# UNIVERSITAS BINA NUSANTARA

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|---|---|---|---|
| **Undergraduate Program** *Program Sarjana (Reguler)* | | | **Semester : Ganjil / ~~Genap~~ / ~~Pendek~~ *)** |
| ☐ **Ujian Tengah Semester** ☐ **Ujian Akhir Semester**<br>☐ **Ujian Semester Pendek** ☐ **Ujian Lain-lain : _____** | | | **Tahun Ajaran : 2013 / 2014** |
| Fak. / Jurusan | : | **SoCS / STIF** | N I M : |
| Mata Kuliah | : | **Kalkulus II ( K0434 )** | |
| Hari / Tanggal | : | **Senin, 25 November 2013** | N a m a : |
| Dosen | : | Team Dosen | |
| Kelas | : | Pagi | Tanda Tangan : |
| Waktu | : | 08.00 – 09.40 WIB (100 menit) | |
| Sifat Ujian | : | Tutup / ~~Buka~~ Buku *) | |
| Perlengkapan | : | Buku Ujian / ~~Kalkulator~~ / ~~Kamus / Kertas Gambar – A3 / Kr. Gambar – A2~~ *) | |
| *) *Coret yang tidak perlu* | | | |
| **Naskah dimasukkan ke dalam buku ujian dan dikumpulkan kembali !!!** | | | |

**Kerjakan semua soal di bawah ini secara lengkap di buku ujian !**

**Bobot setiap nomor soal 25%.**

1. **Jumlah Penduduk sekarang P(0) dan Jumlah Penduduk setelah t tahun P(t). Jika jumlah penduduknya setelah 10 tahun menjadi dua kali lipat, tentukan setelah berapa tahun jumlah penduduknya menjadi tiga kali lipat ?**

2. $\frac{dy}{dx} + \frac{y}{x} = 3x^2 + 7$ **Tentukan Solusi Umum PDnya !**

3. $\frac{d^2y}{dx^2} - 10\frac{dy}{dx} + 25y = e^{4x}$ **dengan syarat batas untuk** $x = 0$, maka $y = 2$ dan untuk $x = 1$, maka $y = e^{4x}$.

   **Tentukan Solusi PDnya !**

4. **Tentukan solusi umum PD Simultan (Sistem Persamaan Diferensial) di bawah ini !**

$$\frac{dx}{dt} = 3x + 4y$$
$$\frac{dy}{dt} = 4x - 3y$$

*Selamat mengerjakan, semoga sukses*

# UTS - KALKULUS II (#0434)

SENIN, 25 NOV. 2013
pk 08.00 - 09.40

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① JUMLAH PENDUDUK SEKARANG = $P(0)$
SETELAH $t$ TAHUN = $P(t)$

$P(10) = 2 \cdot P(0)$
$P(t) = 3 \cdot P(0) \longrightarrow t = ?$

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LAJU PERTUMBUHAN PENDUDUK :

$$\frac{\partial P}{\partial t} = k \cdot P \longrightarrow \frac{\partial P}{P} = k\,dt \longrightarrow \frac{\partial P}{P} - k\,dt = 0$$

$$\int \frac{\partial P}{P} - k\int dt = \int 0 \longrightarrow \ln P - k.t = C$$

$$\ln P - \ln e^{kt} = \ln C \longrightarrow \ln \frac{P(t)}{e^{kt}} = \ln C$$

$$\frac{P(t)}{e^{kt}} = C \longrightarrow \boxed{P(t) = C \cdot e^{kt}}$$

$t = 0 \longrightarrow P(0) = C \cdot e^{k(0)} \longrightarrow P(0) = C \cdot e^{0} \longrightarrow \boxed{P(0) = C}$

$$\Longrightarrow \boxed{P(t) = P(0) \cdot e^{kt}}$$

$P(10) = P(0) \cdot e^{k(10)}$
$2\,P(0) = P(0) \cdot e^{10k}$

$2 = e^{10k}$
$\ln 2 = \ln e^{10k}$
$\ln 2 = 10k \longrightarrow \boxed{k = \frac{\ln 2}{10}}$

