UAS "MATEMATIKA TEKNIK I" (KO104)

SELASA, 4 FEBRUARI 2014
pk 15.00 – 16.40

① TENTUKAN SOLUSI UMUM DARI PD LINEAR EULER-CAUCHY

$$x^2 \cdot \frac{d^2y}{dx^2} + 7x \cdot \frac{dy}{dx} - 7y = x^3$$

MISALKAN : $\boxed{x = e^u}$

$$\ln x = \ln e^u$$

$$\boxed{\ln x = u}$$

$$D(D-1).y + 7.Dy - 7y = x^3$$

$$(D^2 \underline{- D + 7D} - 7).y = (e^u)^3$$

$$\boxed{(D^2 + 6D - 7).y = e^{3u}}$$

I) $(D^2 + 6D - 7).y = 0$

PERS. EIGEN

$$\alpha^2 + 6\alpha - 7 = 0$$

$$(\alpha + 7)(\alpha - 1) = 0$$

$$\alpha_1 = -7, \quad \alpha_2 = 1$$

$$\boxed{Y_h = c_1 . e^{\alpha_1 u} + c_2 e^{\alpha_2 u}}$$

$$Y_h = c_1 . e^{-7u} + c_2 e^u$$

$$Y_h = \frac{c_1}{e^{7u}} + c_2 e^u$$

$$Y_h = \frac{c_1}{(e^u)^7} + c_2 (e^u)$$

$$\boxed{Y_h = \frac{c_1}{x^7} + c_2 x}$$

II) $(D^2 + 6D - 7).y = e^{③u}$

$k = 3$
$D = k = 3$

$$Y_k = \frac{e^{3u}}{D^2 + 6D - 7}$$

$$Y_k = \frac{e^{3u}}{(D+7)(D-1)} = \frac{(e^u)^3}{\underbrace{(3+7)}_{10}\underbrace{(3-1)}_{2}} = \boxed{\frac{x^3}{20}}$$

SOLUSI UMUM PD

$$Y = Y_h + Y_k$$

$$\boxed{Y = \frac{c_1}{x^7} + c_2 x + \frac{x^3}{20}}$$

② TENTUKAN SOLUSI UMUM DARI PD LINEAR ORDE 2 NON HOMOGEN !

$$\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 10y = x \cdot e^{2x}$$

$$D^2y - 7Dy + 10y = x \cdot e^{2x}$$

$$(D^2 - 7D + 10) \cdot y = x \cdot e^{2x}$$

(I) $(D^2 - 7D + 10) \cdot y = 0$

PERS. EIGEN

$$\alpha^2 - 7\alpha + 10 = 0$$

$$(\alpha - 5)(\alpha - 2) = 0$$

$$\alpha_1 = 5 \; , \; \alpha_2 = 2$$

$$y_h = C_1 e^{\alpha_1 x} + C_2 \cdot e^{\alpha_2 x}$$

$$\boxed{y_h = C_1 e^{5x} + C_2 e^{2x}}$$

(II) $(D^2 - 7D + 10) \cdot y = x \cdot e^{2x}$

$a = 2$

$D + a = D + 2$

$$y_K = \frac{1}{D^2 - 7D + 10} \cdot (e^{2x} \cdot x)$$

$$\boxed{\begin{matrix} D(x) = 1 \\ D^2(x) = 0 \end{matrix}}$$

$$y_K = e^{2x} \cdot \left[\frac{1}{(D+2)^2 - 7(D+2) + 10}(x)\right]$$

$$= e^{2x} \left[\frac{1}{D^2 + 4D + \cancel{4} - 7D - \cancel{14} + \cancel{10}}(x)\right]$$

$$= e^{2x} \left[\frac{1}{-3D + D^2}(x)\right]$$

$$= e^{2x} \left[\left(-\frac{1}{3D} - \frac{1}{9} - \frac{1}{27}D\right)(x)\right]$$

$$= e^{2x} \left[-\frac{1}{3}D^{-1}(x) - \frac{1}{9}x - \frac{1}{27}D(x)\right]$$

$$= e^{2x} \left[-\frac{1}{3}\int x\,dx - \frac{1}{9}x - \frac{1}{27}(1)\right]$$

$$y_K = e^{2x}\left(-\frac{1}{3} \cdot \frac{1}{2}x^2 - \frac{1}{9}x - \frac{1}{27}\right)$$

$$\boxed{y_K = e^{2x}\left(-\frac{1}{6}x^2 - \frac{1}{9}x - \frac{1}{27}\right)}$$

$$-\frac{1}{3D} - \frac{1}{9} - \frac{1}{27}D$$

$$-3D + D^2 \;\big/\; 1$$

$$\underline{1 - \frac{1}{3}D -}$$

$$\frac{1}{3}D$$

$$\underline{\frac{1}{3}D - \frac{1}{9}D^2 -}$$

$$\frac{1}{9}D^2$$

$$\underline{\frac{1}{9}D^2 - \frac{1}{27}D^3 -}$$

SOLUSI UMUM PD

$$y = y_h + y_K$$

$$y = C_1 e^{5x} + C_2 e^{2x} + e^{2x}\left(-\frac{1}{6}x^2 - \frac{1}{9}x - \frac{1}{27}\right)$$

