$$\frac{d^4y}{dx^4} + \frac{dy}{dx} + y = \underline{\cos 2x} + \underline{3} \rightarrow \text{So}$$

$$D^4y + Dy + y = \cos 2x + 3$$

$$\boxed{(D^4 + D + 1)\cdot y = \underbrace{\cos 2x}_{\mathrm{I}} + \underbrace{3}_{\mathrm{II}}}$$

I $(D^4 + D + 1)\cdot y = \cos 2x \rightarrow a = 2$

$$D^2 = -a^2$$

$$D^2 = -(2)^2$$

$$\boxed{D^2 = -4}$$

$$y_{k_1} = \frac{\cos 2x}{D^4 + D + 1}$$

$$= \frac{\cos 2x}{(D^2)^2 + D + 1} = \frac{\cos 2x}{\underbrace{(-4)^2}_{16} + D + 1}$$

$$= \frac{\cos 2x}{D + 17} \cdot \frac{D - 17}{D - 17}$$

$$= \frac{D(\cos 2x) - 17\cos 2x}{D^2 - (17)^2}$$

y_{k}

→ SOLUSI KHUSUS PD

$$Y_{k_1} = \frac{-(2)\cdot \sin 2x - 17\cos 2x}{-4-289} = \frac{-2\sin 2x - 17\cos 2x}{-293}$$

$$\boxed{Y_{k_1} = \frac{2}{293}\sin 2x + \frac{17}{293}\cos 2x}$$

(II) $(D^4 + D + 1)\cdot y = 3$

$$Y_{k_2} = \frac{3}{D^4 + D + 1}$$

$$Y_{k_2} = \left(\frac{1}{1 + D + D^4}\right)(3)$$

$$= (1 - D)(3)$$

$$= 3 - D(3)$$

$$= 3 - 0$$

$$\boxed{Y_{k_2} = 3}$$

$\boxed{D(3) = 0}$

$D^2(3) = 0$

$$\begin{array}{r|l} & 1 - D \\ \hline 1 + D + D^4 & 1 \\ & 1 + D \quad + D^4 \\ \hline & -D \quad - D^4 \\ & -D - D^2 \quad - D^4 \\ \hline \end{array}$$

SOLUSI KHUSUS PD

$$y_K = y_{K_1} + y_{K_2}$$

$$y_K = \frac{2}{293}\sin 2x + \frac{17}{293}\cos 2x + 3$$

$$f(x) = \underline{\cos(ax+b)}$$

$$D[\cos(ax+b)] = -a \cdot \sin(ax+b)$$

$$\underbrace{D^2[\cos(ax+b)] = -a^2}_{D^2 = -a^2} \cdot [\cos(ax+b)]$$

OPERATOR INVERS

$$\boxed{y = \frac{1}{D-\alpha} \cdot f(x)}$$

$$= e^{\alpha x} \int e^{-\alpha x} f(x)\, dx$$

$$\boxed{\frac{1}{D-\alpha} \cdot f(x) = e^{\alpha x} \int e^{-\alpha x} \cdot f(x)\, dx}$$

$$\underline{f(x) = \cos(ax+b)}$$

$$D[\cos(ax+b)] = -a \cdot \sin(ax+b)$$

$$D^2[\cos(ax+b)] = -a^2 \cdot [\cos(ax+b)]$$

$$D^2 = -a^2$$

OPERATOR INVERS

$$\boxed{y = \frac{1}{D-\alpha} \cdot f(x)}$$

$$= e^{\alpha x} \int e^{-\alpha x} f(x)\, dx$$

$$\boxed{\frac{1}{D-\alpha} \cdot f(x) = e^{\alpha x} \int e^{-\alpha x} \cdot f(x)\, dx}$$

$$\boxed{y = \frac{1}{D-\alpha} \cdot f(x)}$$

$$(D-\alpha) \cdot y = f(x)$$

$$Dy - \alpha \cdot y = f(x)$$

$$\frac{dy}{dx} - \alpha \cdot y = f(x)$$

$$\frac{dy}{dx} + P(x) \cdot y = Q(x)$$

PD. LINEAR

$$P(x) = -\alpha$$

$$Q(x) = f(x)$$

$$S(x) = e^{\int P(x)\,dx}$$

$$= e^{\int (-\alpha)\,dx} = e^{-\alpha x}$$

SOLUSI UMUM PD

$$y = \frac{1}{S(x)} \left[\int S(x) \cdot Q(x)\,dx + C \right]$$

$$y = \frac{1}{e^{-\alpha x}} \left[\int e^{-\alpha x} \cdot f(x)\,dx + C \right]$$

