6.

$$\frac{dx}{dt} = x + y - z$$

$$\frac{dy}{dt} = x - y + z$$

$$\frac{dz}{dt} = -x + y + z$$

$$\frac{dx}{dt} = x + y - z \longrightarrow (Dx - x) - y + z = 0$$

$$\frac{dy}{dt} = x - y + z \longrightarrow -x + (Dy + y) - z = 0$$

$$\frac{dz}{dt} = -x + y + z \longrightarrow x - y + (Dz - z) = 0$$

$\underline{x = ?}$, $\underline{y = ?}$, $\underline{z = ?}$

$$\begin{aligned} (D-1)x - y + z &= 0 \\ -x + (D+1)y - z &= 0 \\ x - y + (D-1)z &= 0 \end{aligned} \longrightarrow \begin{bmatrix} (D-1) & -1 & 1 \\ -1 & (D+1) & -1 \\ 1 & -1 & (D-1) \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A \cdot \vec{X} = B$$

$|A|$

$|A$

$|A$

$$|A| = \left|\begin{array}{ccc|cc} (D-1) & -1 & 1 & (D-1) & -1 \\ -1 & (D+1) & -1 & -1 & (D+1) \\ 1 & -1 & (D-1) & 1 & -1 \end{array}\right|$$

$$- \quad - \quad - \qquad + \quad + \quad +$$

$$|A| = \left[(D+1)(D-1)^2 + 1 + 1\right] - \left[(D+\cancel{1}) + (D-\cancel{1}) + (D-1)\right]$$

$$|A| = \left[(D+1)(D^2-2D+1) + 2\right] - \left[D+D+D-1\right]$$

$$= D^3 - 2D^2 + D + D^2 - 2D + 1 + 2 - 3D + 1$$

$$|A| = D^3 - D^2 - 4D + 4$$

# ATURAN CRAMER

$$X = \frac{|A_1|}{|A|} = \frac{\begin{vmatrix} 0 & -1 & 1 \\ 0 & (D+1) & -1 \\ 0 & -1 & (D-1) \end{vmatrix}}{\begin{vmatrix} (D-1) & -1 & 1 \\ -1 & (D+1) & -1 \\ 1 & -1 & (D-1) \end{vmatrix}}$$

$$X = \frac{0}{\underline{D^3 - D^2 - 4D + 4}}$$

$$\boxed{(D^3 - D^2 - 4D + 4) \cdot X = 0}$$

PERS. EIGEN

$$\alpha^3 - \alpha^2 - 4\alpha + 4 = 0$$

$$(\alpha^3 - \alpha^2) - (4\alpha - 4) = 0$$

$$\alpha^2(\alpha - 1) - 4(\alpha - 1) = 0$$

$$(\alpha-1)\underbrace{(\alpha^2-4)}=0$$

$$(\alpha-1)(\alpha+2)(\alpha-2)=0$$

$$\underline{\alpha_1=1}, \ \underline{\alpha_2=-2}, \ \underline{\alpha_3=2}$$

$$X = c_1 e^{\alpha_1 t} + c_2 e^{\alpha_2 t} + c_3 e^{\alpha_3 t}$$

$$\boxed{X = c_1 e^{t} + c_2 e^{-2t} + c_3 e^{2t}}$$

---

$$Y = \frac{|A_2|}{|A|} = \frac{\begin{vmatrix} (D-1) & 0 & 1 \\ -1 & 0 & -1 \\ 1 & 0 & (D-1) \end{vmatrix}}{\begin{vmatrix} (D-1) & -1 & 1 \\ -1 & (D+1) & -1 \\ 1 & -1 & (D-1) \end{vmatrix}}$$

