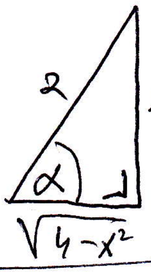
INTEGRAL DENGAN SUBSTITUSI TRIGONOMETRI

$$\int x^3\sqrt{4-x^2}\,.dx$$

$$\boxed{\int \sin^n x.dx = -\frac{1}{n}\sin^{n-1}x.\cos x + \frac{n-1}{n}\int \sin^{n-2}x.dx}$$



$$\sqrt{4-x^2} = \sqrt{2^2-x^2}$$

$$\boxed{\frac{x}{2} = \sin\alpha}$$

$$x = 2\sin\alpha \longrightarrow \boxed{x^3 = (2\sin\alpha)^3 = 2^3\sin^3\alpha = 8\sin^3\alpha}$$

$$\frac{dx}{d\alpha} = 2\cos\alpha$$

$$dx = 2\cos\alpha.d\alpha$$

$$\boxed{\frac{\sqrt{4-x^2}}{2} = \cos\alpha} \longrightarrow \sqrt{4-x^2} = 2\cos\alpha$$

$$\int x^3\sqrt{4-x^2}\,.dx = \int 8\sin^3\alpha\,.(2\cos\alpha).(2\cos\alpha.d\alpha)$$

$$= 32\int \sin^3\alpha\,.\cos^2\alpha.d\alpha$$

$$= 32\int \sin^3\alpha.(1-\sin^2\alpha).d\alpha$$

$$= 32\left[\int \sin^3\alpha\,.d\alpha - \int \sin^5\alpha.d\alpha\right]$$

$$= 32\left[\int \sin^3\alpha.d\alpha - \left\{-\frac{1}{5}\sin^{5-1}\alpha.\cos\alpha + \frac{5-1}{5}\int \sin^{5-2}\alpha.d\alpha\right\}\right]$$

$$= 32\left[\int \sin^3\alpha.d\alpha - \left\{-\frac{1}{5}\sin^4\alpha.\cos\alpha + \frac{4}{5}\int \sin^3\alpha.d\alpha\right\}\right]$$

$$= 32\left[\int \sin^3\alpha.d\alpha + \frac{1}{5}\sin^4\alpha.\cos\alpha - \frac{4}{5}\int \sin^3\alpha.d\alpha\right]$$

$$= 32\left[\frac{1}{5}\int \sin^3\alpha.d\alpha + \frac{1}{5}\sin^4\alpha.\cos\alpha\right]$$

$$= 32\left[\frac{1}{5}\left\{-\frac{1}{3}\sin^{3-1}\alpha.\cos\alpha + \frac{3-1}{3}\int \sin^{3-2}\alpha.d\alpha\right\} + \frac{1}{5}\sin^4\alpha.\cos\alpha\right]$$

$$= 32\left[\frac{1}{5}\left\{-\frac{1}{3}\sin^2\alpha.\cos\alpha + \frac{2}{3}\int \sin\alpha.d\alpha\right\} + \frac{1}{5}\sin^4\alpha.\cos\alpha\right]$$

$$= 32\left[-\frac{1}{15}\sin^2\alpha.\cos\alpha + \frac{2}{15}(-\cos\alpha) + \frac{1}{5}\sin^4\alpha.\cos\alpha + C\right]$$

$$= -\frac{32}{15}\sin^2\alpha.\cos\alpha - \frac{64}{15}\cos\alpha + \frac{1}{5}\sin^4\alpha.\cos\alpha + C$$

$$= -\frac{32}{15}\left(\frac{x}{2}\right)^2.\frac{\sqrt{4-x^2}}{2} - \frac{64}{15}.\frac{\sqrt{4-x^2}}{2} + \frac{1}{5}\left(\frac{x}{2}\right)^4.\frac{\sqrt{4-x^2}}{2} + C$$

$$= -\frac{32}{15}.\frac{x^2}{4}.\frac{\sqrt{4-x^2}}{2} - \frac{32}{15}\sqrt{4-x^2} + \frac{1}{5}.\frac{x^4}{16}.\frac{\sqrt{4-x^2}}{2} + C$$

$$= -\frac{4}{15}x^2\sqrt{4-x^2} - \frac{32}{15}\sqrt{4-x^2} + \frac{1}{160}x^4\sqrt{4-x^2} + C$$

