PD. TINGKAT n DENGAN FUNGSI-FUNGSI KHUSUS DARI X

(PD. LINEAR EULER - CAUCHY)

$$a_0 \cdot x^n \frac{d^n y}{dx^n} + a_1 \cdot x^{n-1} \cdot \frac{d^{n-1} y}{dx^{n-1}} + \dots + a_{n-1} x \frac{dy}{dx} + a_n y = 0$$

CONTOH :

$$\boxed{2 x^3 \frac{d^3 y}{dx^3} + 5 x^2 \frac{d^2 y}{dx^2} + 3 x \frac{dy}{dx} + 6y = 0}$$

MISALKAN : $\boxed{x = e^u}$

$$\ln x = \ln e^u$$

$$\ln x = u$$

$$\rightarrow \boxed{u = \ln x}$$

$$\boxed{\frac{du}{dx} = \frac{1}{x}}$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{1}{x} \quad \cdot (x)$$

$$\boxed{x \cdot \frac{dy}{dx} = \frac{dy}{du}}$$

diferensialkan thd. x

$$(x)' \frac{dy}{dx} + x \cdot \left(\frac{dy}{dx}\right)' = \left(\frac{dy}{du}\right)' \qquad u'$$

$$1 \cdot \frac{dy}{dx} + x \cdot \frac{d^2y}{dx^2} = \frac{d^2y}{du^2} \cdot \frac{du}{dx}$$

$$\frac{dy}{dx} + x \cdot \frac{d^2y}{dx^2} = \frac{d^2y}{du^2} \cdot \frac{1}{x} \quad \cdot (x)$$

$$x \cdot \frac{dy}{dx} + x^2 \cdot \frac{d^2y}{dx^2} = \frac{d^2y}{du^2}$$

$$\frac{dy}{du} + x^2 \cdot \frac{d^2y}{dx^2} = \frac{d^2y}{du^2}$$

$$\boxed{x^2 \cdot \frac{d^2y}{dx^2} = \frac{d^2y}{du^2} - \frac{dy}{du}}$$

$$(x^2)' \frac{d^2y}{dx^2} + x^2 \left(\frac{d^2y}{dx^2}\right)' = \left(\frac{d^2y}{du^2}\right)' - \left(\frac{dy}{du}\right)'$$

$$2x \cdot \frac{d^2y}{dx^2} + x^2 \cdot \frac{d^3y}{dx^3} = \frac{d^3y}{du^3} \cdot \frac{du}{dx} - \frac{d^2y}{du^2} \cdot \frac{du}{dx}$$

$$2x \cdot \frac{d^2y}{dx^2} + x^2 \cdot \frac{d^3y}{dx^3} = \frac{d^3y}{du^3} \cdot \frac{1}{x} - \frac{d^2y}{du^2} \cdot \frac{1}{x}$$

$$2x^2 \cdot \frac{d^2y}{dx^2} + x^3 \cdot \frac{d^3y}{dx^3} = \frac{d^3y}{du^3} - \frac{d^2y}{du^2}$$

$$x^3 \cdot \frac{d^3y}{dx^3} = \frac{d^3y}{du^3} - \frac{d^2y}{du^2} - 2\, x^2 \cdot \frac{d^2y}{dx^2}$$

$$= \frac{d^3y}{du^3} - \frac{d^2y}{du^2} - 2\left(\frac{d^2y}{du^2} - \frac{dy}{du}\right)$$

$$\boxed{x^3 \cdot \frac{d^3y}{dx^3} = \frac{d^3y}{du^3} - 3\frac{d^2y}{du^2} + 2\frac{dy}{du}}$$

$$x \cdot \frac{dy}{dx} = \frac{dy}{du} = Dy = f(u)$$

$$x^2 \cdot \frac{d^2y}{dx^2} = \frac{d^2y}{du^2} - \frac{dy}{du} = D^2y - Dy = (D^2 - D)y = \underline{D(D-1) \cdot y}$$

$$x^3 \cdot \frac{d^3y}{dx^3} = \frac{d^3y}{du^3} - 3\frac{d^2y}{du^2} + 2\frac{dy}{du}$$

$$= D^3y - 3D^2y + 2Dy$$

$$= D(D^2 - 3D + 2)y$$

$$= \underline{D(D-1)(D-2)y}$$

$$x^4 \cdot \frac{d^4y}{dx^4} = D(D-1)(D-2)(D-3)y$$

Hal 147

(1) $x \cdot \frac{d^2y}{dx^2} - 2\frac{dy}{dx} = x^4$ ... (x)

