SISTEM PERSAMAAN DIFERENSIAL
(PD. SIMULTAN)

$$\boxed{\begin{aligned}\frac{dx}{dt} &= 4x + y \\ \frac{dy}{dt} &= y - 2x\end{aligned}} \qquad y = \frac{dx}{dt} - 4x$$

, DENGAN SYARAT AWAL $x(0) = 3$, DAN $y(0) = -5$

$$\Longrightarrow \begin{aligned} x(t) &= ? \\ y(t) &= ? \end{aligned}$$

(I) $\dfrac{dx}{dt} = 4x + y$

$$\longrightarrow \frac{d^2x}{dt^2} = 4\frac{dx}{dt} + \frac{dy}{dt}$$

$$\frac{d^2x}{dt^2} = 4\frac{dx}{dt} + y - 2x$$

$$\frac{d^2x}{dt^2} = 4\frac{dx}{dt} + \frac{dx}{dt} - 4x - 2x$$

$$\frac{d^2x}{dt^2} = 5\frac{dx}{dt} - 6x$$

$$\frac{d^2x}{dt^2} - 5\frac{dx}{dt} + 6x = 0$$

$$D^2 x - 5Dx + 6x = 0$$

$$(D^2 - 5D + 6) \cdot x = 0$$

PERS. EIGEN

$$\alpha^2 - 5\alpha + 6 = 0$$

$$(\alpha - 3)(\alpha - 2) = 0$$

$$\alpha_1 = 3, \ \alpha_2 = 2$$

$$x = C_1 e^{\alpha_1 t} + C_2 e^{\alpha_2 t}$$

$$\boxed{x = C_1 e^{3t} + C_2 e^{2t}}$$

$$x = c_1 e^{3t} + c_2 e^{2t}$$

$$\boxed{\begin{aligned} x &= x(t) \\ y &= y(t) \end{aligned}}$$

$$\frac{dx}{dt} = 3c_1 e^{3t} + 2c_2 e^{2t}$$

$$4x = 4c_1 e^{3t} + 4c_2 e^{2t} \quad -$$

$$\boxed{y = -c_1 e^{3t} - 2c_2 e^{2t}}$$

SOLUSI UMUM PD

$$\boxed{\begin{aligned} x &= c_1 e^{3t} + c_2 e^{2t} \\ y &= -c_1 e^{3t} - 2c_2 e^{2t} \end{aligned}}$$

$$\boxed{x(t) = c_1 e^{3t} + c_2 e^{2t}}$$

$$x(0) = c_1 \underbrace{e^{3(0)}}_{1} + c_2 \underbrace{e^{2(0)}}_{1} = 3$$

$$\boxed{c_1 + c_2 = 3}$$

$$\boxed{y(t) = -c_1 e^{3t} - 2c_2 e^{2t}}$$

$$y(0) = -c_1 \underbrace{e^{3(0)}}_{1} - 2c_2 \underbrace{e^{2(0)}}_{1} = -5$$

$$\boxed{-c_1 - 2c_2 = -5}$$

$$\cancel{c_1} + c_2 = 3$$
$$\cancel{-c_1} - 2c_2 = -5 \quad +$$
$$\overline{\quad -c_2 = -2 \quad}$$
$$\boxed{c_2 = 2}$$

$$c_1 + \cancel{c_2} = 3$$
$$\cancel{c_2} = 2 \quad -$$
$$\overline{\quad \boxed{c_1 = 1} \quad}$$

