APLIKASI TRANSFORMASI LAPLACE PADA STATISTIKA MATEMATIKA (TEORI STATISTIK)

SEBARAN EKSPONENSIAL

$$f(x) = \begin{cases} \frac{1}{\theta} e^{-\frac{x}{\theta}} & \text{, UNTUK } 0 \leq x < \infty \\ 0 & \text{, UNTUK } x \text{ LAINNYA } (-\infty < x < 0) \end{cases}$$

BUKTIKAN : $\boxed{\mu = \theta \quad , \quad \sigma^2 = \theta^2}$

BUKTI

$$\mu = E(x) = \int_{-\infty}^{\infty} x \cdot f(x)\, dx$$

$$= \int_{-\infty}^{0} x \cdot (0)\, dx + \int_{0}^{\infty} x \cdot \left(\frac{1}{\theta} e^{-\frac{x}{\theta}}\right) dx$$

$$= 0 + \frac{1}{\theta} \int_{0}^{\infty} e^{-\left(\frac{1}{\theta}\right)x} \cdot x \cdot dx$$

MISALKAN $\boxed{\frac{1}{\theta} = s}$ DAN $x = t \rightarrow \frac{dx}{dt} = 1$

$\boxed{dx = dt}$

$$\mu = \left(\frac{1}{\theta}\right) \int_{0}^{\infty} e^{-st} (t)\, dt$$

$$\mu = s \cdot \mathcal{L}\{t\}$$

$$\mu = s \cdot \frac{1!}{s^{1+1}} = \cancel{s} \cdot \frac{1}{s^{\cancel{2}}}$$

$$\mu = \frac{1}{s} = \frac{1}{\left(\frac{1}{\theta}\right)}$$

$$\boxed{\mu = \theta}$$

$$E(x^2) = \int_{-\infty}^{\infty} x^2 \cdot f(x)\,dx$$

$$= \int_{-\infty}^{0} x^2 (0)\,dx + \int_{0}^{\infty} x^2 \left(\frac{1}{\theta} e^{-\frac{x}{\theta}}\right) dx$$

$$= 0 + \frac{1}{\theta} \int_0^{\infty} e^{-\frac{1}{\theta}x} (x^2)\,dx$$

$$= s \cdot \int_0^{\infty} e^{-st} (t^2)\,dt$$

$$= s \cdot L\{t^2\}$$

$$= s \cdot \frac{2!}{s^{2+1}}$$

$$= \frac{2s}{s^3}$$

$$= \frac{2}{s^2}$$

$$= \frac{2}{\left(\frac{1}{\theta}\right)^2} = \frac{2}{\frac{1}{\theta^2}} = \frac{2}{1} \cdot \frac{\theta^2}{1}$$

$$\underline{E(x^2) = 2\theta^2}$$

$$\sigma^2 = E(x^2) - \{E(x)\}^2$$

$$\sigma^2 = 2\theta^2 - (\mu)^2$$

$$\sigma^2 = 2\theta^2 - \theta^2$$

$$\boxed{\sigma^2 = \theta^2}$$