$$Y = \frac{0}{D^3 - D^2 - 4D + 4}$$

$$\boxed{(D^3 - D^2 - 4D + 4)\, y = 0}$$

PERS. EIGEN

$$\alpha^3 - \alpha^2 - 4\alpha + 4 = 0$$

$$(\alpha^3 - \alpha^2) - (4\alpha - 4) = 0$$

$$\alpha^2 (\alpha - 1) - 4(\alpha - 1) = 0$$

$$(\alpha - 1)(\alpha^2 - 4) = 0$$

$$(\alpha - 1)(\alpha + 2)(\alpha - 2) = 0$$

$$\alpha_4 = 1, \quad \alpha_5 = -2, \quad \alpha_6 = 2$$

$$y = c_4 e^{\alpha_4 t} + c_5 e^{\alpha_5 t} + c_6 e^{\alpha_6 t}$$

$$\boxed{y = c_4 e^{t} + c_5 e^{-2t} + c_6 e^{2t}}$$

$$Z = \frac{|A_3|}{|A|} = \frac{\begin{vmatrix} (D-1) & -1 & \boxed{0} \\ -1 & (D+1) & 0 \\ 1 & -1 & 0 \end{vmatrix}}{\begin{vmatrix} (D-1) & -1 & 1 \\ -1 & (D+1) & -1 \\ 1 & -1 & (D-1) \end{vmatrix}} = \frac{0}{D^3 - D^2 - 4D + 4}$$

$$\underline{(D^3 - D^2 - 4D + 4)} \cdot Z = 0$$

PERS. EIGEN

$$\alpha^3 - \alpha^2 - 4\alpha + 4 = 0$$

$$(\alpha^3 - \alpha^2) - (4\alpha - 4) = 0$$

$$\alpha^2(\alpha - 1) - 4(\alpha - 1) = 0$$

$$(\alpha - 1)(\alpha^2 - 4) = 0$$

$$(\alpha - 1)(\alpha + 2)(\alpha - 2) = 0$$

$$\alpha_7 = 1, \quad \alpha_8 = -2, \quad \alpha_9 = 2$$

$$Z = C_7 e^{\alpha_7 t} + C_8 e^{\alpha_8 t} + C_9 e^{\alpha_9 t}$$

$$\boxed{Z = C_7 e^{t} + C_8 e^{-2t} + C_9 e^{2t}}$$

Sol. UM. PD (SEMENTARA)

$$X = C_1 e^{t} + C_2 e^{-2t} + C_3 e^{2t}$$

$$Y = C_4 e^{t} + C_5 e^{-2t} + C_6 e^{2t}$$

$$Z = C_7 e^{t} + C_8 e^{-2t} + C_9 e^{2t}$$

$X =$

$\frac{dx}{dt} =$

$$X = c_1 e^{t} + c_2 e^{-2t} + c_3 e^{2t}$$

$$\frac{dx}{dt} = c_1 e^{t} - 2c_2 e^{-2t} + 2c_3 e^{2t}$$

$$X = c_1 e^{t} + c_2 e^{-2t} + c_3 e^{2t}$$

$$Y = c_4 e^{t} + c_5 e^{-2t} + c_6 e^{2t} \quad +$$

$$X + Y = (c_1 + c_4) e^{t} + (c_2 + c_5) e^{-2t} + (c_3 + c_6) e^{2t}$$

$$z = c_7 e^{t} + c_8 e^{-2t} + c_9 e^{2t} \quad -$$

$$X + Y - z = (c_1 + c_4 - c_7) e^{t} + (c_2 + c_5 - c_8) e^{-2t} + (c_3 + c_6 - c_9) e^{2t}$$

$$\frac{dx}{dt} =$$

$$\boxed{c_1 e^{t} - 2c_2 e^{-2t} + 2c_3 e^{2t} = (c_1 + c_4 - c_7) e^{t} + (c_2 + c_5 - c_8) e^{-2t} + (c_3 + c_6 - c_9) e^{2t}}$$

$$\text{Koef. } e^{t}: \quad \cancel{c_1} = \cancel{c_1} + c_4 - c_7 \longrightarrow \boxed{c_7 = c_4} \quad \ldots (1)$$

$$\text{Koef. } e^{-2t}: \quad -2c_2 = c_2 + c_5 - c_8 \longrightarrow \boxed{c_5 - c_8 = -3c_2} \quad \ldots (2)$$

