Hal 108

(CROSS VEKTOR)

SIFAT-SIFAT PERKALIAN SILANG

$$\vec{a} \times \vec{b} = |\vec{a}| \cdot |\vec{b}| \cdot \sin\varphi \cdot \vec{\mu} \rightarrow \text{VEKTOR}$$

(1) $\vec{a} \times \vec{a} = |\vec{a}| \cdot |\vec{a}| \cdot \underbrace{\sin 0}_{=0} \cdot \vec{\mu}$

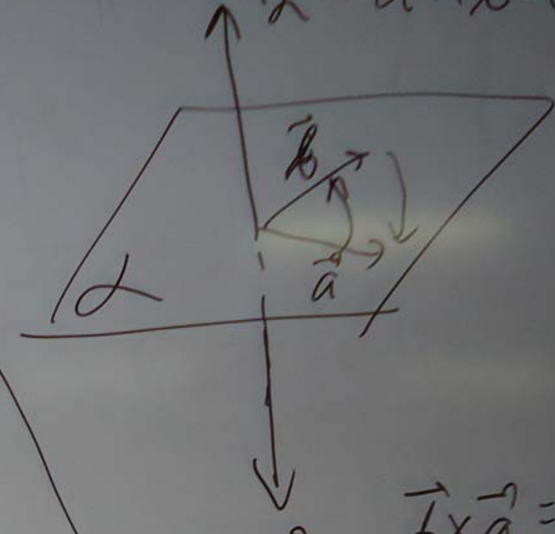
$$\boxed{\vec{a} \times \vec{a} = \vec{0}}$$

(2) $\vec{0} \times \vec{a} = \underbrace{|\vec{0}|}_{=0} \cdot |\vec{a}| \cdot \sin\varphi \cdot \vec{\mu}$

$$\boxed{\vec{0} \times \vec{a} = \vec{0}}$$

(3) $\vec{a} \times \vec{b} = \ominus(\vec{b} \times \vec{a})$

$\vec{n_\alpha} = \vec{a} \times \vec{b} = (1, 2, 3)$



$\vec{n_\alpha} = \vec{b} \times \vec{a} = (-1, -2, -3)$

(4) $\vec{a} \times (\vec{b} + \vec{c})$

$= \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$

(5) $\vec{a} \times \vec{b} = |\vec{a}| \cdot |\vec{b}| \cdot \sin\varphi \cdot \vec{\mu} \rightarrow$ VEKTOR

$$|\vec{a} \times \vec{b}| = |\vec{a}| \cdot |\vec{b}| \cdot \sin\varphi \longrightarrow \text{SKALAR}$$

(6) $$|\vec{a} \times \vec{b}|^2 = \left(|\vec{a}| \cdot |\vec{b}| \cdot \sin\varphi\right)^2$$

$$|\vec{a} \times \vec{b}|^2 = |\vec{a}|^2 \cdot |\vec{b}|^2 \cdot \sin^2\varphi$$

$$= |\vec{a}|^2 \cdot |\vec{b}|^2 \cdot \left(1 - \cos^2\varphi\right)$$

$$= |\vec{a}|^2 \cdot |\vec{b}|^2 - \underline{|\vec{a}|^2 \cdot |\vec{b}|^2 \cdot \cos^2\varphi}$$

$$= |\vec{a}|^2 \cdot |\vec{b}|^2 - \left(\underline{|\vec{a}| \cdot |\vec{b}| \cdot \cos\varphi}\right)^2$$

$$\boxed{|\vec{a} \times \vec{b}|^2 = |\vec{a}|^2 \cdot |\vec{b}|^2 - \left(\vec{a} \cdot \vec{b}\right)^2}$$

7 $\vec{a} \cdot (\vec{b} \times \vec{c}) = (\vec{a} \times \vec{b}) \cdot \vec{e}$

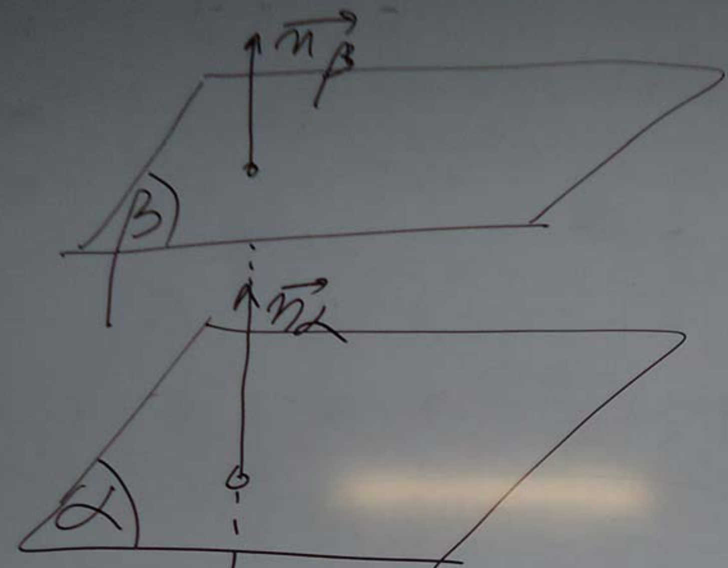
7) $\vec{a} \cdot (\vec{b} \times \vec{c}) = (\vec{a} \times \vec{b}) \cdot c$

