PD. LINEAR EULER-CAUCHY

① $$x^2 \frac{d^2y}{dx^2} + 8x \frac{dy}{dx} - 8y = x^3$$

Misalkan: $x = e^u$ $\rightarrow$ $\underline{x^3 = (e^u)^3 = e^{3u}}$

$\ln x = \ln e^u$

$\ln x = u$

$$D(D-1)y + 8 \cdot Dy - 8y = e^{3u}$$

$$(D^2 - D + 8D - 8)y = e^{3u}$$

$$(D^2 + 7D - 8)y = e^{3u}$$

$$(D-1)(D+8)y = e^{3u}$$

Ⓘ Pers. Eigen

$$(\alpha - 1)(\alpha + 8) = 0$$

$$\alpha_1 = 1 \qquad \alpha_2 = -8$$

$$y_h = C_1 e^{\alpha_1 u} + C_2 e^{\alpha_2 u}$$

$$= C_1 \boxed{e^{u}} + C_2 \boxed{e^{-8u}} \quad , \quad \boxed{e^{-8u} = (e^{u})^{-8} = x^{-8}}$$

$$y_h = C_1 x + C_2 x^{-8}$$

(II) $$(D^2 + 7D - 8)\, y = e^{3u} \xrightarrow{} \overset{k=3}{\boxed{D = k = 3}}$$

$$y_k = \frac{e^{3u}}{(D^2+7D-8)} = \frac{e^{3u}}{(9+21-8)} = \frac{e^{3u}}{22} = \frac{1}{22} x^3$$

(SOLUSI KHUSUS PD)

Sol. umum PD :

$$y = y_h + y_k$$

$$y = C_1 x + C_2 \boxed{x^{-8}} + \frac{1}{22} x^3$$

$$\boxed{y = C_1 x + \frac{C_2}{x^8} + \frac{1}{22} x^3}$$

2.) $\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 15y = xe^{2x}$

$D^2y - 8Dy + 15y = xe^{2x}$

$(D^2 - 8D + 15)y = xe^{2x}$

① Pers. Eigen

$(\alpha^2 - 8\alpha + 15) = 0$

$(\alpha - 5)(\alpha - 3) = 0$

$\underline{\alpha_1 = 5}, \underline{\alpha_2 = 3}$

$$\boxed{y_h = C_1 e^{5x} + C_2 e^{3x}}$$

$2x$

$2x$

II. Solusi Khusus

$$(D^2-8D+15)y = xe^{2x}$$

$$y_k = \boxed{\frac{1}{(D^2-8D+15)}\left(e^{2x}.x\right)}$$

$$= e^{2x}\left[\frac{1}{(D+\underline{2})^2-8(D+\underline{2})+15}.(x)\right]$$

$$= e^{2x}\left[\frac{1}{D^2+4D+4-8D-16+15}(x)\right]$$

$$= e^{2x}\left[\boxed{\frac{1}{D^2-4D+3}}.(x)\right]$$

$$= e^{2x}\left[\left(\frac{1}{3}+\frac{4}{9}D\right)(x)\right]$$

$$= e^{2x}\left[\frac{1}{3}x+\frac{4}{9}\underbrace{D(x)}_{1}\right]$$

$$= e^{2x}\left[\frac{1}{3}x+\frac{4}{9}\right]$$

$$\boxed{Y_k = \frac{1}{3}xe^{2x}+\frac{4}{9}e^{2x}}$$

$$\frac{1}{f(D)}.(e$$

$3-4$

$$\frac{1}{f(D)}\cdot\left(e^{ax}\cdot F\right) = e^{ax}\cdot\left[\frac{1}{f(D+a)}\cdot F\right]$$

$$\boxed{\begin{array}{l} D(x) = 1 \\ D^2(x) = 0 \end{array}}$$

x)]

$$\boxed{\begin{array}{rl} & \frac{1}{3}+\frac{4}{9}D \\ \cline{2-2} 3-4D+D^2 \big) & 1 \\ & 1-\frac{4}{3}D+\frac{1}{3}D^2 \quad - \\ \cline{2-2} & \frac{4}{3}D-\frac{1}{3}D^2 \\ & \frac{4}{3}D - \end{array}}$$

(x)]

x)]