③ TENTUKAN SOLUSI UMUM DAN SOLUSI SINGULAR DARI PD CLAIRAUT!

$$y = xp + p^2 + 3 \quad , \quad p = \frac{dy}{dx}$$

DIDIFERENSIAL TERHADAP X

$$\frac{dy}{dx} = \underset{1}{(x)'} \cdot p + x.(p)' + 2p \cdot (p)' + 0$$

$$\cancel{p} = \cancel{p} + x \cdot \frac{dp}{dx} + 2p \cdot \frac{dp}{dx}$$

$$0 = x \cdot \frac{dp}{dx} + 2p \cdot \frac{dp}{dx}$$

$$\frac{dp}{dx}(x + 2p) = 0$$

$$\frac{dp}{dx} = 0 \quad , \quad x + 2p = 0$$

$$dp = 0.dx$$

$$\int dp = \int 0 . dx$$

$$\boxed{p = C}$$

SOLUSI UMUM

$$y = xp + p^2 + 3$$

$$\boxed{y = cx + c^2 + 3}$$

$$2p = -x$$

$$\boxed{p = -\tfrac{1}{2}x}$$

SOLUSI SINGULAR

$$y = xp + p^2 + 3$$

$$y = x\left(-\tfrac{1}{2}x\right) + \left(-\tfrac{1}{2}x\right)^2 + 3$$

$$y = -\tfrac{1}{2}x^2 + \tfrac{1}{4}x^2 + 3$$

$$\boxed{y = -\tfrac{1}{4}x^2 + 3}$$

④

$$\boxed{\begin{matrix}\frac{dx}{dt} - 5y = 3t \\ \frac{dy}{dt} - 5x = t\end{matrix}} \begin{matrix}\to Dx - 5y = 3t \\ \to -5x + Dy = t\end{matrix} \to \begin{bmatrix} D & -5 \\ -5 & D\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}3t\\t\end{bmatrix}$$

$$A \cdot \vec{X} = B$$

SYARAT AWAL : $\underline{X(0)=0}$ DAN $\underline{Y(0)=0}$.

$$|A| = \begin{vmatrix} D & -5 \\ -5 & D \end{vmatrix} = D^2 - (-5)^2 = \underline{D^2 - 25}$$

$$|A_1| = \begin{vmatrix} 3t & -5 \\ t & D \end{vmatrix} = D(3t) - (-5)(t) = \underline{3 + 5t}$$

$$|A_2| = \begin{vmatrix} D & 3t \\ -5 & t \end{vmatrix} = D(t) - (3t)(-5) = \underline{1 + 15t}$$