VEKTOR NORMAL $(\vec{n_\alpha})$

$$\left.\begin{array}{l} \vec{a} = (x_1, y_1, z_1) \\ \vec{b} = (x_2, y_2, z_2) \end{array}\right\} \rightarrow \vec{n_\alpha} = \vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix}$$

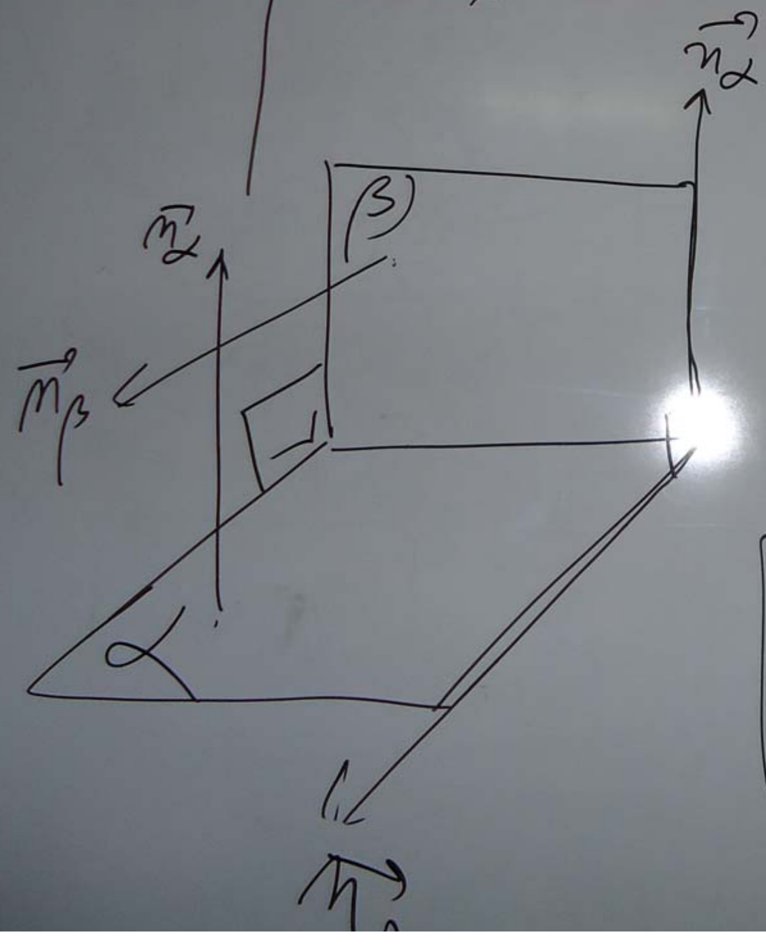
$$\begin{array}{lcccc} & & n_1 & & \\ \vec{a} \rightarrow & \boxed{x_1} & y_1 & z_1 & \boxed{x_1} \\ \vec{b} \rightarrow & \boxed{x_2} & y_2 & z_2 & \boxed{x_2} \\ & n_3 & & n_2 & \end{array}$$

$$\vec{n_\alpha} = \vec{a} \times \vec{b} = \left(\underbrace{\begin{vmatrix} y_1 & z_1 \\ y_2 & z_2 \end{vmatrix}}_{n_1}, \underbrace{\begin{vmatrix} z_1 & x_1 \\ z_2 & x_2 \end{vmatrix}}_{n_2}, \underbrace{\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix}}_{n_3} \right)$$