$x^2 \frac{d^2y}{dx^2} - 2\left(x\frac{dy}{dx}\right) = x^5$

MISALKAN : $x = e^u$ → $x^5 = (e^u)^5 = e^{5u}$

$\ln x = \ln e^u$

$\ln x = u$

$D(D-1)y - 2Dy = e^{5u}$

$(D^2 - D - 2D)y = e^{5u}$

$(D^2 - 3D)y = e^{5u}$

I $(D^2 - 3D)y = 0$

PERS. EIGEN

$\alpha^2 - 3\alpha = 0$

$\alpha(\alpha - 3) = 0$

$\alpha_1 = 0$, $\alpha_2 = 3$

$Y_h = c_1 e^{\alpha_1 u} + c_2 e^{\alpha_2 u}$

$Y_h = c_1 e^{0u} + c_2 e^{3u}$

$Y_h = c_1 + c_2 (e^u)^3$

$\boxed{Y_h = c_1 + c_2 x^3}$

II $(D^2 - 3D)y = e^{5u}$ $\quad k = 5$

$\boxed{D = k = 5}$

$$Y_k = \frac{e^{5u}}{D^2 - 3D} = \frac{e^{5u}}{5^2 - 3(5)} = \frac{e^{5u}}{25 - 15} = \frac{1}{10}(e^u)^5$$

$$= \frac{1}{10} x^5$$

SOL. UM. PD

$Y = Y_h + Y_k$

$Y = c_1 + c_2 \cdot x^3 + \frac{1}{10} x^5$

PD TINGKAT-1 DERAJAT n

$$\left(\frac{dy}{dx}\right)^4 - \frac{dy}{dx} = 0$$

MISALKAN : $\boxed{\frac{dy}{dx} = p}$

$$p^4 - p = 0$$

$$p(p^3 - 1) = 0$$

$\boxed{p = 0}$

$\frac{dy}{dx} = 0$

$dy = 0\,dx$

$\int dy = \int 0\,dx$

$y = c$

$\boxed{y - c = 0}$

$p^3 - 1 = 0$

$p^3 = 1$

$p = \sqrt[3]{1}$

$\boxed{p = 1}$

$\frac{dy}{dx} = 1$

$dy = dx$

$\int dy = \int dx$

$y = x + c$

$\boxed{y - x - c = 0}$

SOL. UM. PD

$$(y-c)\cdot(y-x-c) = 0$$

② $x^2\left(\frac{dy}{dx}\right)^2 + 3xy\left(\frac{dy}{dx}\right) + 2y^2 = 0$.

$2xy + xy = 3xy$

$(2xy) \times (xy) = 2x^2y^2$

MISALKAN : $\boxed{\frac{dy}{dx} = p}$

$x^2 \cdot p^2 + 3xy \cdot p + 2y^2 = 0$

$$\frac{(x^2p + 2xy)(x^2p + xy)}{x^2} = 0$$

$$\frac{x(xp+2y)}{x} \cdot \frac{x(xp+y)}{x} = 0$$

$(xp+2y)(xp+y) = 0$

$xp + 2y = 0$	$xp + y = 0$
$xp = -2y$	$xp = -y$
$p = -\frac{2y}{x}$	$p = -\frac{y}{x}$

$$\frac{dy}{dx} = -\frac{2y}{x}$$

$$\frac{dy}{y} = -\frac{2dx}{x}$$

$$\int \frac{dy}{y} = -2\int \frac{dx}{x}$$

$$\ln y = -2\ln x + c$$

$$\ln y + 2\ln x = \ln c$$

$$\ln y + \ln x^2 = \ln c$$

$$\ln x^2 y = \ln c$$

$$x^2 y = c$$

$$\boxed{x^2 y - c = 0}$$

$$\frac{dy}{dx} = -\frac{y}{x}$$

$$\frac{dy}{y} = -\frac{dx}{x}$$

$$\int \frac{dy}{y} = -\int \frac{dx}{x}$$

$$\ln y = -\ln x + c$$

$$\ln y + \ln x = \ln c$$

$$\ln xy = \ln c$$

$$xy = c$$

$$\boxed{xy - c = 0}$$

Sol. um. PD:

