdot L^{-1}\{F(s)\}$

PERIKSA :

$x(t) = \frac{2}{3} e^{\frac{11}{2}t} \cdot \sinh \frac{3}{2}t$

$x(t) = \frac{\cancel{2}}{3} e^{\frac{11}{2}t} \left(\frac{e^{\frac{3}{2}t} - e^{-\frac{3}{2}t}}{\cancel{2}}\right) = \frac{1}{3} e^{\frac{14}{2}t} - \frac{1}{3} e^{\frac{8}{2}t}$

$$\boxed{x(t) = \frac{1}{3} e^{7t} - \frac{1}{3} e^{4t}}$$

② $L^{-1}\left\{\frac{1}{s^2-6s+8}\right\} = L^{-1}\left\{\frac{1}{(s-4)(s-2)}\right\}$

$$\boxed{\frac{1}{(s-4)(s-2)} = \frac{A}{(s-4)} + \frac{B}{(s-2)}}$$

$$A = \underset{\substack{\downarrow \\ s-4=0 \\ s=4}}{\cancel{(s-4)}} \cdot \frac{1}{\cancel{(s-4)}(s-2)} = \frac{1}{4-2} = \boxed{\frac{1}{2}}$$

$$B = \underset{\substack{\downarrow \\ s-2=0 \\ s=2}}{\cancel{(s-2)}} \cdot \frac{1}{(s-4)\cancel{(s-2)}} = \frac{1}{2-4} = \frac{1}{-2} = \boxed{-\frac{1}{2}}$$

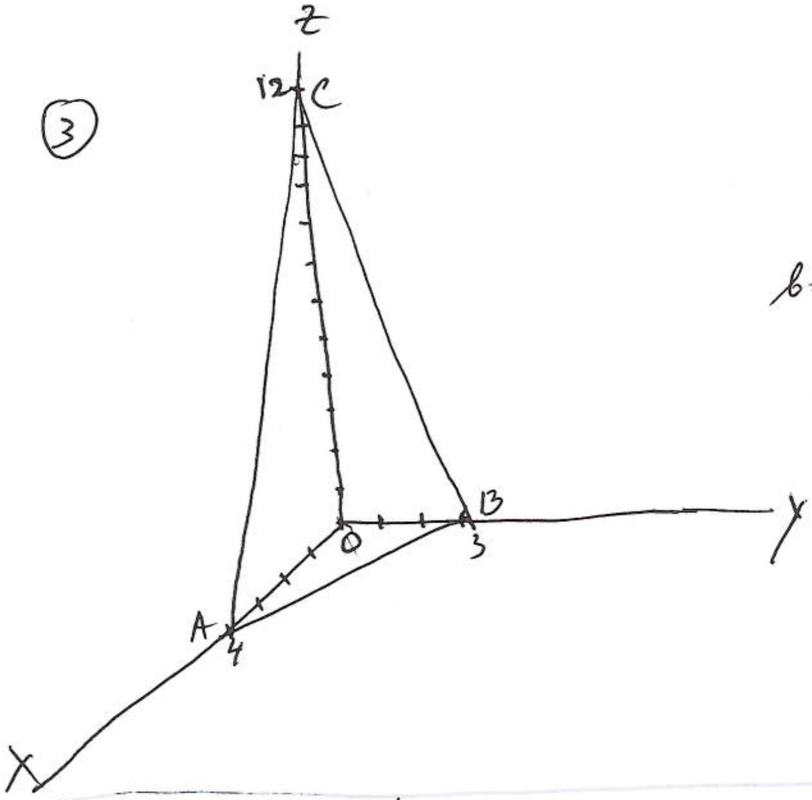
$$L^{-1}\left\{\frac{1}{s^2-6s+8}\right\} = L^{-1}\left\{\frac{A}{s-4} + \frac{B}{s-2}\right\}$$

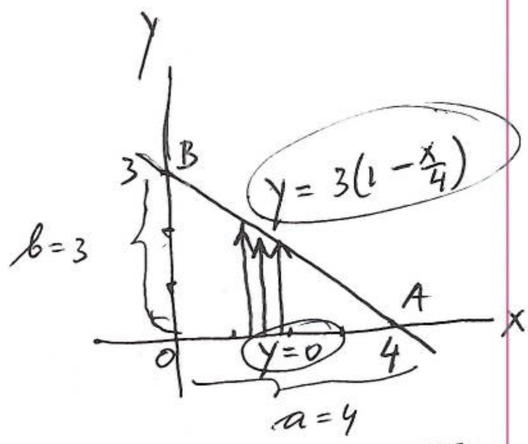
$$= L^{-1}\left\{\frac{\frac{1}{2}}{s-4} + \frac{-\frac{1}{2}}{s-2}\right\}$$

$$= \frac{1}{2} L^{-1}\left\{\frac{1}{s-4}\right\} - \frac{1}{2} L^{-1}\left\{\frac{1}{s-2}\right\}$$

$$= \underline{\underline{\frac{1}{2} e^{4t} - \frac{1}{2} e^{2t}}}$$

(3)





$$\boxed{\frac{x}{a} + \frac{y}{b} = 1}$$

$$\frac{x}{4} + \frac{y}{3} = 1$$

$$\frac{y}{3} = 1 - \frac{x}{4}$$

$$y = 3\left(1 - \frac{x}{4}\right)$$

$$z = 12 - 3x - 4y$$

$$\left.\begin{matrix} y=0 \\ z=0 \end{matrix}\right\} \rightarrow 0 = 12 - 3x - 4(0) \rightarrow 3x = 12 \rightarrow \underline{x = 4} \rightarrow (4, 0, 0)$$