Solusi PD:

$$x(t) = e^{3t} + 2e^{2t}$$
$$y(t) = -e^{3t} - 4e^{2t}$$

SISTEM PERSAMAAN DIFERENSIAL

(PD. SIMULTAN)

$$\frac{dx}{dt} = 4x + y$$

$$\frac{dy}{dt} = y - 2x$$

, DENGAN SYARAT AWAL

$x(0) = 3$, DAN $y(0) = -5$

$$\Rightarrow x(t) = ?\quad y(t) = ?$$

$$2x = y - \frac{dy}{dt}$$

(II) $\frac{dy}{dt} = y - 2x$

$$\rightarrow \frac{d^2y}{dt^2} = \frac{dy}{dt} - 2\frac{dx}{dt}$$

$$\frac{d^2y}{dt^2} = \frac{dy}{dt} - 2(4x + y)$$

$$\frac{d^2y}{dt^2} = \frac{dy}{dt} - \underline{8x} - 2y$$

$$= \frac{dy}{dt} - 4\underline{(2x)} - 2y$$

$$\frac{d^2y}{dt^2} = \frac{dy}{dt} - 4\left(y - \frac{dy}{dt}\right) - 2y$$

$$\frac{d^2y}{dt^2} = \frac{dy}{dt} - 4y + 4\frac{dy}{dt} - 2y$$

$$\frac{d^2y}{dt^2} = 5\frac{dy}{dt} - 6y$$

$$\boxed{\frac{d^2y}{dt^2} - 5\frac{dy}{dt} + 6y = 0}$$

$$D^2 y - 5D y + 6 y = 0$$

$$(D^2 - 5D + 6)\, y = 0$$

PERS. EIGEN

$$\alpha^2 - 5\alpha + 6 = 0$$

$$(\alpha - 3)(\alpha - 2) = 0$$

$$\alpha_1 = 3,\ \alpha_2 = 2$$

$$Y(t) = c_1 e^{\alpha_1 t} + c_2 e^{\alpha_2 t}$$

$$Y(t) = c_1 e^{3t} + c_2 e^{2t}$$

$$\frac{dy}{dt} = 3 c_1 e^{3t} + 2 c_2 e^{2t} \quad -$$

$$2X = -2 c_1 e^{3t} - c_2 e^{2t} \quad \times \frac{1}{2}$$

$$X(t) = -c_1 e^{3t} - \frac{1}{2} c_2 e^{2t}$$

SOLUSI UMUM PD

$$x(t) = -c_1 e^{3t} - \frac{1}{2} c_2 e^{2t}$$

$$y(t) = c_1 e^{3t} + c_2 e^{2t}$$

$$x(t) = -c_1 e^{3t} - \frac{1}{2} c_2 e^{2t}$$

$$x(0) = -c_1 \underbrace{e^{3(0)}}_{1} - \frac{1}{2} c_2 \underbrace{e^{2(0)}}_{1} = 3$$

$$\boxed{-c_1 - \frac{1}{2} c_2 = 3}$$

$$y(t) = c_1 e^{3t} + c_2 e^{2t}$$

$$y(0) = c_1 \underbrace{e^{3(0)}}_{1} + c_2 \underbrace{e^{2(0)}}_{1} = -5$$

$$\boxed{c_1 + c_2 = -5}$$

$$-\cancel{c_1} - \frac{1}{2}c_2 = 3$$

$$\cancel{c_1} + c_2 = -5 \quad +$$

$$\frac{1}{2}c_2 = -2 \quad \times 2$$

$$\boxed{c_2 = -4}$$

$$c_1 + \cancel{c_2} = -5$$

$$\cancel{c_2} = -4 \quad -$$

$$\boxed{c_1 = -1}$$

Solusi PD:

$$x(t) = e^{3t} + 2e^{2t}$$

$$y(t) = -e^{3t} - 4e^{2t}$$

SISTEM PERSAMAAN DIFERENSIAL

(PD. SIMULTAN)

$$\frac{dx}{dt} = 4x + y \longrightarrow Dx = 4x + y$$

$$\frac{dy}{dt} = y - 2x \longrightarrow Dy = y - 2x$$

DENGAN SYARAT AWAL $x(0) = 3$, DAN $y(0) = -5$

$$\longrightarrow \quad Dx - 4x - y = 0 \longrightarrow (D-4)x - y = 0$$

$$2x + Dy - y = 0 \longrightarrow 2x + (D-1)y = 0$$

$$\begin{bmatrix} (D-4) & -1 \\ 2 & (D-1) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$A \cdot \vec{X} = B$$

