

LATIHAN SOAL

① DIK : $\vec{a} = (2, 1, 3)$
 $\vec{b} = (1, 2, 4)$
 $\vec{c} = (3, 4, 5)$

DITANYA :

$$\vec{a} \cdot (\vec{b} \times \vec{c})$$

$$(\vec{a} \times \vec{b}) \cdot \vec{c}$$

$\vec{c}_1 =$ PROYEKSI \vec{a} PADA \vec{b}

\vec{c}_2 , DI MANA $\vec{c}_2 \perp \vec{c}_1$

2) TENTUKAN PERSAMAAN VEKTOR, PERSAMAAN PARAMETER, DAN PERSAMAAN KOORDINAT (KARTESIUS) DARI GARIS YANG MELALUI $A(1, 2, -3)$ DAN $B(3, 1, -2)$. \emptyset

3) TENTUKAN PERSAMAAN VEKTOR DAN PERS. KOORDINAT (KARTESIUS) BIDANG YG MELALUI $A(1, 2, 3)$, $B(2, 1, 3)$ & $C(3, 1, 2)$.

4) a) DIK: $\vec{a} = (1, 2, 3)$
 $\vec{b} = (3, 1, 2)$
 $\vec{c} = (2, x, 1)$

JIKA \vec{c} MERUPAKAN KOMBINASI LINER DARI \vec{a} & \vec{b} ,
MAKA TENTUKAN NILAI x !

b) DIK: $\vec{a} = (1, 2, 3)$
 $\vec{b} = (3, 1, 2)$
 $\vec{c} = (2, 4, 1)$

APAKAH \vec{a} , \vec{b} & \vec{c} BEBAS LINER
ATAU TIDAK BEBAS LINER?

5) DIK : $A = \begin{bmatrix} 1 & 1 \\ 4 & -2 \end{bmatrix}$

TENTUKAN NILAI EIGEN
DAN VEKTOR EIGEN!

$$\textcircled{1} \text{ Dik: } \vec{a} = (2, 1, 3)$$

$$\vec{b} = (1, 2, 4)$$

$$\vec{c} = (3, 4, 5)$$

$$\vec{b} \rightarrow \begin{array}{|c|c|c|} \hline 1 & 2 & 4 \\ \hline \end{array} \begin{array}{|c|} \hline 1 \\ \hline \end{array}$$

$$\vec{c} \rightarrow \begin{array}{|c|c|c|} \hline 3 & 4 & 5 \\ \hline \end{array} \begin{array}{|c|} \hline 3 \\ \hline \end{array}$$

m_1 m_2 m_3

$$\vec{a} \rightarrow \begin{array}{|c|} \hline 2 \\ \hline \end{array}$$

$$\vec{b} \rightarrow \begin{array}{|c|} \hline 1 \\ \hline \end{array}$$

$$\vec{a} \times \vec{b} = \begin{pmatrix} \dots \\ \dots \\ \dots \end{pmatrix}$$

$$\vec{b} \times \vec{c} = \left(\begin{vmatrix} 2 & 4 \\ 4 & 5 \end{vmatrix}, \begin{vmatrix} 4 & 1 \\ 5 & 3 \end{vmatrix}, \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} \right)$$

$$= (10 - 16, 12 - 5, 4 - 6)$$

$$= (-6, 7, -2)$$

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = (2, 1, 3) \cdot (-6, 7, -2)$$

$$= (2)(-6) + (1)(7) + (3)(-2)$$

$$= -12 + 7 - 6$$

$$= -11$$

$$\begin{array}{l} \vec{a} \rightarrow \begin{array}{c} \boxed{2} \\ \times \\ \boxed{1} \end{array} \begin{array}{c} \overbrace{1 \quad 3}^{n_1} \\ \times \\ 2 \quad 4 \end{array} \times \begin{array}{c} \boxed{2} \\ \times \\ \boxed{1} \end{array} \\ \vec{b} \rightarrow \begin{array}{c} \boxed{1} \\ \times \\ \boxed{2} \end{array} \begin{array}{c} \overbrace{2 \quad 4}^{n_2} \\ \times \\ \boxed{3} \end{array} \times \begin{array}{c} \boxed{2} \\ \times \\ \boxed{1} \end{array} \\ \underbrace{\hspace{10em}}_{n_3} \end{array}$$

