

Soal Latihan “Matematika Teknik I” :

1. $(x^2 + y^2)dx + 2xydy = 0$

Tentukan Solusi Umum PDnya !

2. $\frac{dy}{dx} + \frac{y}{x} = e^{2x}$

Tentukan Solusi Umum PDnya !

3. $\frac{dy}{dx} - \frac{y}{x} = y^2 e^x$

Tentukan Solusi Umum PDnya !

4. $y'' - y' - 12y = e^{5x}$, $y(0) = 3\frac{1}{8}$, $y'(0) = 5\frac{5}{8}$

Tentukan Solusi PDnya !

5. $\frac{d^2y}{dx^2} - 10\frac{dy}{dx} + 25y = xe^{5x}$

Tentukan Solusi Umum PDnya !

6. Jika Jumlah Penduduk sekarang $P(0)$ dan Jumlah Penduduk setelah t tahun $P(t)$. Jika jumlah penduduk setelah 10 tahun menjadi tiga kali lipat, tentukan setelah berapa tahun jumlah penduduknya menjadi tujuh kali lipat ?

7. Jika Modal yang ditabung sekarang $M(0)$ dan Modal tabungannya setelah t tahun $M(t)$. Jika tabungannya mendapat bunga 11% per tahun, setelah berapa tahun Modalnya menjadi lima kali lipat

$$1) (x^2 + y^2) dx + 2xy dy = 0$$

$$M(x, y) = x^2 + y^2 \rightarrow$$

$$\frac{\partial M}{\partial y} = 2y \quad \left[\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \right]$$

$$N(x, y) = 2xy \rightarrow \frac{\partial N}{\partial x} = 2y$$

$$\text{II } F(x, y) = \int M(x, y) dx + C(y)$$

$$\text{I } F(x, y) = \int M(x, y) dx + C(y) \rightarrow \left[\frac{\partial F}{\partial y} = N(x, y) \right]$$

$$\frac{\partial F}{\partial x} =$$

$$= \int x^2 dx + \int y^2 dx + C(y)$$

$$= \frac{1}{3} x^3 + x y^2 + C(y)$$

$$\frac{\partial F}{\partial y} = 2xy + C'(y) = 2xy$$

$$C'(y) = 0 \rightarrow C(y) = \int 0 dy = C$$

Solusi

$$\text{PD } \frac{0}{0} \left[\frac{1}{3} x^3 + x y^2 + C(y) = 0 \right]$$

$$\frac{1}{3} x^3 + x y^2 + C = 0$$

solusi

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$\textcircled{II} F(x,y) = \int^N(x,y) dy + c(x) \Rightarrow \left[\frac{\partial F}{\partial x} = M(x,y) \right]$$

$$= \int (2xy) dy + c(x) = 2x \cdot (\frac{1}{2}y^2) + c(x)$$

$$F(x,y) = xy^2 + c(x)$$

$$\frac{\partial F}{\partial x} = x^2 + c'(x) = x^2 + x^2$$

$$c'(x) = x^2 \rightarrow c(x) = \int c'(x) dx = \int x^2 dx$$

$$c(x) = \frac{1}{3}x^3 + C$$

solum PD :

$$xy^2 + c(x) = 0$$

$$xy^2 + \frac{1}{3}x^3 + C = 0$$

$$y = 0$$

2.

$$\frac{dy}{dx} + \frac{y}{x} = e^{2x}$$

$$\frac{dy}{dx} + \frac{1}{x} \cdot y = e^{2x}$$

$$\int p(x) dx$$

$$\int (x) = e^{\int \frac{1}{x} dx}$$

$$= e^{\ln x}$$

$$= x$$

$$\left. \begin{aligned} P(x) &= \frac{1}{x} \\ Q(x) &= e^{2x} \end{aligned} \right]$$

Solusi umum

$$y =$$

$$y =$$

$$y =$$

$$y$$

$$x) = \frac{1}{x}$$

$$x) = e^{2x}$$

Solusi umum P.D

$$y = \frac{1}{s(x)} \left[\int s(x) Q(x) dx + c \right]$$

$$y = \frac{1}{x} \left[\int \underbrace{x \cdot \frac{1}{2} e^{2x}}_{dv} dx + c \right]$$

$$\frac{dv}{dx} = 1$$

$$dv = dx$$

$$\int dv = \int e^{2x} dx$$

$$v = \frac{1}{2} e^{2x}$$

$$y = \frac{1}{x} \left[x \cdot \frac{1}{2} e^{2x} - \int \frac{1}{2} e^{2x} dx + c \right]$$

$$y = \frac{1}{x} \left(x \cdot \frac{1}{2} e^{2x} \right) - \frac{1}{x} \cdot \frac{1}{2} \left(\frac{1}{2} e^{2x} \right) + \frac{c}{x}$$