bid. α // bid. β

$\longrightarrow \boxed{\vec{n}_\alpha = \vec{n}_\beta}$

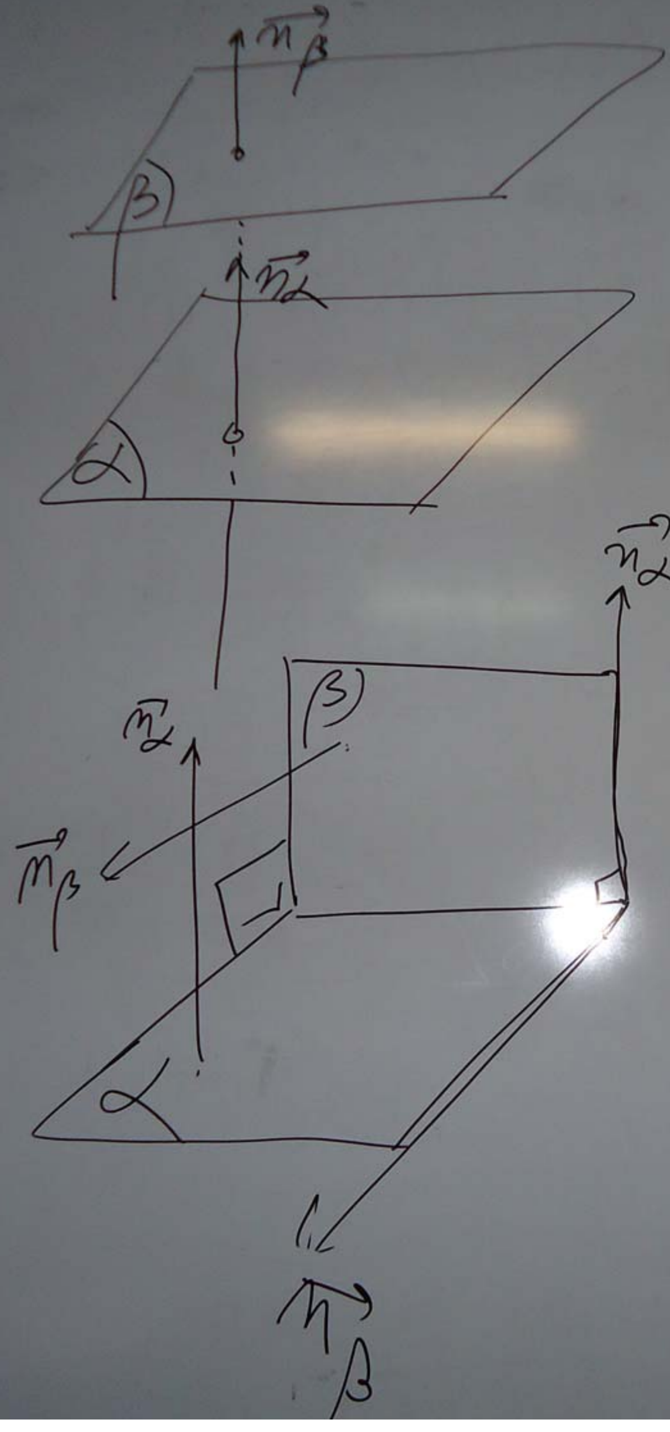


bid. α $\perp$ bid. β

$\longrightarrow \vec{n}_\alpha \perp \vec{n}_\beta \to \varphi = 90^\circ$

$\vec{n}_\alpha \cdot \vec{n}_\beta = |\vec{n}_\alpha| . |\vec{n}_\beta| . \underbrace{\cos 90^\circ}_{= 0}$

$$\boxed{\vec{n}_\alpha \cdot \vec{n}_\beta = 0}$$



bid. α // bid. β

$\longrightarrow \boxed{\vec{n}_\alpha = \vec{n}_\beta}$

bid. α $\perp$ bid. β

$\longrightarrow \quad \vec{n}_\alpha \perp \vec{n}_\beta \rightarrow \varphi = 90^\circ$

$$\vec{n}_\alpha \cdot \vec{n}_\beta = |\vec{n}_\alpha| . |\vec{n}_\beta| . \underbrace{\cos 90^\circ}_{=0}$$

$$\boxed{\vec{n}_\alpha \cdot \vec{n}_\beta = 0}$$

90°

Hal 110

Bid. α $\begin{cases} \text{melalui } T(1,2,3) \\ /\!/ \text{ bid. } \beta : \underline{2x-3y+z=7} \end{cases}$

$\vec{n}_\beta = (2,-3,1)$

bid. α // bid. β $\longrightarrow$ $\boxed{\vec{n}_\alpha = \vec{n}_\beta}$

$\vec{n}_\alpha = (2,-3,1)$

Bid. α $\begin{cases} \text{melalui } T(\overset{x_1}{1},\overset{y_1}{2},\overset{z_1}{3}) \\ \perp \vec{n}_\alpha = (\underset{n_1}{2},\underset{n_2}{-3},\underset{n_3}{1}) \end{cases}$