$$\text{Koef. } e^{2t}: \quad 2c_3 = c_3 + c_6 - c_9 \longrightarrow \boxed{c_6 - c_9 = c_3} \quad \ldots (3)$$

$$y = c_4 e^{t} + c_5 e^{-2t} + c_6 e^{2t}$$

$$\boxed{\frac{dy}{dt} = c_4 e^{t} - 2c_5 e^{-2t} + 2c_6 e^{2t}}$$

$$x = c_1 e^{t} + c_2 e^{-2t} + c_3 e^{2t}$$

$$y = c_4 e^{t} + c_5 e^{-2t} + c_6 e^{2t} \quad (-)$$

$$x - y = (c_1 - c_4)e^{t} + (c_2 - c_5)e^{-2t} + (c_3 - c_6)e^{2t}$$

$$z = c_7 e^{t} + c_8 e^{-2t} + c_9 e^{2t} \quad +$$

$$\underbrace{x - y + z}_{\frac{dy}{dt}} = (c_1 - c_4 + c_7)e^{t} + (c_2 - c_5 + c_8)e^{-2t} + (c_3 - c_6 + c_9)e^{2t}$$

)

$)$...(2)

$$\boxed{C_4 e^{t} - 2C_5 e^{-2t} + 2C_6 e^{2t} = (C_1 - C_4 + C_7)e^{t} + (C_2 - C_5 + C_8)e^{-2t} + (C_3 - C_6 + C_9)e^{2t}}$$

...(3)

Koef. $e^{t}$ : $C_4 = C_1 - C_4 + C_7 \rightarrow \boxed{C_7 - 2C_4 = -C_1}$ .... (4)

Koef. $e^{-2t}$ : $-2C_5 = C_2 - C_5 + C_8 \rightarrow \boxed{C_8 + C_5 = -C_2}$ .... (5)

Koef. $e^{2t}$ ; $2C_6 = C_3 - C_6 + C_9 \rightarrow \boxed{C_9 - 3C_6 = -C_3}$ ....(6)

(4) $\rightarrow$ $C_7 - 2C_4 = -C_1$

$C_4 - 2C_4 = -C_1$

$-C_4 = -C_1$

$\boxed{C_4 = C_1}$

$C_7 = C_4$

$\boxed{C_7 = C_1}$

(2) $\rightarrow$ $C_5 - \cancel{C_8} = -3C_2$

(5) $\rightarrow$ $C_5 + \cancel{C_8} = -C_2$ +

$2C_5 = -4C_2$

$\boxed{C_5 = -2C_2}$

$\cancel{C_5} + C_8 = -C_2$

$\cancel{C_5} \quad = -2C_2$ (−)

$\boxed{C_8 = C_2}$

(3) $\rightarrow$ $C_6 - \cancel{C_9} = C_3$

(6) $\rightarrow$ $-3C_6 + \cancel{C_9} = -C_3$ +

$-2C_6 = 0$

$\boxed{C_6 = 0}$

$C_6 - C_9 = C_3$

$0 - C_9 = C_3$

$\boxed{C_9 = -C_3}$

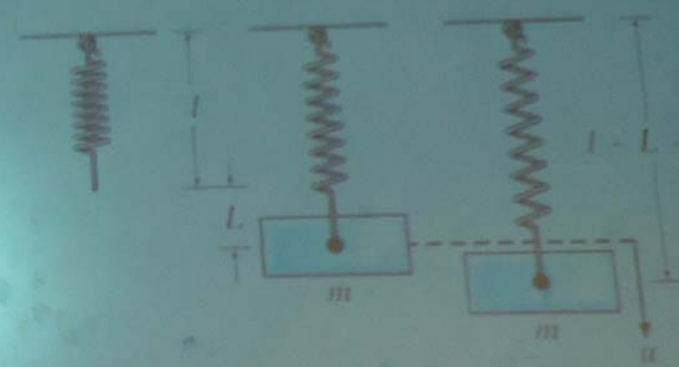
## SOLUSI UMUM PD

$$X = C_1 e^{t} + C_2 e^{-2t} + C_3 e^{2t}$$

$$Y = C_1 e^{t} - 2C_2 e^{-2t}$$

$$Z = C_1 e^{t} + C_2 e^{-2t} - C_3 e^{2t}$$

# Spring – Mass System




- Suppose a mass $m$ hangs from vertical spring of original length $l$. The mass causes an elongation $L$ of the spring.
- The force $F_G$ of gravity pulls mass down. This force has magnitude $mg$, where $g$ is acceleration gravity.