$$(x^2y - c)(xy - c) = 0$$

PD. CLAIRAUT

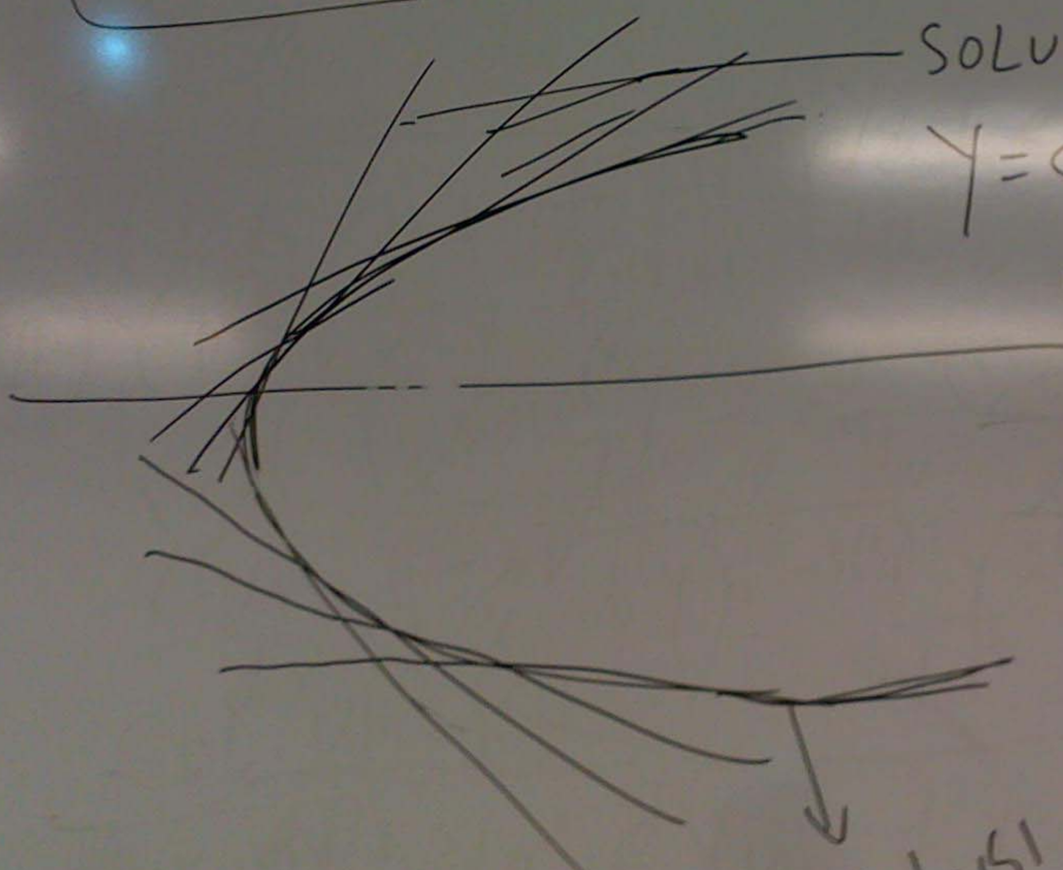
$$y = x \cdot \frac{dy}{dx} + f\left(\frac{dy}{dx}\right), \quad \boxed{\frac{dy}{dx} = p}$$

$$\boxed{y = xp + f(p)}$$

SOLUSI UMUM :

$$y = cx + f(c)$$

$$(p = c)$$



SOLUSI SINGULAR

CONTOH :

$$\boxed{y = xp + \frac{1}{p}} \quad \frac{1}{p} = p^{-1}, \qquad \boxed{\frac{dy}{dx} = p}$$

DIDIFERENSIAL TERHADAP X

$$\frac{dy}{dx} = \underbrace{(x)'}_{1} p + x(p)' + (-1)p^{-2} \cdot p'$$

$$\cancel{p} = \cancel{p} + x \cdot \frac{dp}{dx} - \frac{1}{p^2} \cdot \frac{dp}{dx}$$

$$0 = \frac{dp}{dx}\left(x - \frac{1}{p^2}\right)$$

$$\frac{dp}{dx} = 0$$

$$dp = 0 \cdot dx$$

$$\int dp = \int 0 \cdot dx$$

$$\boxed{p = c}$$

SOLUSI UMUM :

$$y = xp + \frac{1}{p}$$

$$\boxed{y = cx + \frac{1}{c}}$$

$$x - \frac{1}{p^2} = 0$$

$$x = \frac{1}{p^2}$$

$$\boxed{p^2 = \frac{1}{x}}$$

$$y = xp + \frac{1}{p}$$

$$y^2 = \left(xp + \frac{1}{p}\right)^2$$

$$y^2 = x^2p^2 + 2x + \frac{1}{p^2}$$

$$y^2 = x^2\left(\frac{1}{x}\right) + 2x + x$$

$$y^2 = x + 2x + x$$

$$\boxed{y^2 = 4x}$$

SOLUSI SINGULAR

TENTUKAN SOLUSI UMUM PD

HAL 107 PD. LINEAR EULER-CAUCHY

5) $x \cdot \frac{d^2y}{dx^2} - \frac{dy}{dx} = x^2$

HAL 110 (PD TINGKAT-1 DERAJAT-2)