$$\boxed{C = 0}$$

$$\rightarrow y = e^{\alpha x} \left[\int e^{-\alpha x} \cdot f(x)\,dx + 0 \right]$$

$$\boxed{\frac{1}{D-\alpha} \cdot f(x) = e^{\alpha x} \int e^{-\alpha x} f(x)\,dx}$$

$$y = \frac{1}{(D-\alpha_1)(D-\alpha_2)(D-\alpha_3)} \cdot f(x)$$

<u>CONTOH</u>:

$$y = \frac{1}{(D-1)(D-2)(D-3)} \cdot e^{6x}$$

$$y = \frac{1}{D-1} \cdot \frac{1}{D-2} \cdot \left[\frac{1}{D-3} \cdot e^{6x}\right]$$

$$y = \frac{1}{D-1} \cdot \frac{1}{D-2} \cdot e^{3x} \int e^{-3x} \left(e^{6x}\right) dx$$

$$y = \frac{1}{D-1} \cdot \frac{1}{D-2} \cdot e^{3x} \int e^{3x}\, dx$$

$$y = \frac{1}{D-1} \cdot \frac{1}{D-2} \cdot e^{3x} \cdot \left(\frac{1}{3}\right) \int e^{3x} \cdot d(3x) \qquad \frac{3\,dx}{d(3x)}$$

$$y = \frac{1}{D-1} \cdot \frac{1}{D-2} \cdot \left(e^{3x} \cdot \frac{1}{3} \cdot e^{3x}\right)$$

$$y = \frac{1}{D-1}\cdot\left[\frac{1}{D-2}\cdot\overbrace{\left(\frac{1}{3}e^{6x}\right)}^{f(x)}\right]$$

$$y = \frac{1}{D-1}\cdot e^{2x}\int e^{-2x}\left(\frac{1}{3}e^{6x}\right)dx$$

$$= \left(\frac{1}{3}\right)\cdot\frac{1}{D-1}\cdot e^{2x}\int e^{4x}\,dx$$

$$= \left(\frac{1}{3}\right)\cdot\frac{1}{D-1}\cdot e^{2x}\left(\frac{1}{4}\right)\int e^{4x}\cdot\overset{4dx}{d(4x)}$$

$$= \frac{1}{12}\cdot\frac{1}{D-1}\cdot e^{2x}\left(e^{4x}\right)$$

$$= \frac{1}{12}\cdot\left[\frac{1}{D-1}\cdot e^{6x}\right]$$

$$= \frac{1}{12}\cdot e^{x}\int e^{-x}\left(e^{6x}\right)dx$$

$$= \frac{1}{12}\cdot e^{x}\left(\frac{1}{5}\right)\int e^{5x}\cdot\overset{5dx}{d(5x)}$$

$$= \frac{1}{60}e^{x}\cdot\left(e^{5x}\right) = \frac{1}{60}e^{6x}$$

1) $f(x) = e^{ax}$ $\boxed{D = a}$

1 AKAR SAMA

$$Y_K = \frac{1}{\underbrace{(D-1)}_{g(D)}(D-a)} \cdot e^{ax}$$

$$Y_K = \frac{1}{g(D)} \left[\frac{1}{D-a} \cdot e^{ax} \right]$$

$$= \frac{1}{g(D)} \cdot e^{ax} \int e^{-ax}\left(e^{ax}\right) dx$$

$$Y_K = \frac{1}{g(D)} \cdot e^{ax} \cdot x$$

$$= \frac{1}{g(D)} \cdot \left(x \cdot e^{ax}\right)$$

2) $f(x) = e^{ax} \quad D = a. \longrightarrow$ <u>2 AKAR</u> SAMA

$$Y_K = \frac{1}{(D-2)\,(D-a)^2} \cdot e^{ax}$$

$$= \underbrace{\frac{1}{D-2}}_{g(D)} \cdot \left[\frac{1}{(D-a)^2} \cdot e^{ax}\right]$$

$$Y_K = \frac{1}{g(D)} \cdot \frac{1}{D-a}\left[\frac{1}{D-a} \cdot e^{ax}\right]$$

$$= \frac{1}{g(D)} \cdot \frac{1}{D-a} \cdot e^{ax} \int \underbrace{e^{-ax}\left(e^{ax}\right)}_{1} dx$$

$$= \frac{1}{g(D)} \cdot \left[\frac{1}{D-a} \cdot \left(x e^{ax}\right)\right]$$