Bid. α : $n_1(x-x_1)+n_2(y-y_1)+n_3(z-z_1)=0$

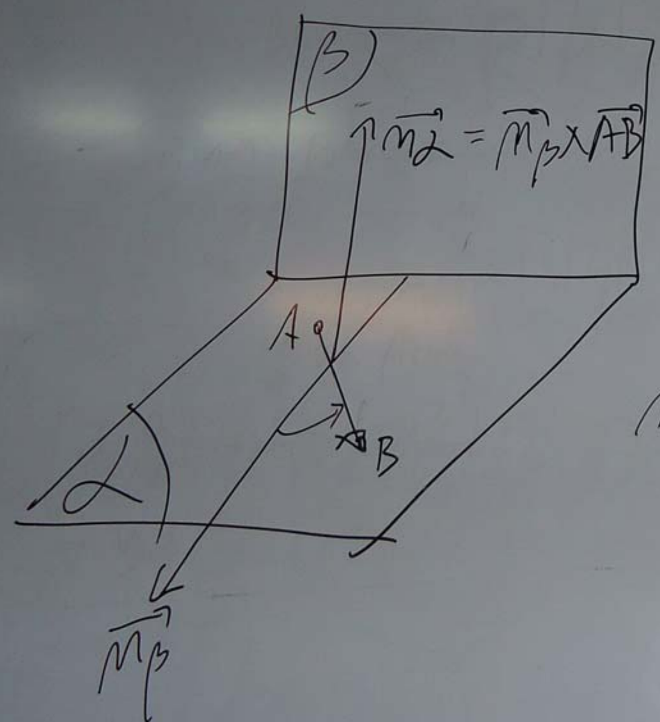
$2(x-1)+(-3)(y-2)+1(z-3)=0$

$2x-2-3y+6+z-3=0$

$2x-3y+z+1=0$

Bid. α $\begin{cases} \text{melalui } A(1,2,3) \longrightarrow \vec{OA} = \vec{a} = (1,2,3) \\ \text{melalui } B(-1,2,1) \longrightarrow \vec{OB} = \vec{b} = (-1,2,1) \\ \perp \text{ bid. } \beta : x - y + 3z - 6 = 0 \end{cases}$

$$\vec{n_\beta} = (1,-1,3)$$



$$\vec{AB} = \vec{b} - \vec{a}$$

$$\vec{AB} = (-1,2,1) - (1,2,3)$$

$$\vec{AB} = (-2,0,-2)$$

$$\vec{n_\alpha} = \vec{n_\beta} \times \vec{AB} = \left| \begin{array}{ccc|cc} \vec{i} & \vec{j} & \vec{k} & \vec{i} & \vec{j} \\ 1 & -1 & 3 & 1 & -1 \\ -2 & 0 & -2 & -2 & 0 \end{array} \right.$$

$$= (2\vec{i} - 6\vec{j} + 0\vec{k}) - (2\vec{k} + 0\vec{i} - 2\vec{j})$$

$$= 2\vec{i} - 4\vec{j} - 2\vec{k} = (2, -4, -2)$$

$\vec{n_\beta}$ —

$\vec{AB}$ —

$\vec{n_\alpha}$

$$\begin{array}{rcccc}
 & & \multicolumn{2}{c}{\overline{n_1}} & \\
\vec{n_\beta} \to & \boxed{1} & -1 & 3 & \boxed{1} \\
\vec{AB} \to & \boxed{-2} & 0 & -2 & \boxed{-2} \\
 & \multicolumn{2}{c}{\overline{}} & \multicolumn{2}{c}{\overline{}} \\
 & \multicolumn{2}{c}{n_3} & \multicolumn{2}{c}{n_2}
\end{array}$$

$$\vec{n_\alpha} = \vec{n_\beta} \times \vec{AB}$$

$$= \left(\begin{vmatrix} -1 & 3 \\ 0 & -2 \end{vmatrix}, \begin{vmatrix} 3 & 1 \\ -2 & -2 \end{vmatrix}, \begin{vmatrix} 1 & -1 \\ -2 & 0 \end{vmatrix} \right)$$

$$= (2-0, -6+2, 0-2)$$

$$= (2, -4, -2)$$

I) Bid. α $\begin{cases} \text{MELALUI } A(\overset{x_1,\,y_1,\,z_1}{1, 2, 3}) \\ \perp \ \vec{n_\alpha} = (\underset{n_1}{2}, \underset{n_2}{-4}, \underset{n_3}{-2}) \end{cases}$

Bid. α : $n_1(x - x_1) + n_2(y - y_1) + n_3(z - z_1) = 0$

$2(x - 1) + (-4)(y - 2) + (-2)(z - 3) = 0$

$\underline{2x} - 2 - \underline{4y} + 8 - \underline{2z} + 6 = 0$

$\boxed{2x - 4y - 2z + 12 = 0}$

II Bid. α $\begin{cases} \text{MELALUI } B(\overset{x_1}{-1}, \overset{y_1}{2}, \overset{z_1}{1}) \\ \perp \ \vec{n_\alpha} = (\underset{n_1}{2}, \underset{n_2}{-4}, \underset{n_3}{-2}) \end{cases}$