2) $(xp + y)^2 = 1$, $\frac{dy}{dx} = p$

TENTUKAN SOLUSI UMUM & SOLUSI SINGULAR

2) $(y - xp)^2 = 1 + p^2$ (PD. CLAIRAUT)

9) $\boxed{x\frac{d^2y}{dx^2} - \frac{dy}{dx} = x}$ $\cdot x$

$x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} = x^2$

substitution

$\boxed{\begin{array}{l} x = e^u \\ \ln x = \ln e^u \\ \ln x = u \end{array}} \rightarrow x^2 = (e^u)^2 = e^{2u}$

$D(D-1)y - Dy = (e^u)^2$

$(D^2 - D - D)y = e^{2u}$

$\boxed{(D^2 - 2D)y = e^{2u}}$

(I)

(II)

I) $(D^2-2D)y=0$

Pasos

$m^2-2m=0$

$m(m-2)=0$

$m_1=0 \quad m_2=2$

$y_h = C_1 e^{0u} + C_2 e^{2u}$

$y_h = C_1 e^{0} + C_2 e^{2u}$

$y_h = C_1 (e^{0}) + C_2 (e^{2u})$

$\boxed{y_h = C_1 + C_2 x^2}$

$e^{2u} = (e^{u})^2 = x^2$

II) $(D^2-2D)y = e^{3u} \Rightarrow \overset{k=3}{D=k=3}$

$$y_k = \frac{e^{3u}}{D^2-2D} = \frac{e^{3u}}{(3^2-2(3))} = \frac{e^{3u}}{9-6} = \frac{e^{3u}}{3} = \frac{1}{3}e^{3u} = \frac{1}{3}x^3$$

Solum PD

$y = y_h + y_k$

$$\boxed{y = C_1 + C_2 x^3 + \frac{1}{3}x^3}$$

2) $(xp+y)^2 = 1$

$(xp+y)^2 - 1^2 = 0$

$(xp+y+1)(xp+y-1) = 0$

$xp+y+1=0$	$xp+y-1=0$
$xp = -(y+1)$	$xp = -(y-1)$
$p = -\dfrac{(y+1)}{x}$	$p = -\dfrac{(y-1)}{x}$
$\dfrac{dy}{dx} = -\dfrac{(y+1)}{x}$	$\dfrac{dy}{dx} = -\dfrac{(y-1)}{x}$
$\dfrac{dy}{y+1} = -\dfrac{dx}{x}$	$\dfrac{dy}{(y-1)} = -\dfrac{dx}{x}$
$\dfrac{d(y+1)}{(y+1)} + \dfrac{dx}{x} = 0$	$\dfrac{d(y-1)}{(y-1)} + \dfrac{dx}{x} = 0$

$$\int \frac{d(y+1)}{(y+1)} + \int \frac{dx}{x} = \int 0$$

$$\ln(y+1) + \ln x = c$$

$$\ln x(y+1) = \ln c$$

$$xy + x = c$$

$$\boxed{xy + x - c = 0}$$

$$\int \frac{d(y-1)}{(y-1)} + \int \frac{dx}{x} = \int 0$$

$$\ln(y-1) + \ln x = c$$

$$\ln x(y-1) = \ln c$$

$$xy - x = c$$

$$xy - x - c = 0$$

Sol. um. PD

$$(xy + x - c)(xy - x - c) = 0$$

PD. CLAIRAUT

$$(y - xp)^2 = 1 + p^2 \quad , \quad \boxed{p = \frac{dy}{dx}}$$

$$y - xp = \pm\sqrt{1+p^2}$$

$$\boxed{y = xp \pm \sqrt{1+p^2}}$$

$$y = x \cdot p \pm (1+p^2)^{1/2}$$

diferensialkan thd. x

$$\frac{dy}{dx} = \frac{(x)'}{1} p + x(p)' \pm \frac{1}{2}(1+p^2)^{\frac{1}{2}-1} \cdot (1+p^2)'$$

$$\cancel{p} = \cancel{p} + x \cdot \frac{dp}{dx} \pm \frac{1}{2}(1+p^2)^{-\frac{1}{2}} (0 + 2p) \cdot p'$$

$$0 = x \cdot \frac{dp}{dx} \pm \frac{1}{\cancel{2}} \cdot \frac{1}{\sqrt{1+p^2}} \cdot \cancel{2}p \cdot \frac{dp}{dx}$$