$$\int x^5 e^{x^2}\,dx \qquad\qquad d(x^2) = 2x\,dx$$

$$= \int x^4 e^{x^2} \cdot (x\,dx)$$

$$= \frac{1}{2}\int x^4 e^{x^2} \cdot \overset{2x\,dx}{d(x^2)}$$

$$= \frac{1}{2}\int \underbrace{x^4}_{U} \cdot d(\underbrace{e^{x^2}}_{V})$$

$$= \frac{1}{2}\left[U \cdot V - \int V \cdot d(U) \right]$$

$$\cancel{= \frac{1}{2}\left[e^{x^2} \cdot x^4 \right.}$$

$$= \frac{1}{2}\left[(x^4)(e^{x^2}) - \int e^{x^2} d(x^4) \right]$$

$$= \frac{1}{2} x^4 e^{x^2} - \frac{1}{2}\int e^{x^2}(4x^3)\,dx$$

$$= \frac{1}{2} x^4 e^{x^2} - \frac{1}{2}(4)\int x^3 e^{x^2}\,dx$$

$$= \frac{1}{2} x^4 e^{x^2} - 2\int x^2 e^{x^2} \cdot (x\,dx)$$

$$= \frac{1}{2} x^4 e^{x^2} - 2\left(\frac{1}{2}\right)\int x^2 e^{x^2} \cdot \overset{2x\,dx}{d(x^2)}$$

$$= \frac{1}{2} x^4 e^{x^2} - \int \underbrace{x^2}_{U}\, d(\underbrace{e^{x^2}}_{V})$$

$$= \frac{1}{2} x^4 e^{x^2} - \left[(x^2)(e^{x^2}) - \int e^{x^2} \cdot d(x^2) \right]$$

$$= \frac{1}{2} x^4 e^{x^2} - x^2 e^{x^2} + e^{x^2} + C$$

$$= \underline{\frac{1}{2} x^4 e^{x^2} - x^2 e^{x^2} + e^{x^2} + C}$$

$$\int \underbrace{e^{at}}_{u} \cdot \underbrace{\sin t \cdot dt}_{dv} = \int e^{at} \cdot d(-\cos t)$$

$dv = \sin t \, dt$

$\int dv = \int \sin t \cdot dt$

$\boxed{V = -\cos t}$

$$= (e^{at})(-\cos t) - \int (-\cos t)\, d(e^{at})$$

$$= -e^{at}\cos t + \int \cos t \cdot e^{at} \cdot (a)\, dt$$

$$= -e^{at}\cos t + a \int \underbrace{e^{at}}_{u} \underbrace{\cos t \cdot dt}_{dv}$$

$dv = \cos t \, dt$

$\int dv = \int \cos t \, dt$

$\boxed{V = \sin t}$

$$= -e^{at} \cdot \cos t + a \int \underbrace{e^{at}}_{u} d(\underbrace{\sin t}_{v})$$

$$= -e^{at} \cdot \cos t + a \left[(e^{at})(\sin t) - \int \sin t \cdot d(e^{at})\right]$$

$$= -e^{at} \cdot \cos t + a e^{at} \sin t - a \int \sin t \cdot e^{at} \cdot (a\, dt)$$

$$\int e^{at} \cdot \sin t \, dt = -e^{at} \cdot \cos t + a \cdot e^{at} \cdot \sin t - a^2 \int e^{at} \cdot \sin t \cdot dt$$

$$\int e^{at} \cdot \sin t \cdot dt + a^2 \int e^{at} \sin t \cdot dt = e^{at}(-\cos t + a \cdot \sin t) + C$$

$$(a^2+1) \int e^{at} \cdot \sin t \cdot dt = e^{at}(a \cdot \sin t - \cos t) + C \qquad \left(\frac{1}{a^2+1}\right)$$

$$\int e^{at} \cdot \sin t \cdot dt = \frac{e^{at}}{a^2+1}(a \cdot \sin t - \cos t) + C$$

ATAU PAKAI RUMUS

$$\int e^{ax} \cdot \cos bx \, dx = \frac{e^{ax}}{a^2+b^2}(a \cdot \cos bx + b \cdot \sin bx) + C$$

$$\int e^{ax} \cdot \sin bx \, dx = \frac{e^{ax}}{a^2+b^2}(a \cdot \sin bx - b \cdot \cos bx) + C$$