$$y = \frac{1}{2} e^{2x} - \frac{1}{4x} e^{2x} + \frac{c}{x}$$

ATIATU

$$y = \frac{1}{x} \left(\frac{1}{2} x e^{2x} - \frac{1}{4} e^{2x} + c \right)$$

$$(3) \frac{dy}{dx} - \frac{y}{x} = (y^2) e^x$$

$$\begin{array}{l} n = 2 \\ Q(x) = e^x \\ P(x) = -\frac{1}{x} \end{array}$$

$$S(x) = e^{(1-n) \int P(x) dx}$$

$$S(x) = e^{(1-2) \int -\frac{1}{x} dx} = e^{\ln x} = x$$

$$y^{1-n} = \frac{1-n}{S(x)} \left[\int S(x) \cdot Q(x) dx + C \right]$$

$$y^{1-2} = \frac{1-2}{x} \left[\int x \cdot e^x dx + C \right]$$

$$y^{-1} = -\frac{1}{x} \left[\int x \cdot d(e^x) + C \right]$$

$$\frac{1}{y} = -\frac{1}{x} \left[x e^x - \int e^x d(x) + C \right]$$

$$\frac{1}{y} = -\frac{1}{x} \left[x e^x - e^x + C \right]$$

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$$y'' - y' - 12y = e^{5x}$$

$$D^2 y - D y - 12y = e^{5x}$$

$$(D^2 - D - 12)y = e^{5x}$$

(I)

$$a = 5$$

Persamaan Eigen

$$(\alpha^2 - \alpha - 12) = 0$$

$$(\alpha - 4)(\alpha + 3) = 0$$

$$\alpha_1 = 4, \alpha_2 = -3$$

Solusi homogen: $y_h = C_1 e^{4x} + C_2 \cdot e^{-3x}$

(II)

Solusi khusus PD

$$y_k = \frac{e^{5x}}{D^2 - D - 12}$$

$$\rightarrow D = a = 5$$

$$= \frac{e^{5x}}{5^2 - 5 - 12}$$

$$= \frac{e^{5x}}{25 - 5 - 12}$$

$$= \frac{e^{5x}}{8}$$

Solusi umum

$$Y = Y_h + Y_p$$

$$Y = C_1 e^{4x} + C_2 e^{-3x} + \frac{e^{5x}}{8}$$

$$Y' = 4C_1 e^{4x} - 3C_2 e^{-3x} + \frac{5}{8} e^{5x}$$

$$Y(0) = 3 \frac{1}{8}$$

$$Y'(0) = 5 \frac{5}{8}$$

$$C_1 + C_2 + \frac{1}{8} = 3 \frac{1}{8}$$

$$4C_1 - 3C_2 + \frac{5}{8} = 5 \frac{5}{8}$$

$$C_1 + C_2 = 3 \dots \textcircled{1}$$

$$4C_1 - 3C_2 = 5 \dots \textcircled{2}$$

$$\begin{array}{l|l} 3\textcircled{1} + \textcircled{2} : 7C_1 = 14 & C_1 + C_2 = 3 \\ C_1 = 2 & 2 + C_2 = 3 \\ & C_2 = 1 \end{array}$$

Solusi PD: $Y = 2e^{4x} + e^{-3x} + \frac{e^{5x}}{8}$

$$3C_1 + 3C_2 = 9$$

$$4C_1 - 3C_2 = 5$$

$$\hline 7C_1 = 14$$

$$C_1 = 2$$

$$5 \quad \frac{d^2 y}{dx^2} - 10 \frac{dy}{dx} + 25 y = x \cdot e^{5x}$$

$$(D^2 y - 10 D + 25) y = x \cdot e^{5x}$$

Pers. Eigen: $(D^2 y - 10 D + 25) \cdot y = 0$

$$(\lambda - 5)^2 = 0 \quad \leftarrow (D - 5)^2 \cdot y = 0$$

$$\lambda_1 = \lambda_2 = 5$$

$$Y_h = (C_1 + C_2 x) \cdot e^{5x}$$

$$Y_k = \frac{1}{(D-5)^2} (e^{5x} \cdot x) \rightarrow \boxed{q=5}$$

$$= e^{5x} \cdot \left[\frac{1}{(\cancel{D+5}-5)^2} \cdot x \right]$$

$$= e^{5x} \cdot \left[\frac{1}{D^2} \cdot x \right]$$

$$\rightarrow e^{5x} \cdot [D^{-1} \cdot D^{-1} \cdot x]$$

$$e^{5x} \cdot [\int \int x \, dx \, dx]$$

$$e^{5x} \cdot \int \frac{1}{2} x^2 \, dx$$

$$e^{5x} \cdot \frac{1}{6} x^3$$

$$Y = Y_h + Y_k$$

$$Y = (C_1 + C_2 x) \cdot e^{5x} + \frac{1}{6} \cdot x^3 \cdot e^{5x} //$$

CARA II

$$(D^2 - 10D + 25) \cdot \textcircled{Y} = X \cdot e^{5x}$$

$$Y_k = \frac{1}{\textcircled{D^2 - 10D + 25}} \cdot (X e^{5x})$$

$$\boxed{\begin{aligned} f(D) &= D^2 - 10D + 25 \\ f'(D) &= 2D - 10 \end{aligned}}$$