$$0 = \frac{dp}{dx} \cdot \left(x \pm \frac{p}{\sqrt{1+p^2}} \right)$$

$$\frac{dp}{dx} = 0$$

$$dp = 0 \cdot dx$$

$$\int dp = \int 0 \cdot dx$$

$$\boxed{p = c}$$

Sol. umum

$$y = xp \pm \sqrt{1+p^2}$$

$$\boxed{y = cx \pm \sqrt{1+c^2}}$$

$$x \pm \frac{p}{\sqrt{1+p^2}} = 0$$

$$x = \mp \frac{p}{\sqrt{1+p^2}}$$

$$x^2 = \left(\mp \frac{p}{\sqrt{1+p^2}}\right)^2$$

$$x^2 = \frac{p^2}{1+p^2}$$

$$x^2(1+p^2) = p^2$$

$$x^2 + p^2x^2 = p^2$$

$$x^2 = p^2 - p^2x^2$$

$$x^2 = p^2(1-x^2)$$

$$\boxed{p^2 = \frac{x^2}{1-x^2}}$$

$$p = \sqrt{\frac{x^2}{1-x^2}} = \frac{x}{\sqrt{1-x^2}}$$

$$(y-xp)^2 = 1+p^2$$

$$\left(y - x \cdot \frac{x}{\sqrt{1-x^2}}\right)^2 = 1 + \frac{x^2}{1-x^2}$$

$$\left(y - \frac{x^2}{\sqrt{1-x^2}}\right)^2 = \frac{1-\cancel{x^2}+\cancel{x^2}}{1-x^2}$$

$$\left(y - \frac{x^2}{\sqrt{1-x^2}}\right)^2 = \frac{1}{1-x^2}$$

$$\boxed{y - \frac{x^2}{\sqrt{1-x^2}} = \pm \frac{1}{\sqrt{1-x^2}}}$$

$$y - \frac{x^2}{\sqrt{1-x^2}} = \frac{1}{\sqrt{1-x^2}}$$

$$y = \frac{1+x^2}{\sqrt{1-x^2}}$$

$$y^2 = \frac{(1+x^2)^2}{1-x^2}$$

(TM)

$$y - \frac{x^2}{\sqrt{1-x^2}} = -\frac{1}{\sqrt{1-x^2}}$$

$$y = \frac{x^2-1}{\sqrt{1-x^2}}$$

$$y^2 = \frac{(x^2-1)^2}{(1-x^2)}$$

$$y^2 = \frac{(x^2-1)^2}{-(x^2-1)}$$

$$-y^2 = x^2-1$$

$$\boxed{1 = x^2+y^2}$$

Solusi Singular

Hal 114

4. $xp^3 - yp^2 + xp - y = 1$

$$(xp^3 - yp^2) + (xp - y) = 1$$

$$p^2(xp - y) + (xp - y) = 1$$

$$(xp - y)(p^2 + 1) = 1$$

$$xp - y = \frac{1}{p^2+1}$$

$$-y = -xp + \frac{1}{p^2+1} \quad \times (-1)$$

$$\boxed{y = xp - \frac{1}{p^2+1}}$$

$$= (p^2+1)^{-1}$$

$$\frac{dy}{dx} = x' p + x\,p' - (-1)\cdot(p^2+1)^{-1-1}\cdot(p^2+1)'$$

$$p = p + x\frac{dp}{dx} + (p^2+1)^{-2}(2p\cdot p' + 0)$$

$$0 = x\frac{dp}{dx} + (p^2+1)^{-2}\cdot 2p\frac{dp}{dx}$$

$$0 = \left\{x + \frac{2p}{(p^2+1)^2}\right\}\frac{dp}{dx}$$

$$\boxed{\frac{dp}{dx} = 0}$$

$$dp = 0\cdot dx$$

$$\int dp = \int 0\cdot dx$$

$$\boxed{p = c}$$

Solusi umum:

$$y = xp - \frac{1}{p^2+1}$$

$$\boxed{y = cx - \frac{1}{c^2+1}}$$

$(P^2+1)'$

$)^{-2}(2P\cdot P'+0)$

$P\frac{dP}{dx}$

SOLUSI SINGULAR

$$X+\frac{2P}{(P^2+1)^2}=0$$

$$\boxed{X=-\frac{2P}{(P^2+1)^2}}$$

$$Y=XP-\frac{1}{P^2+1}$$

$$=\frac{-2P}{(P^2+1)^2}\cdot P-\frac{1}{P^2+1}$$

$$=\frac{-2P^2}{(P^2+1)^2}-\frac{(P^2+1)}{(P^2+1)^2}$$

$$\boxed{Y=\frac{-3P^2-1}{(P^2+1)^2}}$$