$$= \frac{1}{g(D)} \cdot e^{ax} \int e^{-ax}\left(x e^{ax}\right) dx$$

$$= \frac{1}{g(D)} \cdot e^{ax} \cdot \int x\,dx = \frac{1}{g(D)} \cdot e^{ax} \cdot \left(\frac{1}{2}x^2\right)$$

$$= \frac{1}{g(D)} \cdot \left(\frac{x^2}{2} e^{ax}\right)$$

$$\frac{1}{D-a}\cdot e^{ax} = x\,e^{ax} = \frac{x}{1!}e^{ax}$$

$$\frac{1}{(D-a)^2}\cdot e^{ax} = \frac{x^2}{2}e^{ax} = \frac{x^2}{2!}e^{ax}$$

$$\boxed{\frac{1}{(D-a)^n} = \frac{x^n}{n!}e^{ax}}$$

$$\frac{1}{D^2+a^2} \cdot \sin ax$$

$$= \frac{1}{D^2 - i^2 a^2} \cdot \sin ax$$

$$= \frac{1}{(D+ia)(D-ia)} \cdot I_m\left(e^{iax}\right)$$

$$= I_m\left[\frac{1}{(D+ia)(D-ia)} \cdot e^{\overset{K}{ia}x}\right]$$

$$= I_m\left[\frac{1}{ia+ia} \cdot \left\{\frac{1}{D-ia} \cdot e^{iax}\right\}\right]$$

$$= I_m\left[\frac{1}{2ia} \cdot \frac{i}{i}\left(\frac{x}{1!} e^{iax}\right)\right]$$

$$= I_m\left[\frac{i}{2\underset{-1}{i^2}a} \cdot \left(x e^{iax}\right)\right]$$

$$\boxed{i^2 = -1}$$

$$\boxed{e^{i\theta} = \cos\theta + i \cdot \sin\theta}$$

RUMUS EULAR

$$e^{iax} = \cos ax + i \sin ax$$

$$Re\left(e^{iax}\right) = \cos ax$$

$$I_m\left(e^{iax}\right) = \sin ax$$

$$D = K = ia$$

$$z = a + ib$$

$$\mathrm{Re}(z) = a$$

$$\mathrm{Im}(z) = b$$

$$\frac{1}{D^2+a^2}\sin ax = \mathrm{Im}\left[-\frac{i}{2a}\left(x e^{iax}\right)\right]$$

$$= \mathrm{Im}\left[\frac{-ix}{2a}\cdot e^{iax}\right]$$

$$= \mathrm{Im}\left[\frac{-ix}{2a}\left(\cos ax + i\,\cdot\right.\right.$$

$$= \mathrm{Im}\left[-i\,\frac{x}{2a}\cos ax + \frac{x}{2a}\right.$$

$$\frac{1}{D^2+a^2}\cdot\sin ax = -\frac{x}{2a}\cos ax$$

$$Z = a + i b$$

$$Re(z) = a$$

$$Im(z) = b$$

$$\frac{1}{D^2+a^2} \sin ax = Im\left[-\frac{i}{2a}\left(x e^{iax}\right)\right]$$

$$= Im\left[\frac{-ix}{2a} \cdot e^{iax}\right]$$

$$= Im\left[\frac{-ix}{2a}\left(\cos ax + i \cdot \sin ax\right)\right]$$

$$= Im\left[-i \frac{x}{2a} \cos ax + \frac{x}{2a} \sin ax\right]$$

$$\frac{1}{D^2+a^2} \cdot \sin ax = -\frac{x}{2a} \cos ax$$

$$\frac{1}{D^2+a^2}\cdot \cos ax$$

$$= \frac{1}{D^2-i^2a^2}\cdot \cos ax$$

$$= \frac{1}{(D+ia)(D-ia)}\cdot Re\left(e^{iax}\right)$$

$$= Re\left[\frac{1}{(D+ia)(D-ia)}\cdot e^{\overset{K}{iax}}\right]$$

$$= Re\left[\frac{1}{ia+ia}\cdot\left\{\frac{1}{D-ia}\cdot e^{iax}\right\}\right]$$

$$= Re\left[\frac{1}{2ia}\cdot\frac{i}{i}\left(\frac{x}{1!}e^{iax}\right)\right]$$

$$= Re\left[\frac{i}{2\underbrace{i^2}_{-1}a}\cdot\left(x e^{iax}\right)\right]$$