$$\rightarrow Y_k = X \cdot \left[\frac{1}{f(D)} F \right] - \frac{f'(D)}{[f(D)]^2} \cdot F \quad \left. \begin{array}{l} k=5 \\ D=k=5 \end{array} \right\}$$

$$Y_k = X \left[\frac{1}{(D-5)^2} e^{5x} \right] - \frac{2D-10}{[(D-5)^2]^2} e^{5x}$$

$$= X \left[\frac{X^2}{2!} e^{5x} \right] - \frac{2(D-5)}{(D-5)^4} e^{5x}$$

$$= X \left[\frac{X^2}{(2 \cdot 1)} e^{5x} \right] - 2 \left[\frac{1}{(D-5)^3} e^{5x} \right]$$

$$= \frac{1}{2} X^3 e^{5x} - 2 \left[\frac{X^3}{3!} e^{5x} \right]$$

$$= \frac{1}{2} X^3 e^{5x} - \frac{1}{3} X^3 e^{5x}$$
$$= \frac{1}{6} X^3 e^{5x}$$

SOLVING CUMULATIVE PD

$$Y = Y_h + Y_k$$
$$Y = (e_1 + e_2 x) e^{5x} + \frac{1}{6} x^3 e^{5x}$$

LAJU PERTUMBUHAN PENDUDUK

$$\textcircled{6} \left[\frac{dP}{dt} = k \cdot P \right]$$

$$\frac{dP}{P} = k \cdot dt$$

$$\left[\frac{dP}{P} - k \cdot dt = 0 \right]$$

$$\int \frac{dP}{P} - \int k \cdot dt = \int 0$$

$$\ln P - k \cdot t = C$$

$$\ln P - \ln e^{kt} = \ln C$$

$$\ln \frac{P}{e^{kt}} = \ln C$$

$$\frac{P}{e^{kt}} = C$$

$$P(t) = C \cdot e^{kt}$$

$$t=0 \rightarrow P(0) = C \cdot e^{k \cdot 0}$$

$$P(0) = C \rightarrow \boxed{P(t) = P(0) \cdot e^{kt}}$$

$$t=10 \rightarrow \frac{P(t)}{P(0)} = \frac{P(0) \cdot e^{kt}}{P(0)}$$

$$\frac{3P(0)}{3} = \frac{P(0) \cdot e^{kt}}{P(0)}$$

$$3 = e^{kt(10)}$$

$$\ln 3 = \ln e^{10k}$$

$$\ln 3 = 10k$$

$$\boxed{k = \frac{\ln 3}{10}}$$

P(t)

10

$$). e^{kt}$$

$$P(t) = P(0) \cdot e^{k \cdot t}$$

$$7 P(0) = P(0) \cdot e^{\frac{\ln 3}{10} \cdot t}$$

$$7 = e^{\frac{\ln 3}{10} \cdot t}$$

$$\ln 7 = \ln e$$

$$\ln 7 = \frac{\ln 3 \cdot t}{10}$$

$$10 \ln 7 = \ln 3 \cdot (t)$$

$$t = \frac{10 \ln 7}{\ln 3}$$

JADI JUMLAH
PENDUDUK MEN-
JADI 7 X LIPAT
SETELAH $\frac{10 \ln 7}{\ln 3}$ TAHUN

7) Bunga Per tahun = 11%.

$$\left(\frac{dM}{dt} = \frac{11}{100} \cdot M \right)$$

$$\frac{dM}{M} = \frac{11}{100} dt$$

$$\int \frac{dM}{M} - \int \frac{11}{100} dt = \int 0$$

$$\ln M - \ln e^{\frac{11}{100}t} = C$$

$$\ln \frac{M}{e^{\frac{11}{100}t}} = \ln C$$

$$\left(\frac{M}{e^{\frac{11}{100}t}} = C \right)$$

11% . M

$$\rightarrow M(t) = C \cdot e^{\frac{11}{100}t}$$

$$t=0 \rightarrow M(0) = C \cdot e^{\frac{11}{100}(0)}$$

$$M(0) = C$$

$$M(t) = M(0) \cdot e^{\frac{11}{100}t}$$

$$5M(0) = M(0) \cdot e^{\frac{11}{100}t}$$

$$5 = e^{\frac{11}{100}t}$$

$$\ln 5 = \ln e^{\frac{11}{100}t}$$

$$\ln 5 = \frac{11}{100}t$$

$$t = \frac{100 \ln 5}{11}$$

Jadi Modal menjadi 5x lipat
stlh $\frac{100 \ln 5}{11}$ tahun.