Bid. α :

$$n_1(x - x_1) + n_2(y - y_1) + n_3(z - z_1) = 0$$

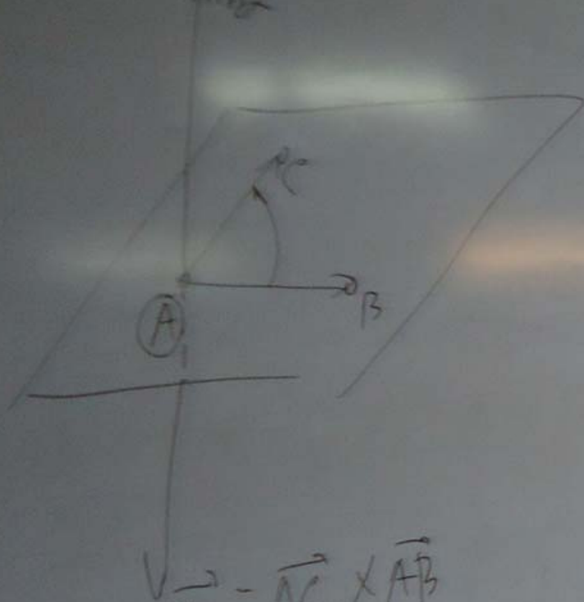
$$2(x+1) + (-4)(y-2) + (-2)(z-1) = 0$$

$$\underline{2x} + 2 - \underline{4y} + 8 - \underline{2z} + 2 = 0$$

$$\boxed{2x - 4y - 2z + 12 = 0}$$

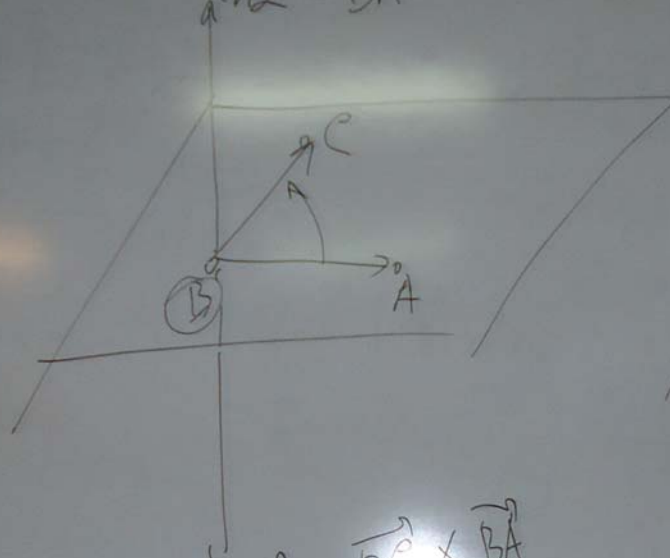
Bidang α MELALUI $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ DAN $C(x_3, y_3, z_3)$.

$\vec{n_\alpha} = \vec{AB} \times \vec{AC}$



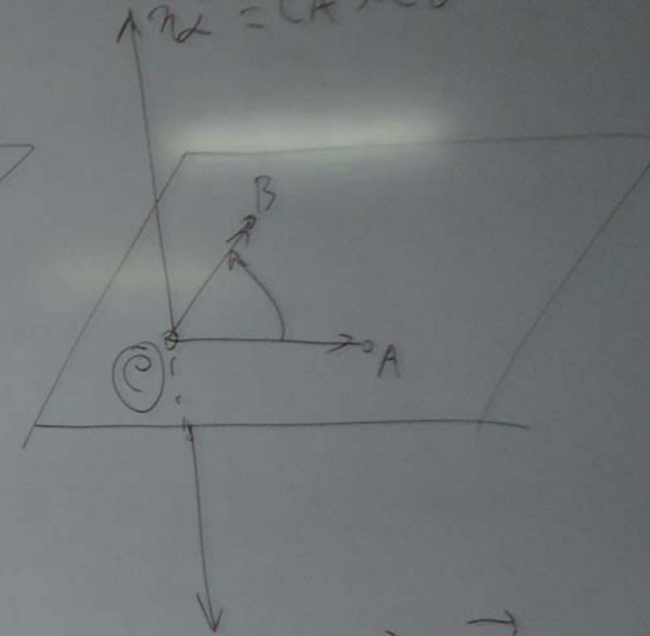
$\vec{m_\alpha} = \vec{AC} \times \vec{AB}$