$$\Longrightarrow \boxed{P(t) = P(0) \cdot e^{\left(\frac{\ln 2}{10}\right)t}}$$

$3\,P(0) = P(0) \cdot e^{\left(\frac{\ln 2}{10}\right).t}$

$3 = e^{\left(\frac{\ln 2}{10}\right)t}$

$\ln 3 = \ln e^{\left(\frac{\ln 2}{10}\right)t}$
$\ln 3 = \frac{\ln 2}{10} \cdot t$
$10 \ln 3 = (\ln 2) \cdot t$

$$\boxed{t = \frac{10 \ln 3}{\ln 2}}$$

JADI JUMLAH PENDUDUKNYA MENJADI TIGA KALI LIPAT, SETELAH $\boxed{\frac{10 \ln 3}{\ln 2}}$ TAHUN

② $\frac{dy}{dx} + \frac{y}{x} = 3x^2 + 7$

$\frac{dy}{dx} + P(x).y = Q(x)$ ⟶ $P(x) = \frac{1}{x}$, $Q(x) = 3x^2 + 7$

PD. LINEAR

$$S(x) = e^{\int P(x)dx} = e^{\int \frac{1}{x}.dx} = e^{\int \frac{dx}{x}} = e^{\ln x} = x$$

Solusi umum PD

$$y = \frac{1}{S(x)}\left[\int S(x).Q(x)dx + C\right]$$

$$y = \frac{1}{x}\left[\int x.(3x^2+7)dx + C\right]$$

$$y = \frac{1}{x}\left[\int (3x^3+7x)dx + C\right]$$

$$y = \frac{1}{x}\left[\frac{3}{4}x^4 + \frac{7}{2}x^2 + C\right]$$

$$\boxed{y = \frac{3}{4}x^3 + \frac{7}{2}x + \frac{C}{x}}$$

(3) $\frac{d^2y}{dx^2} - 10\frac{dy}{dx} + 25y = e^{4x}$

$D^2y - 10Dy + 25y = e^{4x}$

$(D^2 - 10D + 25)\cdot y = e^{4x}$

DENGAN SYARAT BATAS UNTUK $x = 0$, MAKA $y = 2$ DAN UNTUK $x = 1$, MAKA $y = e^4$

(I) $(D^2 - 10D + 25)\cdot y = 0$

PERS. EIGEN

$\alpha^2 - 10\alpha + 25 = 0$

$(\alpha - 5)^2 = 0$

$\alpha_1 = \alpha_2 = 5$

$y_h = (c_1 + c_2 x)e^{\alpha x}$

$\boxed{y_h = (c_1 + c_2 x)e^{5x}}$

(II) $(D^2 - 10D + 25)\cdot y = e^{4x}$ → $k = 4$

$D = k = 4$

$y_k = \frac{e^{4x}}{D^2 - 10D + 25} = \frac{e^{4x}}{(D-5)^2} = \frac{e^{4x}}{(4-5)^2} = \frac{e^{4x}}{1} = \boxed{e^{4x}}$

SOL. KHUSUS PD

SOLUSI UMUM PD

$y = y_h + y_k$

$\boxed{y = (c_1 + c_2 x)e^{5x} + e^{4x}}$

$x = 0 \rightarrow y = 2$

$2 = (c_1 + c_2 \cdot 0)e^{5(0)} + e^{4(0)}$

$2 = (c_1 + 0)\cdot e^0 + e^0$

$2 = c_1 + 1$

$2 - 1 = c_1 \rightarrow \boxed{c_1 = 1}$

$x = 1 \rightarrow y = e^4$

$e^4 = (c_1 + c_2 \cdot 1)e^{5(1)} + e^{4(1)}$

$e^4 = (1 + c_2)e^5 + e^4$

$0 = (1 + c_2)e^5 \rightarrow 1 + c_2 = 0$

$\boxed{c_2 = -1}$

SOLUSI PD

$y = (1 - x)e^{5x} + e^{4x}$

CARA I

(4) $\frac{dx}{dt} = 3x + 4y \rightarrow 4y = \frac{dx}{dt} - 3x$

$\frac{d^2x}{dt^2} = 3\frac{dx}{dt} + 4\frac{dy}{dt}$ , $\frac{dy}{dt} = 4x - 3y$