$$|A| = \begin{vmatrix} (D-4) & -1 \\ 2 & (D-1) \end{vmatrix} = (D-4)(D-1) - (-1)(2)$$

$$= D^2 - 5D + 4 + 2$$

$$= D^2 - 5D + 6$$

ATURAN CRAMER

$$X = \frac{|A_1|}{|A|} = \frac{\begin{vmatrix} \boxed{\begin{matrix} 0 \\ 0 \end{matrix}} & \begin{matrix} -1 \\ (D-1) \end{matrix} \end{vmatrix}}{\begin{vmatrix} (D-4) & -1 \\ 2 & (D-1) \end{vmatrix}} = \frac{0}{D^2 - 5D + 6}$$

$$(D^2 - 5D + 6) \cdot X = 0$$

PERS. EIGEN

$$\alpha^2 - 5\alpha + 6 = 0$$

$$(\alpha - 3)(\alpha - 2) = 0$$

$$\alpha_1 = 3, \quad \alpha_2 = 2$$

$$X(t) = C_1 e^{\alpha_1 t} + C_2 e^{\alpha_2 t}$$

$$\boxed{X(t) = C_1 e^{3t} + C_2 e^{2t}}$$

$$\frac{dx}{dt} = 3C_1 e^{3t} + 2C_2 e^{2t}$$

$$y = \frac{|A_2|}{|A|} = \frac{\begin{vmatrix} (D-4) & \boxed{0} \\ 2 & \boxed{0} \end{vmatrix}}{\begin{vmatrix} (D-4) & -1 \\ 2 & (D-1) \end{vmatrix}} = \frac{0}{D^2-5D+6}$$

$$(D^2-5D+6) \cdot y = 0$$

PERS. EIGEN

$$\alpha^2 - 5\alpha + 6 = 0$$

$$(\alpha-3)(\alpha-2) = 0$$

$$\alpha_3 = 3 \ , \ \alpha_4 = 2$$

$$y(t) = c_3 e^{\alpha_3 t} + c_4 e^{\alpha_4 t}$$

$$y(t) = c_3 e^{3t} + c_4 e^{2t}$$

SOLUSI UMUM PD (SEMENTARA)

$$x(t) = c_1 e^{3t} + c_2 e^{2t}$$

$$y(t) = c_3 e^{3t} + c_4 e^{2t}$$

$$\boxed{\frac{dx}{dt} = 4x + y}$$

$$3c_1 e^{3t} + 2c_2 e^{2t} = 4(c_1 e^{3t} + c_2 e^{2t}) + c_3 e^{3t} + c_4 e^{2t}$$

$$-c_1 e^{3t} - 2c_2 \boxed{e^{2t}} = c_3 e^{3t} + c_4 \boxed{e^{3t}}$$

Koef. e^{3t} : $\boxed{-c_1 = c_3}$

Koef. e^{2t} : $\boxed{-2c_2 = c_4}$

Solusi umum PD

$$x(t) = c_1 e^{3t} + c_2 e^{2t}$$
$$y(t) = -c_1 e^{3t} - 2c_2 e^{2t}$$

$$x(t) = c_1 e^{3t} + c_2 e^{2t}$$
$$x(0) = c_1 \underbrace{e^{3(0)}}_{1} + c_2 \underbrace{e^{2(0)}}_{1} = 3$$
$$\boxed{c_1 + c_2 = 3}$$

$$y(t) = -c_1 e^{3t} - 2c_2 e^{2t}$$
$$y(0) = -c_1 \underbrace{e^{3(0)}}_{1} - 2c_2 \underbrace{e^{2(0)}}_{1} = -5$$

$$\boxed{-\cancel{c_1} - 2c_2 = -5}$$
$$\cancel{c_1} + c_2 = 3 \quad +$$
$$-c_2 = -2$$
$$\boxed{c_2 = 2}$$

$$c_1 + c_2 = 3$$
$$c_2 = 2 \quad -$$
$$\boxed{c_1 = 1}$$

Solusi PD:

