MIXING PROBLEM (Hal, 13 Erwin kreyseig, Advanced Engineering Mathematics)

Mixing Problem occur quite Frequently in Chemical Industry. We explain here how to solve The Basic Model involving a single tank. The tank in Fig.9 contains 1000 gal of water in which initially 100 lb of salt is dissolved. Brine runs in a rate of 10 gal/min, and each galon contains 5 lb of dissolved salt. The Mixture in the tank is kept uniform by stirring. Brine runs out at 10 gal/min.

Find the amount of salt in the tank at any time t.

MASALAH PENCAMPURAN
(Hal, 13 Erwin kreyseig, Advanced Engineering Mathematics)

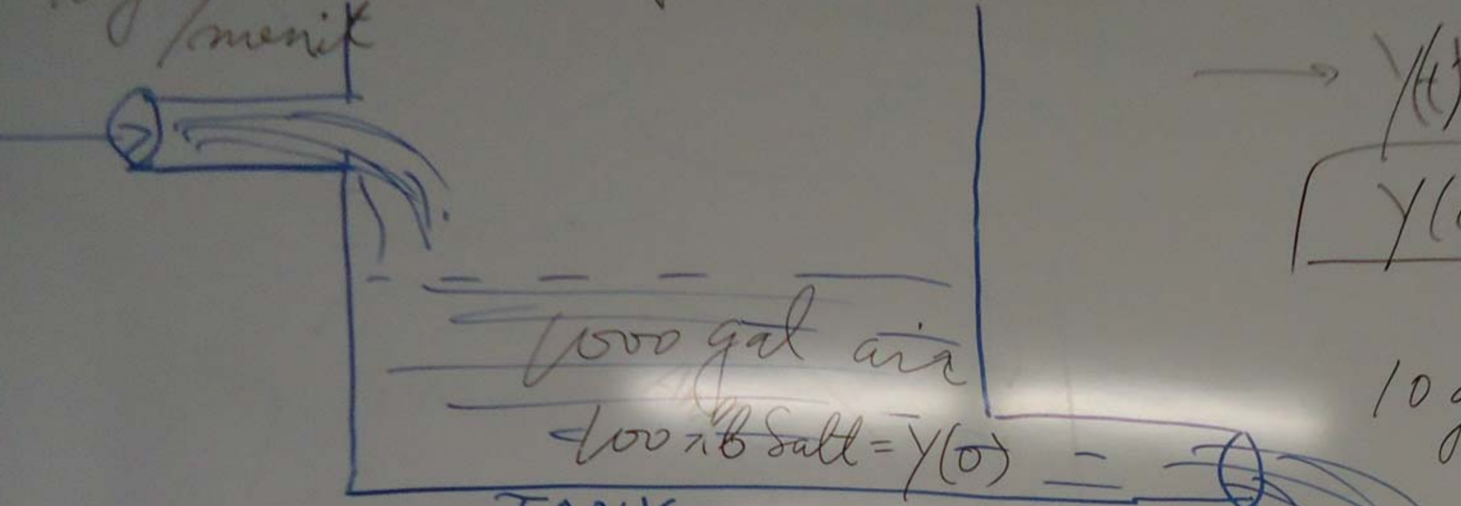
Masalah pencampuran cukup sering terjadi di Industri Kimia. Kami menjelaskan di sini bagaimana memecahkan Model Dasar melibatkan satu tangki. Tangki dalam Gbr.9 mengandung 1.000 gal air yang awalnya 100 lb garam terlarut. Air garam berjalan dengan kecepatan 10 gal / menit, dan setiap galon berisi 5 lb garam terlarut. Campuran di tangki itu diaduk secara merata. Air garam keluar dengan kecepatan 10 gal / mm.

Cari jumlah garam dalam tangki pada setiap waktu t.