$\frac{d^2x}{dt^2} = 3\frac{dx}{dt} + 4(4x - 3y)$

$\frac{d^2x}{dt^2} = 3\frac{dx}{dt} + 16x - 12y$

$\frac{d^2x}{dt^2} = 3\frac{dx}{dt} + 16x - 3(4y)$

$\frac{d^2x}{dt^2} = 3\frac{dx}{dt} + 16x - 3\left(\frac{dx}{dt} - 3x\right)$

$\frac{d^2x}{dt^2} = 3\frac{dx}{dt} + 16x - 3\frac{dx}{dt} + 9x$

$\frac{d^2x}{dt^2} = 25x$

$\frac{d^2x}{dt^2} - 25x = 0$

$D^2x - 25x = 0$

$(D^2 - 25)\cdot x = 0$

PERS. ELGEN

$\alpha^2 - 25 = 0$

$(\alpha + 5)(\alpha - 5) = 0$

$\alpha_1 = -5, \alpha_2 = 5$

$x = c_1 e^{\alpha_1 t} + c_2 e^{\alpha_2 t}$

$x = c_1 e^{-5t} + c_2 e^{5t}$

$\frac{dx}{dt} = -5c_1 e^{-5t} + 5c_2 e^{5t}$

$-3x = -3c_1 e^{-5t} - 3c_2 e^{5t}$ +

$4y = -8c_1 e^{-5t} + 2c_2 e^{5t}$ $\times\left(\frac{1}{4}\right)$

$y = -2c_1 e^{-5t} + \frac{1}{2}c_2 e^{5t}$

Solusi umum PD

$x(t) = c_1 e^{-5t} + c_2 e^{5t}$

$y(t) = -2c_1 e^{-5t} + \frac{1}{2}c_2 e^{5t}$

(4) CARA II

$\frac{dy}{dt} = 4x - 3y$ $\longrightarrow$ $4x = \frac{dy}{dt} + 3y$

$\longrightarrow$ didiferensial terhadap $t$

$\frac{d^2y}{dt^2} = 4\left(\frac{dx}{dt}\right) - 3\frac{dy}{dt}$ , $\frac{dx}{dt} = 3x + 4y$