$a = a$, $b = 1$

$$\int e^{at} \cdot \sin t \cdot dt = \frac{e^{at}}{a^2+1^2}\left[a \cdot \sin t - (1) \cdot \cos t\right] + C$$

$$= \frac{e^{at}}{a^2+1}(a \cdot \sin t - \cos t) + C$$

$$\int \underbrace{\ln^2 x^{20}}_{u} \cdot \underbrace{dx}_{dv} =$$

$$dv = dx$$

$$\int dv = \int dx$$

$$\int \boxed{v = x}$$

$$= \int \underbrace{\ln^2 x^{20}}_{u} \cdot d(\underbrace{x}_{v})$$

$$= u.v - \int v \cdot d(u)$$

$$(\ln x^{20})^2$$

$$= (\ln^2 x^{20})(x) - \int x \cdot d(\ln^2 x^{20})$$

$$= x \cdot \ln^2 x^{20} - \int x \cdot 2(\ln x^{20})^{2-1} \cdot d(\ln x^{20})$$

$$= x \cdot \ln^2 x^{20} - 2 \int x (\ln x^{20}) \cdot \frac{1}{x^{\cancel{20}\,19}} \cdot d(x^{20})$$

$$= x \cdot \ln^2 x^{20} - 2 \int \cancel{x} (\ln x^{20}) \cdot \frac{20 \cancel{x^{19}} dx}{\cancel{x^{19}}}$$

$$= x \cdot \ln^2 x^{20} - 2(20) \int \underbrace{(\ln x^{20})}_{u} \cdot \underbrace{dx}_{dv}$$

$$= x \cdot \ln^2 x^{20} - 40 \int \underbrace{(\ln x^{20})}_{u} \, d(\underbrace{x}_{v})$$

$$= x \ln^2 x^{20} - 40 \left[(\ln x^{20})(x) - \int x \cdot d(\ln x^{20}) \right]$$

$$= x \cdot \ln^2 x^{20} - 40 x \cdot \ln x^{20} + 40 \int x \cdot \frac{1}{x^{20}} \cdot d(x^{20})$$

$$= x \cdot \ln^2 x^{20} - 40 x \cdot \ln x^{20} + 40 \int \frac{1}{\cancel{x^{19}}} \cdot (20 \cancel{x^{19}} \cdot dx)$$

$$= x \cdot \ln^2 x^{20} - 40 x \cdot \ln x^{20} + 40 \cdot (20) \int dx$$

$$= x \cdot \ln^2 x^{20} - 40 x \cdot \ln x^{20} + 800 x + C$$

$$\boxed{\begin{array}{l}\text{INTEGRAL PARSIAL}\\ \int U.\,d(V) = U.V - \int V.\,d(U)\end{array}}$$

$$\int \frac{\ln 2x^5}{x^2}\,dx$$

$$dV = \frac{dx}{x^2}$$

$$\int dV = \int x^{-2}\,dx$$

$$V = \frac{1}{-2+1}\,x^{-2+1}$$

$$\boxed{V = -x^{-1} = -\frac{1}{x}}$$

$$= \int \underbrace{\ln 2x^5}_{U} \cdot \underbrace{\frac{dx}{x^2}}_{dV}$$

$$= \int \underbrace{\ln 2x^5}_{U} \cdot d\Big(\underbrace{-\frac{1}{x}}_{V}\Big)$$

$$= U.V - \int V.\,d(U)$$

$$= (\ln 2x^5)\left(-\frac{1}{x}\right) - \int\left(-\frac{1}{x}\right) d(\ln 2x^5)$$

$$= -\frac{\ln 2x^5}{x} + \int \frac{1}{x}\cdot\frac{1}{2x^5}\,d(2x^5)$$

$$= -\frac{\ln 2x^5}{x} + \int \frac{1}{2\cancel{x^6}\,x^2}\cdot(10\cancel{x^4}\,dx)$$

$$= -\frac{\ln 2x^5}{x} + 5\int \frac{dx}{x^2}$$

$$= -\frac{\ln 2x^5}{x} + 5\int x^{-2}\,dx$$

$$= -\frac{\ln 2x^5}{x} + 5\cdot\left(\frac{1}{-2+1}\,x^{-2+1}\right) + C$$

$$= -\frac{\ln 2x^5}{x} + 5\,(-1\cdot x^{-1}) + C$$

$$= -\frac{\ln 2x^5}{x} - \frac{5}{x} + C$$

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