$$\boxed{i^2=-1}$$

$$\boxed{e^{i\theta} = \cos\theta + i.\sin\theta}$$

RUMUS EULAR

$$e^{iax} = \cos ax + i \sin ax$$

$$Re\left(e^{iax}\right) = \cos ax$$

$$Im\left(e^{iax}\right) = \sin ax$$

$$D = K = ia$$

$$Z = a + i b$$

$$Re(z) = a$$

$$Im(z) = b$$

$$\frac{1}{D^2+a^2}\cos ax = Re\left[-\frac{i}{2a}\left(x e^{iax}\right)\right]$$

$$= Re\left[\frac{-ix}{2a}\cdot e^{iax}\right]$$

$$= Re\left[\frac{-ix}{2a}\left(\cos ax + i\cdot\sin ax\right)\right]$$

$$= Re\left[-i\frac{x}{2a}\cos ax + \frac{x}{2a}\sin ax\right]$$

$$\frac{1}{D^2+a^2}\cdot\cos ax = \frac{x}{2a}\sin ax$$

$$f(x) = e^{ax} \cdot F(x) \to \begin{cases} \cos bx \\ \sin bx \\ ax^2+bx+c \end{cases}$$

$$\boxed{\frac{1}{f(D)} \cdot \left[e^{ax} \cdot F(x)\right] = e^{ax} \cdot \left[\frac{1}{f(D+a)} \cdot F(x)\right]}$$

$$D\left[e^{ax}F\right] = D(e^{ax}) \cdot F + e^{ax}(DF)$$
$$= a \cdot e^{ax} \cdot F + e^{ax} D F$$
$$= e^{ax}(a+D)F$$
$$D\left[e^{ax}F\right] = e^{ax}(D+a) \cdot F$$

$$D^2\left[e^{ax}F\right] = D(e^{ax}) \cdot (D+a) \cdot F + e^{ax} D\left[(D+a) \cdot F\right]$$
$$= a \cdot e^{ax} \cdot (D+a) \cdot F + e^{ax} D \cdot (D+a) F$$
$$= e^{ax}(D+a) F \cdot (a+D)$$
$$= e^{ax}(D+a)^2 F$$

ANALOG

$$\boxed{D^n\left[e^{ax}\cdot F\right] = e^{ax}\cdot(D+a)^n\cdot F}$$

JIKA $D^n = f(D)$

$\longrightarrow (D+a)^n = f(D+a)$

$$\boxed{f(D)\cdot\left[e^{ax}\cdot F\right] = e^{ax}\, f(D+a)\cdot F}$$

$$f(D)\cdot\left[e^{ax}\cdot F_1\right] = e^{ax}\,\underbrace{f(D+a)\cdot F_1}_{F}$$

MISALKAN: $\boxed{f(D+a)\cdot F_1 = F}$

$$F_1 = \frac{1}{f(D+a)}\cdot F$$

$$f(D)\cdot\left[e^{ax}\cdot F_1\right] = e^{ax}\cdot \overbrace{f(D+a)\cdot F_1}^{F}$$

$$f(D)\cdot e^{ax}\cdot\frac{1}{f(D+a)}\cdot F = e^{ax}\cdot F$$

$$e^{ax}\cdot\frac{1}{f(D+a)}\cdot F = \frac{1}{f(D)}\cdot\left[e^{ax}F\right]$$

$$\rightarrow\ \frac{1}{f(D)}\cdot\left[e^{ax}\cdot F\right] = e^{ax}\cdot\left[\frac{1}{f(D+a)}\cdot F\right]$$

```
> dsolve(ode2);
```

$$y(x) = \left(e^x \left(-1 + \_C1 + x\right) + \_C2\right) e^x \quad (200)$$

```
> restart:
> ode2 := diff(y(x),x,x)-4*diff(y(x),x)+4*y(x)=(x^2)*exp
  (3*x);
```

$$ode2 := \frac{d^2}{dx^2} y(x) - 4 \left(\frac{d}{dx} y(x)\right) + 4\, y(x) = x^2\, e^{(3\,x)} \quad (20$$

```
odeadvisor(ode2);
```

$$odeadvisor\left(\frac{d^2}{dx^2} y(x) - 4 \left(\frac{d}{dx} y(x)\right) + 4\, y(x) = x^2\, e^{(3\,x)}\right) \quad (2$$

```
dsolve(ode2);'
```

$$y(x) = e^{(2\,x)}\ \_C2 + e^{(2\,x)}\, x\ \_C1 + \left(6 - 4\,x + x^2\right) e^{(3\,x)}$$