$$x(t) = e^{3t} + 2e^{2t}$$
$$y(t) = -e^{3t} - 4e^{2t}$$

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(3) $$\boxed{\begin{aligned} \frac{dx}{dt} &= x - y + t^2 \\ \frac{dy}{dt} &= x + y - t \end{aligned}}$$

$$Dx = x - y + t^2$$
$$Dy = x + y - t$$

$$Dx - x + y = t^2$$
$$x + y - Dy = t$$

$$(D-1)x + y = t^2$$
$$x + (1-D)y = t$$

$$\begin{bmatrix} (D-1) & 1 \\ 1 & (1-D) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} t^2 \\ t \end{bmatrix}$$

$$A \cdot \vec{X} = B$$

$$|A| = \begin{vmatrix} (D-1) & 1 \\ 1 & (1-D) \end{vmatrix} = (D-1)(1-D) - (1)(1)$$
$$= D - D^2 - 1 + D - 1$$
$$= -D^2 + 2D - 2$$
$$= -(D^2 - 2D + 2)$$

ATURAN CRAMER

$$X = \frac{|A_1|}{|A|} = \frac{\begin{vmatrix} \boxed{\begin{matrix} t^2 \\ t \end{matrix}} & \begin{matrix} 1 \\ (1-D) \end{matrix} \end{vmatrix}}{\begin{vmatrix} (D-1) & 1 \\ 1 & (1-D) \end{vmatrix}} = \frac{(1-D)(t^2) - t}{-(D^2-2D+2)} = \frac{t^2 - D(t^2) - \ldots}{-(D^2-2D+2)}$$

$$-(D^2-2D+2)\, X = t^2 - 2t - t$$
$$-(D^2-2D+2)\, X = t^2 - 3t \quad \times(-1)$$
$$(D^2-2D+2)\, X = -t^2 + 3t$$

(I) $(D^2 - 2D + 2) \cdot X = 0$

PERS. EIGEN

$$\alpha^2 - 2\alpha + 2 = 0$$

$a = 1,\ b = -2,\ c = 2$

$\alpha_{1,2} = \frac{-b}{\ldots}$

$\alpha_{1,2} =$

α

(1)

$$\alpha_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\alpha_{1,2} = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)}$$

$$\alpha_{1,2} = \frac{2 \pm \sqrt{4-8}}{2}$$

$$= \frac{t^2 - D(t^2) - t}{-(D^2 - 2D + 2)}$$

$$\alpha_{1,2} = \frac{2 \pm \sqrt{-4}}{2}$$

$$\alpha_{1,2} = \frac{2 \pm \sqrt{4}\sqrt{-1}}{2}$$

$$\alpha_{1,2} = \frac{2 \pm 2i}{2}$$

$$\alpha_{1,2} = 1 \pm i$$

$a = 1$ $\quad$ $b = 1$

$$x_h = e^{at}(c_1 \cos bt + c_2 \sin bt)$$

$$x_h = e^{t}(c_1 \cos t + c_2 \sin t)$$

II $(D^2-2D+2)X = -t^2+3t$

$$X_K = \frac{1}{2-2D+D^2}(-t^2+3t)$$

$$X_K = \left(\frac{1}{2}+\frac{1}{2}D+\frac{1}{4}D^2\right)(-t^2+3t)$$

$$X_K = \frac{1}{2}(-t^2+3t)+\frac{1}{2}D(-t^2+3t)+\frac{1}{4}D^2(-t^2+3t)$$

$$X_K = -\frac{1}{2}t^2+\frac{3}{2}t+\frac{1}{2}(-2t+3)+\frac{1}{4}(-2)$$

$$X_K = -\frac{1}{2}t^2+\frac{3}{2}t-t+\frac{3}{2}-\frac{1}{2}$$

$$X_K = -\frac{1}{2}t^2+\frac{1}{2}t+1$$

$$X(t) = X_h + X_K$$

$$\boxed{X(t) = e^{t}(c_1\cos t + c_2\sin t) - \frac{1}{2}t^2+\frac{1}{2}t+1}$$

$D(-t^2+3t) = -2t+3$

$D^2(-t^2+3t) = -2$

$$\begin{array}{r|l} & \frac{1}{2}+\frac{1}{2}D+\frac{1}{4}D^2 \\ \hline 2-2D+D^2 & 1 \\ & 1-D+\frac{1}{2}D^2 \\ \hline & D-\frac{1}{2}D^2 \\ & D-D^2+\frac{1}{2}D^3 \\ \hline & \frac{1}{2}D^2-\frac{1}{2}D^3 \end{array}$$