$\vec{n_\alpha} = \vec{BA} \times \vec{BC}$



$\vec{m_\alpha} = \vec{BC} \times \vec{BA}$

$\vec{n_\alpha} = \vec{CA} \times \vec{CB}$



$\vec{m_\alpha} = \vec{CB} \times \vec{CA}$

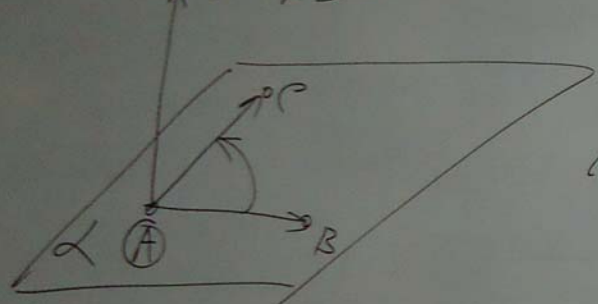
HAL 118

(5) Bid. α melalui $\left\{\begin{array}{l} A(1,2,3) \longrightarrow \vec{OA} = \vec{a} = (1,2,3) \\ B(3,2,1) \longrightarrow \vec{OB} = \vec{b} = (3,2,1) \\ C(2,1,3) \longrightarrow \vec{OC} = \vec{c} = (2,1,3) \end{array}\right.$

$\vec{AB} = \vec{b} - \vec{a} = (3,2,1) - (1,2,3) = (2,0,-2)$

$\vec{AC} = \vec{c} - \vec{a} = (2,1,3) - (1,2,3) = (1,-1,0)$

$\vec{n_\alpha} = \vec{AB} \times \vec{AC}$



$$\begin{array}{l} \vec{AB} \longrightarrow \\ \vec{AC} \longrightarrow \end{array} \boxed{\begin{array}{c} 2 \\ 1 \end{array}} \times \begin{array}{c} 0 \\ -1 \end{array} \times \begin{array}{c} -2 \\ 0 \end{array} \times \boxed{\begin{array}{c} 2 \\ 1 \end{array}}$$

n_1, n_3, n_2

$\vec{n_\alpha} = \vec{AB} \times \vec{AC}$

$= \left(\begin{vmatrix} 0 & -2 \\ -1 & 0 \end{vmatrix}, \begin{vmatrix} -2 & 2 \\ 0 & 1 \end{vmatrix}, \begin{vmatrix} 2 & 0 \\ 1 & -1 \end{vmatrix} \right)$

$= (0-2, -2-0, [illegible]-0)$

$= (-2, -2, -2)$

Bid. α $\left\{\begin{array}{l} \text{MELALUI } A(\overset{x_1}{1}, \overset{y_1}{2}, \overset{z_1}{3}) \\ \perp \vec{n_\alpha} = (\underset{n_1}{-2}, \underset{n_2}{-2}, \underset{n_3}{-2}) \end{array}\right.$

Bid. α :

$n_1(x - x_1) + n_2(y - y_1) + n_3(z - z_1) = 0$

$-2(x-1) - 2(y-2) - 2(z-3) = 0$

$-2x + 2 - 2y + 4 - 2z + 6 = 0$

$-2x - 2y - 2z + 12 = 0$

$x + y + z - 6 = 0$

$(: -2)$

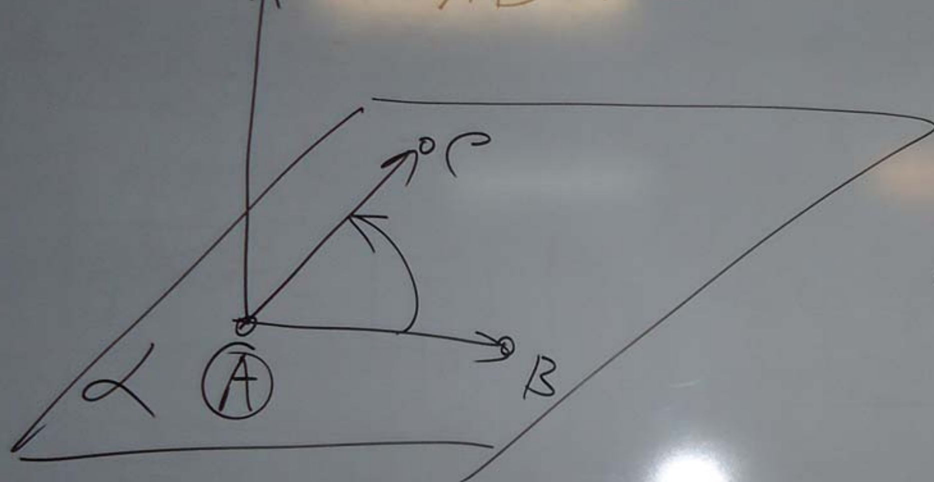
HAL 118

(5) Bid. α melalui $\left\{\begin{array}{l} A(1,2,3) \longrightarrow \vec{OA} = \vec{a} = (1,2,3) \\ B(3,2,1) \longrightarrow \vec{OB} = \vec{b} = (3,2,1) \\ C(2,1,3) \longrightarrow \vec{OC} = \vec{c} = (2,1,3) \end{array}\right.$