10 gal/menit → 1 gal = 5 lb Salt

→ $y(t) = ?$

$\boxed{y(0) = 100 \text{ lb}}$



10 gal/menit Salt

TANK

FIG. 9

$\boxed{y' = \text{SALT INFLOW RATE} - \text{SALT OUTFLOW RATE}}$

$$\frac{dy}{dt} = 10\ \cancel{\text{gal}}/\text{menit} \cdot \frac{5 \text{ lb Salt}}{1\ \cancel{\text{gal}}} - \frac{10 \text{ gal/mnt}}{1000 \text{ gal}} \cdot y$$

$$\boxed{\frac{dy}{dt} = 50 - 0{,}01\,y}$$

$$\frac{dy}{dt} + 0{,}01\, y = 50$$

$$\boxed{\frac{dy}{dt} + P(t) \cdot y = Q(t)}$$

$P(t) = 0{,}01$

$Q(t) = 50$

$$S(t) = e^{\int P(t)\,dt} = e^{\int 0{,}01\,dt} = e^{0{,}01t}$$

SOL. UM. PD

$$y(t) = \frac{1}{S(t)}\left[\int S(t) \cdot Q(t)\,dt + C\right]$$

$$y(t) = \frac{1}{e^{0{,}01t}}\left[\int e^{0{,}01t} \cdot (50)\,dt + C\right]$$

$$y(t) = e^{-0{,}01t}\left[50\left(\frac{1}{0{,}01}\right)\int \underline{e^{0{,}01t} \cdot \frac{0{,}01\,dt}{d(0{,}01t)}} + C\right]$$

$$y(t) = e^{-0{,}01t}\left[\frac{50}{0{,}01}\underline{e^{0{,}01t}} + C\right]$$

$$Y(0) = e^{-0,01.(0)} \left[\frac{50}{0,01} e^{0,01(0)} + C \right]$$

$$100 = \underbrace{e^{0}}_{1} \left[5000 \underbrace{e^{0}}_{1} + C \right]$$

$$100 = 5000 + C$$

$$100 - 5000 = C$$

$$C = -4.900$$

JUMLAH GARAM DLM. TANGKI PADA SETIAP WAKTU t

$$Y(t) = e^{-0,01t} \left(5000\, e^{0,01t} - 4900 \right)$$

$$\boxed{Y(t) = 5000 - 4900 . e^{-0,01t}}$$

lb

PD. LINEAR EULER-CAUCHY

1. $$x^2 \cdot \frac{d^2y}{dx^2} + 3x \cdot \frac{dy}{dx} + y = x^4$$

2. $$\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 12y = x.e^{2x}$$

3. PD. CLAIRAUT

$$y = xp + p^2 - 15$$

4. PD. SIMULTAN (SISTEM P.D.)

$$\frac{dx}{dt} = x - y + t^2$$

$$\frac{dy}{dt} = x + y - t$$

PD Linear Euler Cauchy

1) $x^2 \dfrac{d^2y}{dx^2} + 3x \dfrac{dy}{dx} + y = x^4$

misalkan

$$\boxed{\begin{aligned} x &= e^u \\ \ln x &= \ln e^u \\ \ln x &= u \end{aligned}} \longrightarrow x^4 = e^{4u}$$

$$D(D-1)y + 3Dy + y = e^{4u}$$

$$(D^2 - D + 3D + 1)y = e^{4u}$$

$$(D^2 + 2D + 1)y = e^{4u}$$

I) $(D^2+2D+1)y=0$

Pers. eigen

$$(\alpha^2+2\alpha+1)=0$$
$$(\alpha+1)^2=0$$
$$\boxed{\alpha_{1,2}=-1}$$

$$y_h=(c_1+c_2 \cdot u)e^{\alpha u}$$
$$=(c_1+c_2 u)e^{-u} \quad , e^{-u}=\frac{1}{e^u}=\frac{1}{x}$$
$$=(c_1+c_2 \ln x)\frac{1}{x}$$
$$=\frac{c_1+c_2 \ln x}{x}$$

II $(D^2 + 2D + 1)y = e^{4u} \Rightarrow \dfrac{k = 4}{D = k = 4}$

$$y_k = \frac{e^{4u}}{(D^2+2D+1)} = \frac{e^{4u}}{(4^2+2(4)+1)} = \frac{e^{4u}}{(16+8+1)}$$

$$= \frac{e^{4u}}{25} = \frac{1}{25}\left(e^{4u}\right)$$

$= \frac{1}{x}$

$$y_k = \frac{1}{25}x^4$$

solun. PD

$$y = y_h + y_k$$

$$\boxed{y = \frac{(c_1 + c_2 \ln x)}{x} + \frac{1}{25}x^4}$$

$$\frac{d^2y}{dx^2} - \frac{7\,dy}{dx} + 12y = x \cdot e^{2x}$$

$$(D^2 - 7D + 12)\,y = x \cdot e^{2x}$$

$$(D-4)(D-3)\,y = x \cdot e^{2x} = e^{2x} \cdot x$$

Pers Eigen

$$(\alpha - 4)(\alpha - 3) = 0$$

$$\alpha_1 = 4,\ \alpha_2 = 3$$

$$y_h = C_1 e^{4x} + C_2 e^{3x}$$

$a = 2$

$$y_{khusus} = \frac{1}{D^2 - 7D + 12} \cdot \left(e^{2x} \cdot x\right) \rightarrow a = 2$$