$$y = \frac{|A_2|}{|A|} = \frac{\begin{vmatrix} (D-1) & t^2 \\ 1 & t \end{vmatrix}}{\begin{vmatrix} (D-1) & 1 \\ 1 & (1-D) \end{vmatrix}} = \frac{(D-1)(t) - t^2}{-(D^2-2D+2)} = \frac{D(t) - t - t^2}{-(D^2-2D+2)}$$

$$-(D^2-2D+2)\cdot y = 1 - t - t^2 \quad \times -1$$

$$\boxed{(D^2-2D+2)\cdot y = t^2+t-1}$$

I) $(D^2-2D+2)\cdot y = 0$

PERS. EIGEN

$\alpha^2 - 2\alpha + 2 = 0$

$\boxed{\alpha_{3,4} = 1 \pm i}$ (ANALOG)

$a=1 \qquad b=1$

$y_h = e^{at}(C_3 \cos bt + C_4 \sin bt)$

$$\boxed{y_h = e^{t}(C_3 \cos t + C_4 \sin t)}$$

II $(D^2-2D+2)y = t^2+t-1$

$$y_k = \frac{1}{2-2D+D^2}(t^2+t-1)$$

$$y_k = \left(\frac{1}{2}+\frac{1}{2}D+\frac{1}{4}D^2\right)(t^2+t-1)$$

$$y_k = \frac{1}{2}(t^2+t-1)+\frac{1}{2}D(t^2+t-1)+\frac{1}{4}D^2(t^2+t-1)$$

$$y_k = \frac{1}{2}t^2+\frac{1}{2}t-\frac{1}{2}+\frac{1}{2}(2t+1)+\frac{1}{4}(2)$$

$$y_k = \frac{1}{2}t^2+\frac{1}{2}t-\cancel{\frac{1}{2}}+t+\cancel{\frac{1}{2}}+\frac{1}{2}$$

$$y_k = \frac{1}{2}t^2+1\frac{1}{2}t+\frac{1}{2}$$

$$y(t) = y_h + y_k$$

$$\boxed{y(t) = e^t(c_3\cos t + c_4\sin t)+\frac{1}{2}t^2+1\frac{1}{2}t+\frac{1}{2}}$$

$D(t^2+t$

$D^2(t^2+$

$2-2D+D^2$

$$D(t^2+t-1) = 2t+1$$

$$D^2(t^2+t-1) = 2$$

$$\frac{1}{2} + \frac{1}{2}D + \frac{1}{4}D^2$$

$$2 - 2D + D^2 \,\big)\, 1$$

$$1 - D^2 + \frac{1}{2}D^2 \quad -$$

$$D^2 - \frac{1}{2}D^2$$

$$D^2 - D^2 + \frac{1}{2}D^3 \quad -$$

$$\frac{1}{2}D^2 - \frac{1}{2}D^3$$

SOLUSI UMUM PD (SEMENTARA)

$$X(t) = e^t(C_1 \cos t + C_2 \sin t) - \frac{1}{2}t^2 + \frac{1}{2}t + 1$$

$$Y(t) = e^t(C_3 \cos t + C_4 \sin t) + \frac{1}{2}t^2 + 1\frac{1}{2}t + \frac{1}{2}$$

$$\rightarrow \frac{dy}{dt} = (e^t)'(C_3 \cos t + C_4 \sin t) + e^t(C_3 \cos t + C_4 \sin t)' + \frac{1}{\cancel{2}}(\cancel{2}t) + 1\frac{1}{2}(1) + 0$$