$\vec{AB} = \vec{b} - \vec{a} =$

$\vec{AC} = \vec{c} - \vec{a} =$

$\vec{n_\alpha} = \vec{AB} \times \vec{AC}$



$$\begin{array}{c|cccc} & & n_1 & & \\ \vec{AB} \rightarrow & \boxed{2} & 0 & -2 & \boxed{2} \\ \vec{AC} \rightarrow & \boxed{1} & -1 & 0 & \boxed{1} \\ & n_3 & & n_2 & \end{array}$$

$$\vec{n_\alpha} = \vec{AB} \times \vec{AC}$$

$$= \left(\begin{vmatrix} 0 & -2 \\ -1 & 0 \end{vmatrix}, \begin{vmatrix} -2 & 2 \\ 0 & 1 \end{vmatrix}, \begin{vmatrix} 2 & 0 \\ 1 & -1 \end{vmatrix} \right)$$

$$= (0-2,\ -2-0,\ -2-0)$$

$$= (-2, -2, -2)$$

$(2,1,3)-(1,2,3)=(1,-1,0)$

Bid. α $\begin{cases} \text{MELALUI } A(\overset{x_1}{1},\overset{y_1}{2},\overset{z_1}{3}) \\ \perp \vec{n_\alpha} = (\underset{n_1}{-2},\underset{n_2}{-2},\underset{n_3}{-2}) \end{cases}$

Bid. α :

$n_1(x-x_1)+n_2(y-y_1)+n_3(z-z_1)=0$

$-2(x-1)-2(y-2)-2(z-3)=0$

$-2x+2-2y+4-2z+6=0$

$-2x-2y-2z+12=0$

$\boxed{x+y+z-6=0}$ $:-2$

KOMBINASI LINEAR

$\vec{c}$ MERUPAKAN KOMBINASI LINEAR DARI $\vec{a}$ & $\vec{b}$

$$\boxed{\vec{c} = \lambda_1 . \vec{a} + \lambda_2 \vec{b}}$$

di mana : $\lambda_1 \neq 0$ atau $\lambda_2 \neq 0$

CONTOH :

$$\left.\begin{aligned} \vec{a} &= ([illegible], 2, 4, 5) \\ \vec{b} &= 2, 1, 3, -4) \\ \vec{c} &= (16, 10, 22, 1) \end{aligned}\right\} \text{di } R^4$$

APAKAH $\vec{c}$ MERUPAKAN KOMBINASI LINEAR DARI $\vec{a}$ & $\vec{b}$?

SOLUSI

$$\boxed{\vec{c} = \lambda_1 \cdot \vec{a} + \lambda_2 \vec{b}}$$

$$\begin{bmatrix} 16 \\ 10 \\ 22 \\ 1 \end{bmatrix} = \lambda_1 \cdot \begin{bmatrix} 3 \\ 2 \\ 4 \\ 5 \end{bmatrix} + \lambda_2 \cdot \begin{bmatrix} 2 \\ 1 \\ 3 \\ -4 \end{bmatrix}$$

$$\boxed{\begin{aligned} 16 &= 3\lambda_1 + 2\lambda_2 \\ 10 &= 2\lambda_1 + \lambda_2 \end{aligned}}$$

$$22 = 4\lambda_1 + 3\lambda_2$$

$$1 = 5\lambda_1 - 4\lambda_2$$

$$\begin{aligned} 3\lambda_1 + 2\lambda_2 &= 16 \quad |\times 1 \\ 2\lambda + \lambda_2 &= 10 \quad |\times 2 \end{aligned}$$

$$\begin{aligned} 3\lambda_1 + 2\lambda_2 &= 16 \\ 4\lambda_1 + 2\lambda_2 &= 20 \quad - \\ \hline -\lambda_1 &= -4 \end{aligned}$$

$$\boxed{\lambda_1 = 4}$$

$$2\lambda_1 + \lambda_2 = 10$$

$$2(4) + \lambda_2 = 10$$

$$\lambda_2 = 10 - 8$$

$$\boxed{\lambda_2 = 2}$$

$$\begin{aligned} 22 &= 4\lambda_1 + 3\lambda_2 \\ 22 &= 4(4) + 3(2) \\ 22 &= 16 + 6 \\ 22 &= 22 \ (M) \end{aligned}$$

$$\begin{aligned} 1 &= 5\lambda_1 - 4\lambda_2 \\ 1 &= 5(4) - 4(2) \\ 1 &= 20 - 8 \\ 1 &\neq 12 \ (TM) \end{aligned}$$

$\vec{c}$ BUKAN MERUPAKAN KOMBINASI LINEAR DARI $\vec{a}$ & $\vec{b}$.