$$= e^{2x}\left[\frac{1}{(D+2)^2 - 7(D+2) + 12}(x)\right]$$

$$= e^{2x}\left[\frac{1}{D^2+\underline{4D}+4-\underline{7D}-14+12}(x)\right]$$

$$= e^{2x}\left[\frac{1}{D^2-3D+2}(x)\right]$$

$$= e^{2x}\left[\left(\frac{1}{2}+\frac{3}{4}D\right)(x)\right] = e^{2x}\left[\frac{1}{2}x+\frac{3}{4}\underbrace{D(x)}_{1}\right]$$

$$= e^{2x}\cdot\left[\frac{1}{2}x+\frac{3}{4}\right]$$

$$yk = \frac{1}{2}x\,e^{2x}+\frac{3}{4}e^{2x}$$

$$\boxed{\begin{aligned} D(x) &= 1 \\ D^2(x) &= 0 \end{aligned}}$$

$$\begin{array}{r|l} & \frac{1}{2}+\frac{3}{4}D \\ \hline 2-3D+D^2 & 1 \\ & 1-\frac{3}{2}D+\frac{1}{2}D^2 \\ \hline & \frac{3}{2}D-\frac{1}{2}D^2 \\ & \frac{3}{2}D-\frac{9}{4}D^2 \\ \hline \end{array}$$

Solusi umum PD: $\boxed{y = yh + yk}$

$$y = C_1\cdot e^{4x}+C_2e^{3x}+\frac{1}{2}x\cdot e^{2x}+\frac{3}{4}e^{2x}$$

3) $y = xp + p^2 - 15$ diturunkan thd x $p = \frac{dy}{dx}$

$$\frac{dy}{dx} = x'p + xp' + 2p \cdot p'$$

$$\cancel{p} = \cancel{p} + xp' + 2p \cdot \left(\frac{dp}{dx}\right)$$

$$0 = x\left(\frac{dp}{dx}\right) + 2p\left(\frac{dp}{dx}\right)$$

$$0 = (x+2p)\left(\frac{dp}{dx}\right)$$

Solum:

$$\frac{dp}{dx} = 0$$

$$dp = 0\,dx$$

$$\int dp = \int 0\,dx$$

$$\boxed{p = C}$$

SOLUSI UMUM:

$$y = xp + p^2 - 15$$

$$\boxed{y = cx + c^2 - 15}$$

Solusi Singular $\longrightarrow$ $\boxed{x + 2p = 0}$

$$2p = -x$$

$$p = -x/2$$

$$\boxed{y = xp + p^2 - 15} = x\left(-\frac{x}{2}\right) + \left(-\frac{x}{2}\right)^2 - 15$$

$$y = \left(-\frac{x^2}{2}\right) + \left(\frac{1}{4}x^2\right) - 15$$

$$y = -\frac{1}{2}x^2 + \frac{1}{4}x^2 - 15$$

$$y = -\frac{2}{4}x^2 + \frac{1}{4}x^2 - 15$$

$$y = \boxed{-\frac{1}{4}x^2 - 15}$$

① x^2

②

③

④ $$\frac{dx}{dt} = x - y + t^2$$

$$y = -\frac{dx}{dt} + x + t^2$$

$$\frac{dy}{dt} = x + y - t$$

I.) $$\frac{dx}{dt} = x - y + t^2$$

$$\frac{d^2x}{dt^2} = \frac{dx}{dt} - \frac{dy}{dt} + 2t$$

$$\frac{d^2x}{dt^2} = \frac{dx}{dt} - x - y + t + 2t$$

$$\frac{d^2x}{dt^2} = \frac{dx}{dt} - x + \frac{dx}{dt} - x - t^2 + 3t$$

$$\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + 2x = -t^2 + 3t$$

$$D^2x - 2Dx + 2x = -t^2 + 3t$$

$$(D^2 - 2D + 2)x = -t^2 + 3t$$

$+t^2$

Pers. Eigen

$D^2 - 2D + 2 = 0$

$\alpha^2 - 2\alpha + 2 = 0$

$$\alpha_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{2 \pm \sqrt{(-2)^2-4(1)(2)}}{2\cdot 1}$$