$$\frac{dy}{dt} = e^t(C_3 \cos t + C_4 \sin t) + e^t(-C_3 \sin t + C_4 \cos t) + t + 1\frac{1}{2}.$$

$$\frac{dy}{dt} = e^t\left[(C_3 + C_4)\cos t + (C_4 - C_3)\sin t\right] + t + 1\frac{1}{2}$$

$$X = e^t(C_1 \cos t + C_2 \sin t) - \frac{1}{2}t^2 + \frac{1}{2}t + 1$$

$$Y = e^t(C_3 \cos t + C_4 \sin t) + \frac{1}{2}t^2 + 1\frac{1}{2}t + \frac{1}{2}$$

$$X + Y = e^t\left[(C_1 + C_3)\cos t + (C_2 + C_4)\sin t\right] + 2t + 1\frac{1}{2}$$

$$\boxed{\frac{dy}{dt} = x + y - t}$$

$$e^t\left[(c_3+c_4)\underline{\cos t} + (c_4-c_3)\underline{\underline{\sin t}}\right] + \underline{t + 1\tfrac{1}{2}} = e^t\left[(c_1+c_3)\cos t + (c_2+c_4)\sin t\right] + \underline{2t} + 1\tfrac{1}{2} - \underline{t}$$

$$= e^t\left[(c_1+c_3)\underline{\cos t} + (c_2+c_4)\underline{\underline{\sin t}}\right] + \underline{t + 1\tfrac{1}{2}}$$

Koef. $\cos t$: $\cancel{c_3} + c_4 = c_1 + \cancel{c_3} \longrightarrow \boxed{c_4 = c_1}$

Koef. $\sin t$: $\cancel{c_4} - c_3 = c_2 + \cancel{c_4} \longrightarrow -c_3 = c_2$

$$\boxed{c_3 = -c_2}$$

Solusi umum PD

$$x(t) = e^t(c_1\cos t + c_2\sin t) - \frac{1}{2}t^2 + \frac{1}{2}t + 1$$

$$y(t) = e^t(-c_2\cos t + c_1\sin t) + \frac{1}{2}t^2 + 1\frac{1}{2}t + \frac{1}{2}$$

$... ka)$

$...^2 + \frac{1}{2}t + 1$

$... + 1\frac{1}{2}t + \frac{1}{2}$

$... t + c_4 \sin t)'$

$... + c_4 \cos t)$

$... + 1\frac{1}{2}$

$$\boxed{\frac{dy}{dt} = x + y - t}$$

$$e^t\left[(c_3+c_4)\cos t + (c_4-c_3)\sin t\right] + t + 1\tfrac{1}{2} = e^t\left[(c_1+c_3)\cos t + (c_2+c_4)\sin t\right] + 2t + 1\tfrac{1}{2} - t$$

$$= e^t\left[(c_1+c_3)\cos t + (c_2+c_4)\sin t\right] + t + 1\tfrac{1}{2}$$

Koef. cos t : $\cancel{c_3} + c_4 = c_1 + \cancel{c_3} \rightarrow \boxed{c_4 = c_1}$

Koef. sin t : $\cancel{c_4} - c_3 = c_2 + \cancel{c_4} \rightarrow -c_3 = c_2$

$$\boxed{c_3 = -c_2}$$

Solusi umum PD

$$x(t) = e^t\left(c_1\cos t + c_2\sin t\right) - \frac{1}{2}t^2 + \frac{1}{2}t + 1$$

$$y(t) = e^t\left(-c_2\cos t + c_1\sin t\right) + \frac{1}{2}t^2 + 1\tfrac{1}{2}t + \frac{1}{2}$$

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(4)
$$4\frac{dx}{dt}+5x-3y=4e^{t}$$
$$4\frac{dy}{dt}-5y+3x=4e^{t}$$

Solusi:

$$x(t)=c_1e^{-t}+c_2e^{t}+\frac{1}{4}te^{t}$$

$$y(t)=\frac{1}{3}c_1e^{-t}+(3c_2-1)e^{t}+\frac{3}{4}te^{t}$$