- The force $F_S$ of spring stiffness pulls mass up. For small elongations $L$, this force is proportional to $L$. That is, $F_s = kL$ (Hooke's Law).
- Since mass is in equilibrium, the forces balance each other:

$$mg = kL$$

# Spring Model

Let $u(t)$ denote the displacement of the mass from its equilibrium position at time $t$, measured downward. Let $f$ be the net force acting on mass. Newton's 2nd Law:

$$mu''(t) = f(t)$$

In determining $f$, there are four separate forces to consider:

- Weight: $w = mg$ (downward force)
- Spring force: $F_s = -k(L + u)$ (up or down force)
- Damping force: $F_d(t) = -\gamma u'(t)$ (up or down)
- External force: $F(t)$ (up or down force)

Taking into account these forces, Newton's Law becomes:

$$mu''(t) = mg + F_s(t) + F_d(t) + F(t) = mg - k\left[L + u(t)\right] - \gamma u'(t) + F(t)$$

Recalling that $mg = kL$, this equation reduces to

$$mu''(t) + \gamma u'(t) + ku(t) = F(t)$$

## HUKUM NEWTON II

$$F = m \cdot a$$

$$\boxed{f(t) = m \cdot u''(t)}$$

JARAK $\longrightarrow x = u(t)$

$$V = \frac{dx}{dt} = x' = u'(t)$$

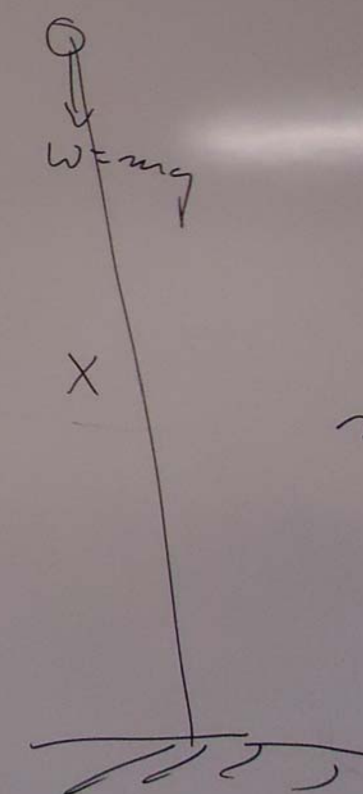
$$a = \frac{d^2x}{dt^2} = x'' = u''(t)$$

$$F = W - \text{GAYA GESEK}$$

$$m \cdot a = mg - K \cdot V^2$$

$$m \cdot \frac{d^2x}{dt^2} = m \cdot g - K \cdot \left(\frac{dx}{dt}\right)^2$$

$$m \cdot x''(t) = m \cdot g - K \cdot \left[x'(t)\right]^2$$




$mg = k($

)

t)

(t)

$IR = E$

$$I = \frac{\overset{V}{E}}{R}$$

$$u(t) = x$$

$$u'(t) = x' = \frac{dv}{dt}$$

$$\underset{m \cdot a}{F} = W + F_s(t) + F_d(t) + F(t)$$

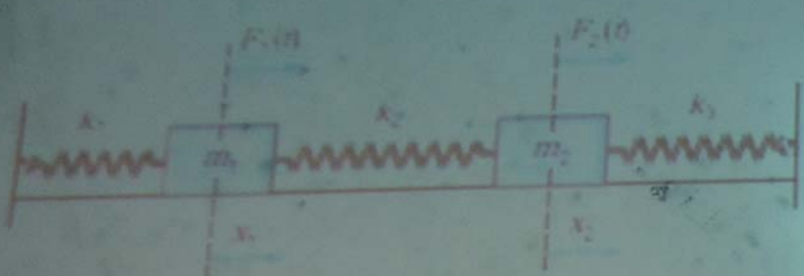
$$m \cdot u''(t) = m \cdot g + [-\underline{k(L+u)}] + \gamma \cdot u'(t) + F(t)$$