<u>solusi umum PD</u>

$$y = y_h + y_k$$

$$Y = C_1e^{5x} + C_2e^{3x} + \frac{1}{3}xe^{2x} + \frac{4}{9}e^{2x}$$

ATAU : $\frac{1}{f(D)} \cdot (X.F) = X.\left[\frac{1}{f(D)} \cdot F\right] - \frac{f'(D)}{[f(D)]^2} \cdot F$

$$\left(\begin{array}{l} f(D) = D^2 - 8D + 15 \\ f'(D) = 2D - 8 \end{array} \right)$$

$$Y_K = \frac{1}{\boxed{D^2-8D+15}} \cdot (x . \overset{F}{\boxed{e^{2x}}})$$

$$= x . \left[\frac{1}{D^2-8D+15} \cdot e^{2x}\right] - \frac{(2D-8)}{(D^2-8D+15)^2} \cdot e^{\overset{K}{\textcircled{2}}x}$$

$$\boxed{\begin{array}{l} K=2 \\ D=K=2 \end{array}}$$

$$= x \left[\frac{1}{2^2 - \underset{16}{8(2)} + 15} \cdot e^{2x}\right] - \frac{2(2)-8}{(2^2-8(2)+15)^2} \cdot e^{2x}$$

$$= x \left(\frac{1}{3} e^{2x}\right) - \frac{-4}{(3)^2} e^{2x}$$

$$Y_K = e^{2x}\left(\frac{1}{3}x + \frac{4}{9}\right)$$

$$= \frac{1}{3} x e^{2x} + \frac{4}{9} e^{2x}$$

③ $y = xp + p^2 + 9$, $\boxed{p = \frac{dy}{dx}}$

(PD. CLAIRAUT)

diff thd x

p'

$$\frac{dy}{dx} = \underset{1}{(x)'}\, p + x(p)' + 2p\,\frac{dp}{dx}$$

$$\cancel{p} = \cancel{p} + x\frac{dp}{dx} + 2p\frac{dp}{dx}$$

$$0 = \frac{dp}{dx}(x+2p)$$

$$\boxed{\frac{dp}{dx} = 0}$$

$dp = 0\,dx$

$\int dp = \int 0\,dx$

$$\boxed{p = c}$$

Solum:

$y = xp + p^2 + 9$

$y = xc + c^2 + 9$

$$\boxed{x + 2p = 0}$$

$x = -2p$

$$\boxed{p = -\frac{x}{2}}$$

$y = xp + p^2 + 9$

$= x\left(\frac{-x}{2}\right) + \left(\frac{-x}{2}\right)^2 + 9$

$= -\frac{x^2}{2} + \frac{x^2}{4} + 9$

$$\boxed{y = -\frac{1}{4}x^2 + 9}$$ Solusi Singular

④

$$\frac{dx}{dt} - 2y = 6t \longrightarrow Dx - 2y = 6t$$

$$-2x + \frac{dy}{dt} = t^2 \longrightarrow -2x + Dy = t^2$$

$$\underbrace{\begin{bmatrix} D & -2 \\ -2 & D \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \end{bmatrix}}_{\vec{x}} = \underbrace{\begin{bmatrix} 6t \\ t^2 \end{bmatrix}}_{B}$$

$$|A| = \begin{vmatrix} D & -2 \\ -2 & D \end{vmatrix}$$

$$\boxed{|A| = D^2 - 4}$$

$$x = \frac{|A_1|}{|A|} = \frac{\begin{vmatrix} \boxed{\begin{matrix} 6t \\ t^2 \end{matrix}} & \begin{matrix} -2 \\ D \end{matrix} \end{vmatrix}}{D^2 - 4} = \frac{D(6t) - (-2)t^2}{D^2 - 4} = \frac{6 + 2t^2}{D^2 - 4}$$