$\vec{a}, \vec{b}$ & $\vec{c}$ <u>BEBAS LINEAR</u>

(TIDAK BERGANTUNG LINEAR)

$$\lambda_1 . \vec{a} + \lambda_2 . \vec{b} + \lambda_3 . \vec{c} = \vec{0}$$

$\lambda_1 = \lambda_2 = \lambda_3 = 0$ (MEMPUNYAI SOLUSI TRIVIAL)

$\rightarrow \vec{a}, \vec{b}$ & $\vec{c}$ <u>BEBAS LINEAR</u>

$\vec{a}, \vec{b}$ & $\vec{c}$ TIDAK BEBAS LINEAR

(BERGANTUNG LINEAR)

$$\lambda_1 . \vec{a} + \lambda_2 . \vec{b} + \lambda_3 . \vec{c} = \vec{0}$$

$\left.\begin{array}{l} \lambda_1 = 2\lambda_3 \\ \lambda_2 = 7\lambda_3 \end{array}\right\}$ MEMP. SOLUSI NON TRIVIAL

$\longrightarrow \vec{a}, \vec{b}$ & $\vec{c}$ TIDAK BEBAS LINEAR

$\vec{a}, \vec{b}$ & $\vec{c}$ BEBAS LINEAR

(TIDAK BERGANTUNG LINEAR)

$$\lambda_1 . \vec{a} + \lambda_2 . \vec{b} + \lambda_3 . \vec{c} = \vec{0}$$

$\lambda_1 = \lambda_2 = \lambda_3 = 0$ (MEMPUNYAI SOLUSI TRIVIAL)

→ $\vec{a}, \vec{b}$ & $\vec{c}$ BEBAS LINEAR

$$A . \vec{x} = B$$

$$\boxed{|A| \neq 0}$$

→ $\vec{a}, \vec{b}$ & $\vec{c}$ BEBAS LINEAR

$, \vec{b}$ & $\vec{c}$ TIDAK BEBAS LINEAR

(BERGANTUNG LINEAR)

$\lambda_1 . \vec{a} + \lambda_2 . \vec{b} + \lambda_3 . \vec{c} = \vec{0}$

$\left.\begin{aligned} \lambda_1 &= 2\lambda_3 \\ \lambda_2 &= 7\lambda_3 \end{aligned}\right\}$ MEMP. SOLUSI NON TRIVIAL

$\rightarrow \vec{a}, \vec{b}$ & $\vec{c}$ TIDAK BEBAS LINEAR

$A . \vec{x} = B$

$|A| = 0 \rightarrow \vec{a}, \vec{b}$ & $\vec{c}$ TIDAK BEBAS LINEAR

... $\vec{a}$ & $\vec{b}$ BEBAS / TIDAK BEBAS LINEAR ?

$$\vec{a} = (1,3)$$
$$\vec{b} = (2,6) = 2.(1,3) = 2.\vec{a}$$

I $\longrightarrow$ $\vec{a}$ & $\vec{b}$ TIDAK BEBAS LINEAR.

$$\lambda_1 \vec{a} + \lambda_2 \vec{b} = \vec{0}$$

II
$$\lambda_1 \begin{bmatrix} 1 \\ 3 \end{bmatrix} + \lambda_2 \begin{bmatrix} 2 \\ 6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\lambda_1 + 2\lambda_2 = 0 \longrightarrow \lambda_1 = -2\lambda_2$$
$$3\lambda_1 + 6\lambda_2 = 0 \quad :3$$
$$\lambda_1 + 2\lambda_2 = 0 \longrightarrow \lambda_1 = -2\lambda_2$$

$$\left.\begin{array}{l} \lambda_1 = -2\lambda_2 \\ \lambda_2 = \lambda_2 \end{array}\right\} \longrightarrow \text{MEMP. SOLUSI NONTRIVIAL}$$

$\longrightarrow$ $\vec{a}$ & $\vec{b}$ TIDAK BEBAS LINEAR

III) $\lambda_1 + 2$ ...

$3\lambda_1 +$ ...

$\begin{bmatrix} 1 \\ 3 \end{bmatrix}$ ...

[A ...

AR.

III) $\lambda_1 + 2\lambda_2 = 0$

$3\lambda_1 + 6\lambda_2 = 0$

$$\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$A \cdot \vec{x} = B$$

$$|A| = \begin{vmatrix} 1 & 2 \\ 3 & 6 \end{vmatrix} = 6 - 6 = 0$$

$\rightarrow$ $\vec{a}$ & $\vec{b}$ TIDAK BEBAS LINEAR