$$\vec{a} \times \vec{b} = \left(\begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix}, \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}, \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} \right)$$

$$= (4 - 6, 3 - 8, 4 - 1)$$

$$= (-2, -5, 3)$$

$$(\vec{a} \times \vec{b}) \cdot \vec{c} = (-2, -5, 3) \cdot (3, 4, 5)$$

$$= -6 - 20 + 15$$

$$\underline{\underline{-11}}$$

$$\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$$

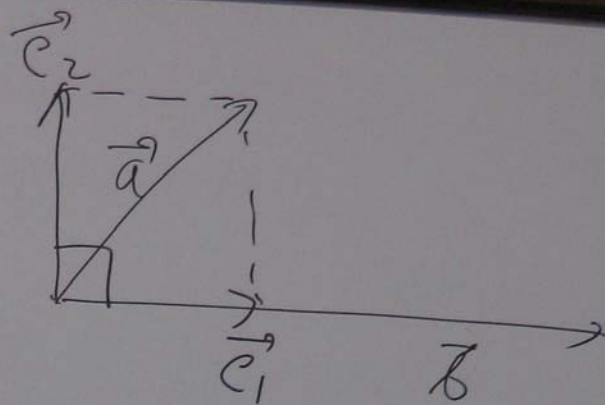
$$(-2)$$

$$(-2)$$

$$\vec{a} = (2, 1, 3)$$

$$\vec{b} = (1, 2, 4)$$

$$\vec{c} = (3, 4, 5)$$



$$\vec{c}_1 = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|^2} \vec{b}$$

$$\vec{c}_1 = \frac{16}{21} (1, 2, 4)$$

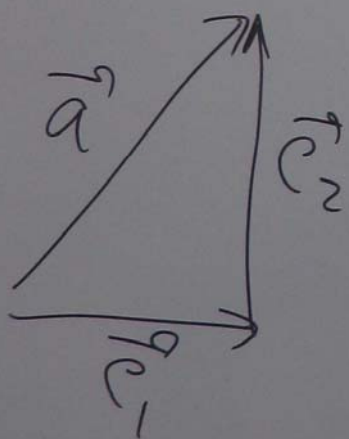
$$= \left(\frac{16}{21}, \frac{32}{21}, \frac{64}{21} \right)$$