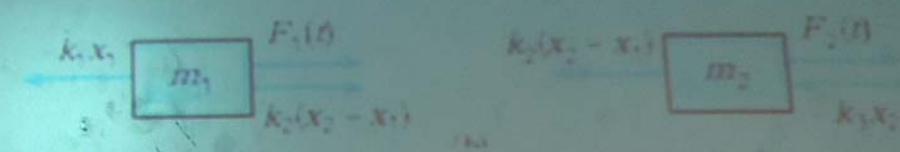
$$m \cdot u''(t) = \cancel{m \cdot g} - \cancel{kL} - \underline{k \cdot u(t)} + \underline{\gamma \cdot u'(t)} + F(t)$$

$$kL \rightarrow \boxed{m \cdot u''(t) - \gamma \cdot u'(t) + k \cdot u(t) = F(t)}$$

PD

# Spring Mass System

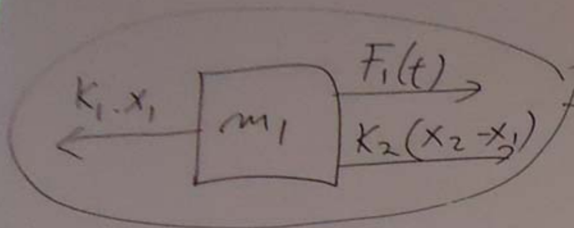




Two masses move on frictionless surface under the influence of external force $F_1(t)$ and $F_2(t)$ and they are also constrained by the three springs whose constant are $k_1$, $k_2$, $k_3$ respectively. We find the following equation for the coordinates $x_1$ and $x_2$ of the two mass.

$$m_1 \frac{d^2 x_1}{dt^2} = -(k_1 + k_2)x_1 + k_2 x_2 + F_1(t)$$

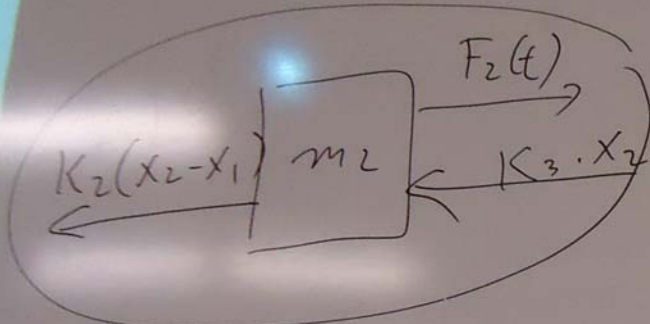
$$m_2 \frac{d^2 x_2}{dt^2} = k_2 x_1 - (k_2 + k_3)x_2 + F_2(t)$$




$F_1 = F_1(t) + K_2(x_2 - x_1) - K_1 x_1$

$m_1 . a_1 = F_1(t) + K_2 . x_2 - K_2 x_1 - K_1 x_1$

$$\boxed{m_1 . \frac{d^2 x_1}{dt^2} = -(K_1 + K_2) x_1 + K_2 . x_2 + F_1(t)}$$




$F_2 = F_2(t) - K_3 . x_2 - K_2(x_2 - x_1)$

$m_2 . a_2 = F_2(t) - K_3 . x_2 - K_2 . x_2 + K_2 . x_1$

$$\boxed{m_2 . \frac{d^2 x_2}{dt^2} = K_2 . x_1 - (K_2 + K_3) x_2 + F_2(t)}$$

# Example 3 (1 of 2)

Solve $(D-3)x+2(D+2)y=2\sin t$ .....(1)

$2(D+1)x+(D-1)y=\cos t$ .....(2)

First we want to find the diff