$$\longrightarrow (D^2 - 4)x = 6 + 2t^2$$

Solusi homogen

$(D^2-4)x=0$

pers. Eigen

$\alpha^2-4=0$

$(\alpha-2)(\alpha+2)=0$

$\alpha_1=2$, $\alpha_2=-2$

$X_h=c_1e^{\alpha_1 t}+c_2e^{\alpha_2 t}$

$\boxed{X_h=c_1e^{2t}+c_2e^{-2t}}$

Solusi khusus

$$X_k=\frac{6+2t^2}{D^2-4}$$

$$=\left(-\frac{1}{4}-\frac{1}{16}D^2\right)(6+2t^2)$$

$$=-\frac{1}{4}(6+2t^2)-\frac{1}{16}D^2(6+2t^2)$$

$$=-\frac{3}{2}-\frac{1}{2}t^2-\frac{1}{4}$$

$$\boxed{X_k=-\frac{7}{4}-\frac{1}{2}t^2}$$

$X=X_h+X_k$

$$\boxed{X=c_1e^{2t}+c_2e^{-2t}-\frac{7}{4}-\frac{1}{2}t^2}$$

$D(6+2t^2)=4t$

$D^2(6+2t^2)=4$

$D^3(6+2t^2)=0$

$-\frac{1}{4}-\frac{1}{16}D^2$

4

$-4+D^2 \;\overline{)\;1\;}$

$1-\frac{1}{4}D^2$

$\frac{1}{4}D^2-\frac{1}{16}D^4$

$\frac{1}{4}D^2$

$Y=\frac{|A_2|}{|A|}$

$\rightarrow (D$

Solusi H

$Y_h=($

Solusi

$Y_k=$

$Y_k=$

2

$+2t^2$

$-4)$

$$y = \frac{|A_2|}{|A|} = \frac{\begin{vmatrix} D & \boxed{6t} \\ -2 & \boxed{t^2} \end{vmatrix}}{D^2-4} = \frac{D(t^2)+12t}{D^2-4} = \frac{2t+12t}{D^2-4}$$

$$\longrightarrow (D^2-4)\,y = 14t$$

Solusi Homogen

$$\boxed{Y_h = C_3 e^{2t} + C_4 e^{-2t}}$$

Solusi Khusus

$$\boxed{Y_k = \frac{14t}{D^2-4}}$$

$$\boxed{\begin{array}{l} D(14t) = 14 \\ \hline D^2(14t) = 0 \end{array}}$$

$$Y_K = \frac{1}{-4+D^2}(14t)$$

$$\begin{array}{r|l} & -\frac{1}{4} \\ \hline -4+D^2 & 1 \\ & 1-\frac{1}{4}D^2 \\ \hline & \frac{1}{4}D^2 \end{array}$$

$$Y_k = \left(-\frac{1}{4}\right)14t$$

$$= -\frac{7}{2}t$$

$$y = Y_h + Y_k$$

$$\boxed{y = C_3 e^{2t} + C_4 e^{-2t} - \frac{7}{2}t}$$

$\frac{1}{16}D^4$

$$x = C_1 e^{2t} + C_2 e^{-2t} - \frac{7}{4} - \frac{1}{2}t^2$$

$$\boxed{Dx = 2C_1 e^{2t} - 2C_2 e^{-2t} - t}$$

$$2y = 2C_3 e^{2t} + 2C_4 e^{-2t} - 7t$$

$$\boxed{Dx - 2y = 6t}$$

$$Dx = 2y + 6t$$

$$Dx = 2C_3 e^{2t} + 2C_4 e^{-2t} - 7t + 6t$$

$$Dx = 2C_3 e^{2t} + 2C_4 e^{-2t} - t$$

$$\searrow$$

$$2C_1 e^{2t} - 2C_2 \boxed{e^{-2t}} - \cancel{t} = 2C_3 e^{2t} + 2C_4 \boxed{e^{-2t}} - \cancel{t}$$

Koefisien e^{2t} : $\cancel{2}C_1 = \cancel{2}C_3 \longrightarrow \boxed{C_3 = C_1}$

Koefisien e^{-2t} : $-\cancel{2}C_2 = \cancel{2}C_4 \longrightarrow \boxed{C_4 = -C_2}$

$$x = C_1 e^{2t} + C_2 e^{-2t} - \frac{7}{4} - \frac{1}{2}t^2$$

$$y = C_1 e^{2t} - C_2 e^{-2t} - \frac{7}{2}t$$

$x(0) = 0$

$$C_1 e^{2(0)} + C_2 e^{-2(0)} - \frac{7}{4} - \frac{1}{2}(0)^2 = 0$$

$$\boxed{C_1 + C_2 = \frac{7}{4}} \quad ①$$

$y(0) = 0$

$$C_1 e^{2(0)} - C_2 e^{-2(0)} - \cancel{\frac{7}{2}(0)} = 0$$

$$\boxed{C_1 - C_2 = 0} \quad ②$$

① $C_1 + \cancel{C_2} = \frac{7}{4}$

② $C_1 - \cancel{C_2} = 0$ $+$ $\longrightarrow$ $2C_1 = \frac{7}{4}$

$\boxed{C_1 = \frac{7}{8}}$

$C_1 = C_2$

$C_2 = C_1$

$\boxed{C_2 = \frac{7}{8}}$

$$\boxed{C_1 = \frac{7}{8},\ C_2 = \frac{7}{8}}$$

$\cancel{t}$

Solusi umum

$$\boxed{\begin{aligned} x &= \frac{7}{8}e^{2t} + \frac{7}{8}e^{-2t} - \frac{7}{4} - \frac{1}{2}t^2 \\ y &= \frac{7}{8}e^{2t} - \frac{7}{8}e^{-2t} - \frac{7}{2}t \end{aligned}}$$