$$\vec{a} \cdot \vec{b} = (2, 1, 3) \cdot (1, 2, 4)$$

$$= 2 + 2 + 12 = 16$$

$$|\vec{b}| = \sqrt{1^2 + 2^2 + 4^2} = \sqrt{21}$$

$$|\vec{b}|^2 = (\sqrt{21})^2 = 21$$



$$\vec{c}_1 + \vec{c}_2 = \vec{a}$$

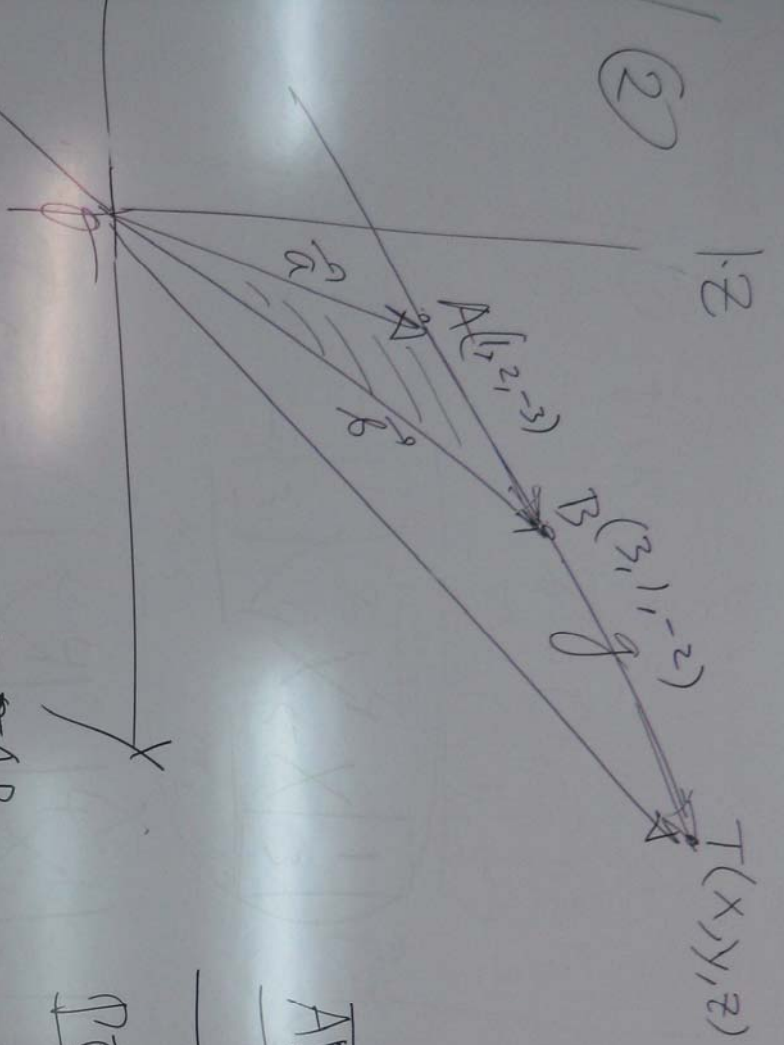
$$\vec{c}_2 = \vec{a} - \vec{c}_1$$

$$\vec{c}_2 = (2, 1, 3) - \left(\frac{16}{21}, \frac{32}{21}, \frac{64}{21} \right)$$

$$= \left(\frac{42}{21}, \frac{21}{21}, \frac{63}{21} \right) - \left(\frac{16}{21}, \frac{32}{21}, \frac{64}{21} \right)$$

$$= \left(\frac{26}{21}, -\frac{11}{21}, -\frac{1}{21} \right)$$

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PERHATIKAN ΔOAB

$$\vec{OA} + \vec{AB} = \vec{OB}$$

$$\vec{a} + \vec{AB} = \vec{b}$$

$$\vec{AB} = \vec{b} - \vec{a}$$

$$= (3, 1, -2) - (1, 2, -3)$$

$$= (2, -1, 1)$$

$$\vec{OA} = \vec{a} = (1, 2, -3)$$

$$\vec{OB} = \vec{b} = (3, 1, -2)$$

$$\vec{OT} = (x, y, z)$$

\vec{AB} & \vec{AT} SEGARIS & SAMA

$$\vec{AT} = \lambda \cdot \vec{AB}$$

PERHATIKAN ΔOAT

$$\vec{OT} = \vec{OA} + \vec{AT}$$

$$(x, y, z) = \vec{a} + \lambda \cdot \vec{AB}$$

$$(x, y, z) = (1, 2, -3) + \lambda \cdot (2, -1, 1)$$

$$(x, y, z) = (1 + 2\lambda, 2 - \lambda, -3 + \lambda)$$

PERK. VEKTOR DARI GARIS g

DAN VEKTOR

$$\begin{array}{l} x = 1 + 2\lambda \\ y = 2 - \lambda \\ z = -3 + \lambda \end{array} \rightarrow \begin{array}{l} x-1 = 2\lambda \rightarrow \lambda = \frac{x-1}{2} \\ y-2 = -\lambda \rightarrow \lambda = \frac{y-2}{-1} \\ z+3 = \lambda \rightarrow \lambda = \frac{z+3}{1} \end{array}$$

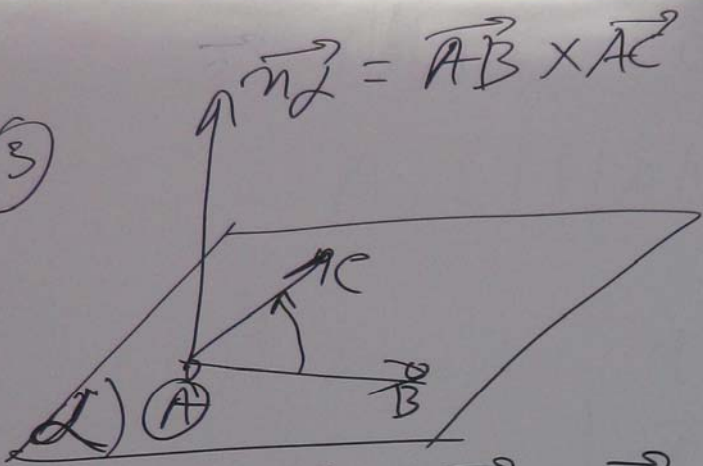
PERS. PARAMETER
DARI GARIS g .

