

UNIVERSITAS BINA NUSANTARA

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| Undergraduate Program <i>Program Sarjana (Reguler)</i> | Semester : Ganjil / Genap / -Pendek *) |
| <input type="checkbox"/> Ujian Tengah Semester <input type="checkbox"/> Ujian Akhir Semester <input type="checkbox"/> Ujian Semester Pendek <input type="checkbox"/> Ujian Lain-lain : _____ | Tahun Ajaran : 2013 / 2014 |
| Fak. / Jurusan : SoCS / MTIF | N I M : _____ |
| Mata Kuliah : Matematika Teknik I (K0104) | |
| Hari / Tanggal : Selasa, 20 November 2013 | N a m a : _____ |
| Dosen : Wikaria Gazali | |
| Kelas : Pagi | Tanda : _____ Tangan : _____ |
| Waktu : 08.00 – 09.40 WIB (100 menit) | |
| Sifat Ujian : Tutup / Buka Buku *) | |
| Perlengkapan : Buku Ujian / Kalkulator / Kamus / Kertas Gambar – A3 / Kr. Gambar – A2 *) | |
| *) Coret yang tidak perlu | |
| Naskah dimasukkan ke dalam buku ujian dan dikumpulkan kembali !!! | |

Kerjakan semua soal di bawah ini secara lengkap di buku ujian !

Bobot setiap nomor soal 25%.

1. $(4x^2 + 3xy^2)dx + (2y - 4y^2 + 3x^2y)dy = 0$

Tentukan Solusi Umum PDnya !

2. $y'' - 2y' - 3y = e^{5x}$

Tentukan Solusi Umum PDnya !

3. $\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 16y = xe^{4x}$

Tentukan Solusi Umum PDnya !

4. **Jumlah Penduduk sekarang P(0) dan Jumlah Penduduk setelah t tahun P(t). Jika jumlah penduduk setelah 4 tahun menjadi tiga kali lipat, tentukan setelah berapa tahun jumlah penduduknya menjadi tujuh kali lipat ?**

Selamat mengerjakan, semoga sukses

UTS "MATEMATIKA TEKNIK I"

SEMESTER GENJIL 2013/2014

$$\textcircled{1} \quad \underbrace{(4x^2 + 3xy^2)}_{M(x,y)} dx + \underbrace{(2y - 4y^2 + 3x^2y)}_{N(x,y)} dy = 0$$

$$M(x,y) = 4x^2 + 3xy^2 \quad \left| \quad N(x,y) = 2y - 4y^2 + 3x^2y \right.$$

$$\frac{\partial M}{\partial y} = 0 + 3x(2y) = 6xy \quad \left| \quad \frac{\partial N}{\partial x} = 0 - 0 + 3y(2x) = 6xy \right.$$

$$\rightarrow \boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}} \Rightarrow \text{PD. ERSAK}$$

$$\text{I) } F(x,y) = \int M(x,y) dx + c(y) \quad \rightarrow \boxed{\frac{\partial F}{\partial y} = N(x,y)}$$

$$F(x,y) = \int (4x^2 + 3xy^2) dx + c(y)$$

$$F(x,y) = 4 \int x^2 dx + 3y^2 \int x dx + c(y)$$

$$F(x,y) = 4 \left(\frac{1}{3} x^3 \right) + 3y^2 \left(\frac{1}{2} x^2 \right) + c(y)$$

$$\boxed{F(x,y) = \frac{4}{3} x^3 + \frac{3}{2} x^2 y^2 + c(y)}$$

$$\rightarrow \frac{\partial F}{\partial y} = 0 + \frac{3}{2} x^2 (2y) + c'(y) = N(x,y)$$

$$\cancel{3x^2y} + c'(y) = 2y - 4y^2 + \cancel{3x^2y}$$

$$c'(y) = 2y - 4y^2$$

$$c(y) = \int c'(y) dy = \int (2y - 4y^2) dy = \frac{2}{2} y^2 - \frac{4}{3} y^3 + C$$