$\frac{d^2y}{dt^2} = 4(3x+4y) - 3\frac{dy}{dt}$

$\frac{d^2y}{dt^2} = 12x + 16y - 3\frac{dy}{dt}$

$\frac{d^2y}{dt^2} = 3(4x) + 16y - 3\frac{dy}{dt}$

$\frac{d^2y}{dt^2} = 3\left(\frac{dy}{dt} + 3y\right) + 16y - 3\frac{dy}{dt}$

$\frac{d^2y}{dt^2} = 3\frac{dy}{dt} + 9y + 16y - 3\frac{dy}{dt}$

$\frac{d^2y}{dt^2} = 25y$

$\frac{d^2y}{dt^2} - 25.y = 0$

$D^2.y - 25.y = 0$

$(D^2 - 25).y = 0$

PERS. EIGEN

$\alpha^2 - 25 = 0$

$(\alpha + 5)(\alpha - 5) = 0$

$\alpha_1 = -5$, $\alpha_2 = 5$

$y = c_1 e^{\alpha_1 t} + c_2 e^{\alpha_2 t}$

$y(t) = c_1 e^{-5t} + c_2 e^{5t}$

$\frac{dy}{dt} = -5c_1 e^{-5t} + 5c_2 e^{5t}$

$4x = \frac{dy}{dt} + 3y$ $\longrightarrow$

$\frac{dy}{dt} = -5c_1 e^{-5t} + 5c_2 e^{5t}$

$3y = 3c_1 e^{-5t} + 3c_2 e^{5t}$ +

$4x = -2c_1 e^{-5t} + 8c_2 e^{5t}$ $\times(\frac{1}{4})$

$x(t) = -\frac{1}{2}c_1 e^{-5t} + 2c_2 e^{5t}$

SOLUSI UMUM PD

$x(t) = -\frac{1}{2}c_1 e^{-5t} + 2c_2 e^{5t}$

$y(t) = c_1 e^{-5t} + c_2 e^{5t}$

(4) CARA III

$\frac{dx}{dt} = 3x+4y \rightarrow Dx - 3x - 4y = 0 \rightarrow (D-3)x - 4y = 0$

$\frac{dy}{dt} = 4x-3y \rightarrow -4x + Dy + 3y = 0 \rightarrow -4x + (D+3)y = 0$

$$\begin{bmatrix} (D-3) & -4 \\ -4 & (D+3) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \rightarrow A = \begin{bmatrix} (D-3) & -4 \\ -4 & (D+3) \end{bmatrix}, \quad \vec{x} = \begin{bmatrix} x \\ y \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$A \cdot \vec{x} = B$

$$|A| = \begin{vmatrix} (D-3) & -4 \\ -4 & (D+3) \end{vmatrix} = (D-3)(D+3) - (-4)(-4)$$

$= D^2 - 9 - 16 = D^2 - 25$

$$|A_1| = \begin{vmatrix} 0 & -4 \\ 0 & (D+3) \end{vmatrix} = 0, \quad |A_2| = \begin{vmatrix} (D-3) & 0 \\ -4 & 0 \end{vmatrix} = 0$$

$x = \frac{|A_1|}{|A|} = \frac{0}{D^2-25} \rightarrow (D^2-25).x = 0$

PERS. EIGEN

$\alpha^2 - 25 = 0$

$(\alpha+5)(\alpha-5) = 0$

$\alpha_1 = -5, \alpha_2 = 5$

$x = c_1 e^{\alpha_1 t} + c_2 e^{\alpha_2 t}$

$x(t) = c_1 e^{-5t} + c_2 e^{5t}$

$\frac{dx}{dt} = -5c_1 e^{-5t} + 5c_2 e^{5t}$

$y = \frac{|A_2|}{|A|} = \frac{0}{D^2-25} \rightarrow (D^2-25).y = 0$

PERS. EIGEN

$\alpha^2 - 25 = 0$

$(\alpha+5)(\alpha-5) = 0$

$\alpha_3 = -5, \alpha_4 = 5$

$y = c_3 . e^{\alpha_3 t} + c_4 . e^{\alpha_4 t}$

$y(t) = c_3 e^{-5t} + c_4 e^{5t}$

SOLUSI UMUM PD (SEMENTARA)

$x(t) = c_1 e^{-5t} + c_2 e^{5t}$

$y(t) = c_3 e^{-5t} + c_4 e^{5t}$

$$\frac{dx}{dt} = 3x + 4y$$

$$3x = 3c_1 e^{-5t} + 3c_2 e^{5t}$$
$$4y = 4c_3 e^{-5t} + 4c_4 e^{5t}$$

$$\frac{dx}{dt} = (3c_1 + 4c_3) e^{-5t} + (3c_2 + 4c_4) e^{5t}$$

$$-5c_1 e^{-5t} + 5c_2 e^{5t} = (3c_1 + 4c_3) e^{-5t} + (3c_2 + 4c_4) e^{5t}$$

Koef. $e^{-5t}$ : $-5c_1 = 3c_1 + 4c_3 \rightarrow -5c_1 - 3c_1 = 4c_3$

$$-8c_1 = 4c_3 \rightarrow \boxed{c_3 = -2c_1}$$

Koef. $e^{5t}$ : $5c_2 = 3c_2 + 4c_4 \rightarrow 5c_2 - 3c_2 = 4c_4$

$$2c_2 = 4c_4 \rightarrow \boxed{c_4 = \tfrac{1}{2} c_2}$$

Solusi umum PD

$$x(t) = c_1 e^{-5t} + c_2 e^{5t}$$
$$y(t) = -2c_1 e^{-5t} + \tfrac{1}{2} c_2 e^{5t}$$