$$\lambda = \lambda = \lambda$$

$$\frac{x-1}{2} = \frac{y-2}{-1} = \frac{z+3}{1}$$

PERS. KOORDINAT (KARTESISUS)
DARI PERS. g

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$$\vec{OA} = \vec{a} = (1, 2, 3)$$

$$\vec{OB} = \vec{b} = (2, 1, 3)$$

$$\vec{OC} = \vec{c} = (3, 1, 2)$$

$$\vec{AB} = \vec{b} - \vec{a} = (2, 1, 3) - (1, 2, 3) = (1, -1, 0)$$

$$\vec{AC} = \vec{c} - \vec{a} = (3, 1, 2) - (1, 2, 3) = (2, -1, -1)$$

$$\vec{AB} \rightarrow \begin{array}{|c|} \hline 1 \\ \hline \end{array} \begin{array}{|c|} \hline -1 \\ \hline \end{array} \begin{array}{|c|} \hline 0 \\ \hline \end{array} \begin{array}{|c|} \hline 1 \\ \hline \end{array}$$

$$\vec{AC} \rightarrow \begin{array}{|c|} \hline 2 \\ \hline \end{array} \begin{array}{|c|} \hline -1 \\ \hline \end{array} \begin{array}{|c|} \hline -1 \\ \hline \end{array} \begin{array}{|c|} \hline 2 \\ \hline \end{array}$$

n_1 (above the first column), n_3 (below the first column), n_2 (below the second column)

$$\vec{n} = \begin{pmatrix} \begin{vmatrix} -1 & 0 \\ -1 & -1 \end{vmatrix}, \begin{vmatrix} 1 & 1 \\ -1 & 2 \end{vmatrix}, \begin{vmatrix} 1 & -1 \\ 2 & -1 \end{vmatrix} \end{pmatrix}$$

$$= (1 - 0, 0 + 1, -1 + 2)$$

$$= (1, 1, 1)$$

$$\text{Bid. } \alpha \quad \left\{ \begin{array}{l} \text{melalui } A \begin{pmatrix} x_1 & y_1 & z_1 \\ 1 & 1 & 2 \\ 2 & 3 \end{pmatrix} \\ \perp \vec{M} = (1, 1, 1) \\ n_1, n_2, n_3 \end{array} \right.$$

PEKS. VEKTOR DARI Bid. α

$$\begin{aligned} (n_1, n_2, n_3) \cdot (x - x_1, y - y_1, z - z_1) &= 0 \\ (1, 1, 1) \cdot (x - 1, y - 2, z - 3) &= 0 \end{aligned}$$

PEKS. KOORDINAT (PARTISIVUS) DARI Bid. α

$$\begin{aligned} n_1(x - x_1) + n_2(y - y_1) + n_3(z - z_1) &= 0 \\ (x - 1) + 1(y - 2) + 1(z - 3) &= 0 \\ \underline{x - 1} + \underline{y - 2} + \underline{z - 3} = 0 &\rightarrow \boxed{X + Y + Z - 6 = 0} \end{aligned}$$

$$4a) \quad \vec{a} = (1, 2, 3) \\ \vec{b} = (3, 1, 2) \\ \vec{c} = (2, x, 1)$$

\vec{c} MERUPAKAN KOMBINASI
LINEAR DARI \vec{a} & \vec{b}

$$\rightarrow \vec{c} = \lambda_1 \cdot \vec{a} + \lambda_2 \vec{b}$$

$$\begin{bmatrix} 2 \\ x \\ 1 \end{bmatrix} = \lambda_1 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + \lambda_2 \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

$$\begin{array}{l} 2 = \lambda_1 + 3\lambda_2 \quad \dots (1) \\ x = 2\lambda_1 + \lambda_2 \quad \dots (2) \\ 1 = 3\lambda_1 + 2\lambda_2 \quad \dots (3) \end{array}$$

$$\begin{aligned} (1) &\rightarrow \lambda_1 + 3\lambda_2 = 2 \quad \left[\begin{array}{l} x_3 \\ x_1 \end{array} \right] \\ (3) &\rightarrow 3\lambda_1 + 2\lambda_2 = 1 \end{aligned}$$