$$= y^2 - \frac{4}{3} y^3 + C$$

Solusi umum PD

$$\boxed{F(x,y) = 0}$$

$$\frac{4}{3} x^3 + \frac{3}{2} x^2 y^2 + c(y) = 0$$

$$\boxed{\frac{4}{3} x^3 + \frac{3}{2} x^2 y^2 + y^2 - \frac{4}{3} y^3 + C = 0} \quad \textcircled{6}$$

$$8x^3 + 9x^2y^2 + 6y^2 - 8y^3 + C = 0$$

$$\textcircled{2} \quad y'' - 2y' - 3y = e^{5x}$$

$$D^2 y - 2Dy - 3y = e^{5x}$$

$$\boxed{(D^2 - 2D - 3)y = e^{5x}}$$

$$\text{I) } (D^2 - 2D - 3) \cdot y = 0$$

PERS. EIGEN

$$\alpha^2 - 2\alpha - 3 = 0$$

$$(\alpha - 3)(\alpha + 1) = 0$$

$$\alpha_1 = 3, \quad \alpha_2 = -1$$

$$y_h = C_1 e^{\alpha_1 x} + C_2 e^{\alpha_2 x}$$

$$\boxed{y_h = C_1 e^{3x} + C_2 e^{-x}}$$

$$\boxed{y_h = C_1 e^{3x} + \frac{C_2}{e^x}}$$

SOLUSI UMUM PD

$$y = y_h + y_k$$

$$\boxed{y = C_1 e^{3x} + \frac{C_2}{e^x} + \frac{1}{12} e^{5x}}$$

$$\text{II) } (D^2 - 2D - 3) \cdot y = e^{5x} \quad \begin{array}{l} \rightarrow k=5 \\ D=k=5 \end{array}$$

$$y_k = \frac{e^{5x}}{D^2 - 2D - 3}$$

$$y_k = \frac{e^{5x}}{(D-3)(D+1)} = \frac{e^{5x}}{\frac{(5-3)(5+1)}{2 \cdot 6}}$$

$$\boxed{y_k = \frac{1}{12} e^{5x}}$$

\rightarrow SOLUSI KHUSUS PD

$$(3) \frac{d^2 y}{dx^2} - 8 \frac{dy}{dx} + 16y = x \cdot e^{4x}$$

$$D^2 y - 8Dy + 16y = e^{4x} \cdot x$$

$$(D^2 - 8D + 16) \cdot y = e^{4x} \cdot x$$

$$(I) \frac{(D^2 - 8D + 16) \cdot y = 0}{\text{PERS. EIGEN}}$$

$$\alpha^2 - 8\alpha + 16 = 0$$

$$(\alpha - 4)^2 = 0$$

$$\alpha_1 = \alpha_2 = 4$$

$$y_h = (c_1 + c_2 x) e^{4x}$$

$$y_h = (c_1 + c_2 x) e^{4x}$$

$$(II) (D^2 - 8D + 16) \cdot y = e^{4x} \cdot x$$

$$y_k = \frac{1}{D^2 - 8D + 16} (e^{4x} \cdot x) \rightarrow a=4, D+a = D+4$$

~~$$y_k = \frac{1}{(D-4)^2} (e^{4x} \cdot x)$$~~

$$y_k = e^{4x} \cdot \left[\frac{1}{(D+4)^2 - 8(D+4) + 16} (x) \right]$$

$$y_k = e^{4x} \cdot \left[\frac{1}{D^2 + 8D + 16 - 8D - 32 + 16} (x) \right]$$

$$y_k = e^{4x} \left[\frac{1}{D^2} (x) \right] = e^{4x} \cdot D^{-2} (x)$$

$$= e^{4x} D^+ D^- (x) = e^{4x} \int \int x \, dx \, dx$$

$$= e^{4x} \int \frac{1}{2} x^2 \, dx = \frac{1}{2} e^{4x} \int x^2 \, dx$$

$$= \frac{1}{2} e^{4x} \left(\frac{1}{3} x^3 \right) = \frac{1}{6} x^3 e^{4x} \quad (\text{Solokhus PD})$$

Solusi umum PD

$$y = y_h + y_k$$

$$y = (c_1 + c_2 x) e^{4x} + \frac{1}{6} x^3 e^{4x}$$