$$X = \frac{|A_1|}{|A|} = \frac{3+5t}{D^2-25}$$

$$(D^2-25).X = 5t+3$$

I) $(D^2-25).X = 0$

$\underline{\text{PERS. EIGEN}}$

$\alpha^2 - 25 = 0$

$(\alpha+5)(\alpha-5) = 0$

$\underline{\alpha_1 = -5}, \underline{\alpha_2 = 5}$

$$X_h = C_1 e^{\alpha_1 t} + C_2 e^{\alpha_2 t}$$

$$\boxed{X_h = C_1 e^{-5t} + C_2 e^{5t}}$$

II) $(D^2-25).X = 5t+3$

$$X_K = \frac{1}{-25+D^2}(5t+3)$$

$$X_K = \left(-\frac{1}{25} - \frac{1}{625}D^2\right)(5t+3)$$

$$X_K = -\frac{1}{25}(5t+3) - \frac{1}{625}\underbrace{D^2(5t+3)}_{0}$$

$$\boxed{X_K = -\frac{1}{25}(5t+3) = -\frac{1}{5}t - \frac{3}{25}}$$

$$\boxed{D(5t+3) = 5(1)+0 = 5}$$

$$D^2(5t+3) = 0$$

$$-25+D^2 \,\big)\, 1 \quad -\frac{1}{25} - \frac{1}{625}D^2$$

$$1 - \frac{1}{25}D^2 -$$

$$\frac{1}{25}D^2$$

$$\boxed{x(t) = X_h + X_K}$$

$$\underline{X(t) = C_1 e^{-5t} + C_2 e^{5t} - \frac{1}{5}t - \frac{3}{25}}$$

$$y = \frac{|A_2|}{|A|} = \frac{1+15t}{D^2-25}$$

$$(D^2-25).y = 15t+1$$

I) $(D^2-25).y = 0$

PERS. EIGEN

$$\alpha^2-25=0$$
$$(\alpha+5)(\alpha-5)=0$$
$$\alpha_1=-5,\ \alpha_2=5$$

$$y_h = c_3 e^{\alpha_3 t} + c_4 e^{\alpha_4 t}$$

$$\boxed{y_h = c_3 e^{-5t} + c_4 e^{5t}}$$

II) $(D^2-25).y = 15t+1$

$$\boxed{D(15t+1) = 15(1)+0 = 15}$$
$$D^2(15t+1) = 0$$

$$y_k = \frac{1}{-25+D^2}(15t+1)$$

$$y_k = \left(-\frac{1}{25} - \frac{1}{625}D^2\right)(15t+1)$$

$$y_k = -\frac{1}{25}(15t+1) - \frac{1}{625}\underbrace{\cancel{D^2(15t+1)}}_{0}$$

$$\boxed{y_k = -\frac{3}{5}t - \frac{1}{25}}$$

$$\boxed{y(t) = y_h + y_k}$$

$$\boxed{y(t) = c_3 e^{-5t} + c_4 e^{5t} - \frac{3}{5}t - \frac{1}{25}}$$

SOLUSI UMUM PD SIMULTAN (SEMENTARA)

$$x(t) = c_1 e^{-5t} + c_2 e^{5t} - \frac{1}{5}t - \frac{3}{25}$$

$$y(t) = c_3 e^{-5t} + c_4 e^{5t} - \frac{3}{5}t - \frac{1}{25}$$

$$\frac{dx}{dt} = (-5)c_1 e^{-5t} + 5.c_2 e^{5t} - \frac{1}{5}(1) - 0$$

$$\boxed{\frac{dx}{dt} = -5c_1 e^{-5t} + 5.c_2 e^{5t} - \frac{1}{5}}$$

$$\frac{dx}{dt} = -5.c_1\, e^{-5t} + 5.c_2\, e^{5t} - \cancel{\tfrac{1}{5}}$$

$$-5y = -5.c_3\, e^{-5t} - 5\, c_4\, e^{5t} + 3t + \cancel{\tfrac{1}{5}} \quad +$$

$$\cancel{3t} = (-5c_1 - 5c_3)e^{-5t} + (5c_2 - 5c_4)e^{5t} + \cancel{3t}$$

$$0 = -5(c_1 + c_3)\, e^{-5t} + 5(c_2 - c_4)\, e^{5t}$$

$$\text{Koef. } e^{-5t} : \quad 0 = -5(c_1 + c_3) \longrightarrow c_1 + c_3 = 0$$

$$c_1 = -c_3 \longrightarrow \boxed{c_3 = -c_1}$$

$$\text{Koef. } e^{5t} : \quad 0 = 5(c_2 - c_4) \longrightarrow c_2 - c_4 = 0$$

$$c_2 = c_4 \longrightarrow \boxed{c_4 = c_2}$$

SOLUSI UMUM PD. SIMULTAN

$$x(t) = c_1\, e^{-5t} + c_2\, e^{5t} - \tfrac{1}{5}t - \tfrac{3}{25}$$

$$y(t) = -c_1\, e^{-5t} + c_2\, e^{5t} - \tfrac{3}{5}t - \tfrac{1}{25}$$

$$x(0) = c_1 . \underset{1}{e^{0}} + c_2 . \underset{1}{e^{0}} - \tfrac{1}{5}(0) - \tfrac{3}{25} = 0 \longrightarrow c_1 + c_2 = \tfrac{3}{25}$$

$$y(0) = -c_1 . \underset{1}{e^{0}} + c_2 . \underset{1}{e^{0}} - \tfrac{3}{5}(0) - \tfrac{1}{25} = 0 \longrightarrow -c_1 + c_2 = \tfrac{1}{25} \quad +$$

$$2c_2 = \tfrac{4}{25} \quad (\times \tfrac{1}{2})$$

$$\boxed{c_2 = \tfrac{2}{25}}$$

$$c_1 + c_2 = \tfrac{3}{25}$$

$$c_2 = \tfrac{2}{25} \quad -$$

$$\boxed{c_1 = \tfrac{1}{25}}$$

SOLUSI PD. SIMULTAN

JARAK YG DITEMPUH MOBIL A : $x(t) = \frac{1}{25}e^{-5t} + \frac{2}{25}e^{5t} - \frac{1}{5}t - \frac{3}{25}$

JARAK YG DITEMPUH MOBIL B : $y(t) = -\frac{1}{25}e^{-5t} + \frac{2}{25}e^{5t} - \frac{3}{5}t - \frac{1}{25}$