$$\left.\begin{matrix} x=0 \\ z=0 \end{matrix}\right\} \rightarrow 0 = 12 - 3(0) - 4y \rightarrow 4y = 12 \rightarrow \underline{y = 3} \rightarrow (0, 3, 0)$$

$$\left.\begin{matrix} x=0 \\ y=0 \end{matrix}\right\} \rightarrow z = 12 - 3(0) - 4(0) \rightarrow z = 12 \rightarrow (0, 0, 12)$$

$$\boxed{\begin{matrix} x \text{ KONSTAN} \\ 0 \leq x \leq 4 \\ 0 \leq y \leq 3\left(1 - \frac{x}{4}\right) \end{matrix}}$$

$$V_{\text{BIDANG-4}} = \iint z \cdot dA$$

$$= \int_{x=0}^{4} \int_{y=0}^{3(1-\frac{x}{4})} (12 - 3x - 4y) \cdot \overbrace{dy \cdot dx}^{dA}$$

$$= \int_{x=0}^{4} dx \int_{y=0}^{3(1-\frac{x}{4})} [(12 - 3x) - 4y]\, dy = \int_{x=0}^{4} dx \cdot \left[(12 - 3x) \cdot y - \frac{4}{2} y^2 \Big|_0^{3(1-\frac{x}{4})} \right]$$

$$= \int_{x=0}^{4} dx \left[(12 - 3x)\left\{3\left(1 - \frac{x}{4}\right) - 0\right\} - 2\left\{\left\{3\left(1 - \frac{x}{4}\right)\right\}^2 - 0^2\right\} \right]$$

$$= \int_{x=0}^{4} \left[12\left(1 - \frac{x}{4}\right) \cdot 3\left(1 - \frac{x}{4}\right) - 2 \cdot 9 \cdot \left(1 - \frac{x}{4}\right)^2 \right] dx$$

$$= \int_{x=0}^{4} \left[36\left(1 - \frac{x}{4}\right)^2 - 18\left(1 - \frac{x}{4}\right)^2 \right] dx = \int_{x=0}^{4} 18\left(1 - \frac{x}{4}\right)^2 dx = 18\left(\frac{-4}{1}\right) \int_0^4 \left(1 - \frac{x}{4}\right)^2 \cdot \overbrace{d\left(1 - \frac{x}{4}\right)}^{-\frac{1}{4}dx}$$

$$= -72\left(\frac{1}{3}\right)\left(1 - \frac{x}{4}\right)^3 \Big|_0^4 = -24\left[\left(1 - \frac{4}{4}\right)^3 - \left(1 - \frac{0}{4}\right)^3\right] = -24\left[(0)^3 - 1^3\right]$$

$$= -24(0 - 1) = \boxed{24}$$

=

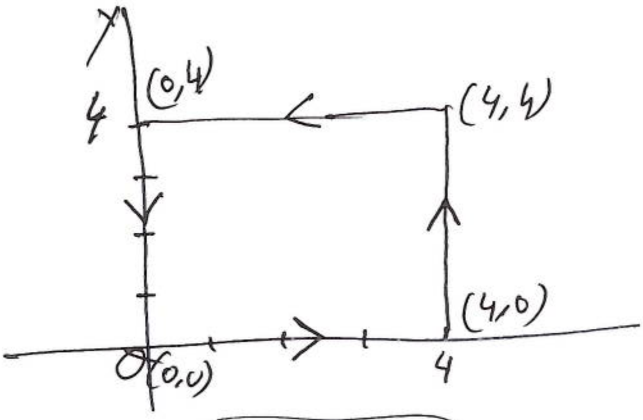
④ $\oint_C (x^2 - xy^3)\,dx + (y^2 - 2xy)\,dy$;

C ADALAH BUJUR SANGKAR DENGAN TITIK-TITIK SUDUT (0,0), (4,0), (4,4), (0,4).

SOLUSI

TEOREMA GREEN

$$\oint_C P(x,y)\,dx + Q(x,y)\,dy = \int_x \int_y \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) \overbrace{dy \cdot dx}^{dA}$$



$$\boxed{\begin{array}{l} 0 \leq x \leq 4 \\ 0 \leq y \leq 4 \end{array}}$$

$$P(x,y) = x^2 - xy^3$$

$$\frac{\partial P}{\partial y} = 0 - x(3y^2) = -3xy^2$$

$$Q(x,y) = y^2 - 2xy$$

$$\frac{\partial Q}{\partial x} = 0 - 2y(1) = -2y$$

$$\oint_C (x^2 - xy^3)\,dx + (y^2 - 2xy)\,dy = \int_{x=0}^{4} \int_{y=0}^{4} \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) \overbrace{dy \cdot dx}^{dA}$$

$$= \int_{x=0}^{4} dx \int_{y=0}^{4} \left[(-2y) - (-3xy^2)\right] dy = \int_{x=0}^{4} dx \left[\int_{y=0}^{4} (-2y + 3xy^2)\,dy\right]$$

$$= \int_{x=0}^{4} dx \left[-\cancel{\frac{2}{2}} y^2 + \cancel{\frac{3x}{3}} \cdot y^3 \Big|_0^4\right] = \int_{x=0}^{4} dx \left[-y^2 + x \cdot y^3 \Big|_0^4\right]$$

$$= \int_{x=0}^{4} dx \left[-(4^2 - 0^2) + x(4^3 - 0^3)\right] = \int_{x=0}^{4} (-16 + 64x)\,dx$$

$$= -16x + \frac{64}{2} x^2 \Big|_0^4$$

$$= -16(4-0) + 32(4^2 - 0^2)$$

$$= -64 + 512$$

$$= \underline{\underline{448}}$$