$$\alpha_{1,2} = \frac{2 \pm \sqrt{-4}}{2}$$

$$\alpha_{1,2} = \frac{2 \pm 2i}{2}$$

$$\alpha_{1,2} = 1 \pm i$$

$a = 1;\ b = 1$

c

$4(1)(2)$

$$x_h = e^{at}(c_1 \cos bt + c_2 \sin bt)$$

$$= e^{t}(c_1 \cos t + c_2 \sin t)$$

$$(D^2 - 2D + 2)x = -t^2 + 3t$$

$$x_k = \frac{1}{(D^2 - 2D + 2)}(-t^2 + 3t)$$

$$x_k = \left(\frac{1}{2} + \frac{1}{2}D + \frac{1}{4}D^2\right)(-t^2 + 3t)$$

$$= -\frac{1}{2}t^2 + \frac{3}{2}t + \frac{1}{2}D(-t^2 + 3t) + \frac{1}{4}D^2(-t^2 + 3t)$$

$$= -\frac{1}{2}t^2 + \frac{3}{2}t + \frac{1}{2}(-2t + 3) + \frac{1}{4}(-2)$$

$$= -\frac{1}{2}t^2 + \frac{3}{2}t - t + \frac{3}{2} - \frac{1}{2}$$

$$x_k = -\frac{1}{2}t^2 + \frac{1}{2}t + 1 \quad //$$

$$x = x_h + x_k$$

$$= e^{t}(c_1 \cos t + c_2 \sin t) - \frac{1}{2}t^2 + \frac{1}{2}t + 1$$

$$\frac{1}{2}+\frac{1}{2}D+\frac{1}{4}D^2$$

$$2-2D+D^2 \,\big/\, 1$$

$$1 - D + \frac{1}{2}D^2 \quad -$$

$$D - \frac{1}{2}D^2$$

$$D - D^2 + \frac{1}{2}D^3 \quad -$$

$$\frac{1}{2}D^2 - \frac{1}{2}D^3$$

$$\frac{1}{2}D^2 - \frac{1}{2}D^3 + \frac{1}{4}D^4$$

$$D(-t^2+3t) = -2t+3$$

$$D^2(-t^2+3t) = -2$$

$$x = \underset{u}{e^t}\underbrace{(c_1 \cos t + c_2 \sin t)}_{v} - \frac{1}{2}t^2 + \frac{1}{2}t + 1$$

$$\frac{dx}{dt} = u'v + uv' - t + \frac{1}{2}$$

$$\frac{dx}{dt} = e^t(c_1 \cos t + c_2 \sin t) + e^t(-c_1 \sin t + c_2 \cos t) - t + \frac{1}{2}$$

$$\frac{dx}{dt} = e^t\left(\cos t.(c_1+c_2) + \sin t(-c_1+c_2)\right] - t + \frac{1}{2}$$

$$-\frac{dx}{dt}$$

X

t^2

$Y =$

$Y =$

x

$-2t+3$

-2

$$\boxed{Y = -\frac{dx}{dt} + x + t^2}$$

$$-\frac{dx}{dt} = e^t\left[\cos t(\cancel{-}c_1 - c_2) + \sin t(+c_1 - \cancel{c_2})\right] - t + \frac{1}{2}$$

$$X = e^t\left[\cancel{c_1\cos t} + \cancel{c_2\sin t}\right] - \frac{1}{2}t^2 + \frac{1}{2}t + 1$$

$$t^2 = \qquad t^2$$

$$Y = e^t\left[-c_2 \cos t + c_1 \sin t\right] + \frac{1}{2}t^2 - \frac{1}{2}t + \frac{3}{2}$$

Solum PD:

$$x = e^t(c_1\cos t + c_2 \sin t) - \frac{1}{2}t^2 + \frac{1}{2}t + 1$$

$$y = e^t\left[-c_2\cos t + c_1 \sin t\right) + \frac{1}{2}t^2 - \frac{1}{2}t + \frac{3}{2}$$

$)-t+\frac{1}{2}$