~~$$3\lambda_1 + 9\lambda_2 = 6$$~~

~~$$3\lambda_1 + 2\lambda_2 = 1 \quad (-)$$~~

$$7\lambda_2 = 5 \rightarrow \lambda_2 = \frac{5}{7}$$

$$\lambda_1 + 3\lambda_2 = 2$$

$$\lambda_1 + 3\left(\frac{5}{7}\right) = 2$$

$$\lambda_1 = \frac{14}{7} - \frac{15}{7} = -\frac{1}{7}$$

$$(2) \rightarrow x = 2\lambda_1 + \lambda_2$$

$$x = 2\left(-\frac{1}{7}\right) + \frac{5}{7}$$

$$x = -\frac{2}{7} + \frac{5}{7}$$

$$x = \frac{3}{7}$$

⑤ DIK: $A = \begin{bmatrix} 1 & 1 \\ 4 & -2 \end{bmatrix}$

TENTUKAN NILAI EIGEN
DAN VEKTOR EIGEN!

$$\begin{aligned} \lambda I - A &= \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 4 & -2 \end{bmatrix} \\ &= \begin{bmatrix} \lambda - 1 & -1 \\ -4 & \lambda + 2 \end{bmatrix} \end{aligned}$$

PERS. EIGEN: $|\lambda I - A| = 0$

$$\begin{vmatrix} \lambda - 1 & -1 \\ -4 & \lambda + 2 \end{vmatrix} = 0$$

$$\underbrace{(\lambda - 1)} \underbrace{(\lambda + 2)} + (-1)(-4) = 0$$

$$\lambda^2 + \lambda - 2 - 4 = 0$$

$$\lambda^2 + \lambda - 6 = 0$$

$$(\lambda + 3)(\lambda - 2) = 0$$

$$\underline{\lambda_1 = -3}, \underline{\lambda_2 = 2}$$

MISALKAN VEKTOR EIGEN

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

RUANG EIGEN :

$$(\lambda \mathbf{I} - A) \cdot \vec{x} = \vec{0}$$

$$\begin{bmatrix} \lambda - 1 & -1 \\ -4 & \lambda + 2 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

H

$$\lambda = -3$$

$$\begin{bmatrix} -3-1 & -1 \\ -4 & -3+2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -4 & -1 \\ -4 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{aligned} -4x_1 - x_2 &= 0 \\ -4x_1 - x_2 &= 0 \end{aligned}$$

KESIMPULAN:

$$-4x_1 - x_2 = 0$$

$$-4x_1 = x_2$$

$$\rightarrow x_2 = -4x_1$$

$$HP = \left\{ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \mid x_1, x_2 \in \mathbb{R} \right\}$$

$$= \left\{ \begin{bmatrix} x_1 \\ -4x_1 \end{bmatrix} \mid x_1 \in \mathbb{R} \right\}$$

$$= x_1 \begin{bmatrix} 1 \\ -4 \end{bmatrix} \mid x_1 \in \mathbb{R} \quad \vec{v} = s \begin{bmatrix} 1 \\ -4 \end{bmatrix} \text{ untuk } \lambda = -3$$

$$\lambda = 2$$

$$\begin{bmatrix} 2-1 & -1 \\ -4 & 2+2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 \\ -4 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$x_1 - x_2 = 0 \rightarrow x_1 = x_2$$

$$-4x_1 + 4x_2 = 0$$

$$x_1 - x_2 = 0 \quad \div (-1)$$

$$x_1 = x_2$$

KESIMPULAN

$$x_1 = x_2$$

$$HP = \left\{ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \mid x_1, x_2 \in \mathbb{R} \right\}$$

$$= \left\{ \begin{bmatrix} x_2 \\ x_2 \end{bmatrix} \mid x_2 \in \mathbb{R} \right\}$$

$$= \left\{ x_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} \mid x_2 \in \mathbb{R} \right\}$$

$$\rightarrow \vec{x} = s \begin{bmatrix} 1 \\ 1 \end{bmatrix} \text{ untuk } s=2$$