ning, premature end of input, use <Shift> + <Enter

$$\boxed{\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = x^2 e^{3x}}$$

$$D^2y - 4Dy + 4y = e^{3x} \cdot x^2$$

$$\boxed{(D^2 - 4D + 4) \cdot y = e^{3x} \cdot x^2}$$

I) $(D^2 - 4D + 4) \cdot y = 0$

PERS. EIGEN

$$\alpha^2 - 4\alpha + 4 = 0$$

$$(\alpha - 2)^2 = 0$$

$$\alpha_1 = \alpha_2 = 2$$

$$y_h = (c_1 + c_2 x) e^{\alpha x}$$

$$y_h = (c_1 + c_2 x) e^{2x}$$

$$(D^2-4D+4)\cdot y = e^{3x}\cdot x^2$$

$$Y_K = \frac{1}{D^2-4D+4}\cdot\left(e^{3x}\cdot x^2\right)$$

$a=3$

$D+a = D+3$

$$= e^{3x}\cdot\left[\frac{1}{(D+3)^2-4(D+3)+4}\cdot(x^2)\right]$$

$$= e^{3x}\cdot\left[\frac{1}{D^2+6D+9-4D-12+4}(x^2)\right]$$

$$= e^{3x}\left[\frac{1}{D^2+2D+1}(x^2)\right]$$

$$= e^{3x}\left[\frac{1}{1+2D+D^2}(x^2)\right]$$

$$= e^{3x}\left[(1-2D+3D^2)(x^2)\right]$$

$$= e^{3x}\left[x^2-2D(x^2)+3D^2(x^2)\right]$$

$$= e^{3x}\left[x^2-2(2x)+3(2)\right] = e^{3x}(x^2-4x+6)$$

$$\boxed{\begin{array}{l} D(x^2) = 2x \\ D^2(x^2) = 2 \end{array}}$$

$$D^3(x^2) = 0$$

$$
\begin{array}{r|l}
 & 1 - 2D + 3D^2 \\
\hline
1 + 2D + D^2 & 1 \\
 & \underline{1 + 2D + D^2} \; - \\
 & -2D - D^2 \\
 & \underline{-2D - 4D^2 - 2D^3} \; - \\
 & 3D^2 + 2D^3
\end{array}
$$

Solusi umum PD

$$y = y_h + y_k$$

$$y = (c_1 + c_2 x)\, e^{2x} + e^{3x}(x^2 - 4x + 6)$$

```
>
Warning, premature end of input, use <Shift> + <Enter> to
avoid this message.
> restart:
> ode2 := diff(y(x),x,x)-4*diff(y(x),x)+4*y(x)=exp(3*x)*
sin(5*x);
```

$$ode2 := \frac{d^2}{dx^2}y(x) - 4\left(\frac{d}{dx}y(x)\right) + 4\,y(x) = e^{(3\,x)}\sin(5\,x) \qquad (203)$$

```
> odeadvisor(ode2);
```

$$odeadvisor\left(\frac{d^2}{dx^2}y(x) - 4\left(\frac{d}{dx}y(x)\right) + 4\,y(x) = e^{(3\,x)}\sin(5\,x)\right) \qquad (204)$$

```
> dsolve(ode2);'
>
```

$$y(x) = e^{(2\,x)}\,\_C2 + e^{(2\,x)}\,x\,\_C1 - \frac{1}{338}e^{(3\,x)}\,(5\cos(5\,x) + 12\sin(5\,x))$$

```
>
```

$$\boxed{\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = e^{3x} \cdot \sin 5x}$$

$$D^2y - 4Dy + 4y = e^{3x} \cdot \sin 5x$$

$$\boxed{(D^2 - 4D + 4) \cdot y = e^{3x} \cdot \sin 5x}$$

I) $(D^2 - 4D + 4) \cdot y = 0$

PERS. EIGEN

$$\alpha^2 - 4\alpha + 4 = 0$$

$$(\alpha - 2)^2 = 0$$

$$\alpha_1 = \alpha_2 = 2$$

$$y_h = (c_1 + c_2 x) e^{\alpha x}$$

$$y_h = (c_1 + c_2 x) e^{2x}$$

$$(D^2-4D+4)\cdot y = e^{3x}\cdot \sin 5x$$

$$y_K = \frac{1}{D^2-4D+4}\cdot(e^{3x}\cdot \sin 5x)$$

$a=3$

$\boxed{D+a = D+3}$

$$= e^{3x}\cdot\left[\frac{1}{(D+3)^2-4(D+3)+4}\cdot(\sin 5x)\right]$$

$$= e^{3x}\cdot\left[\frac{1}{D^2+6D+9-4D-12+4}(\sin 5x)\right]$$

$$= e^{3x}\left[\frac{1}{D^2+2D+1}(\sin 5x)\right]$$

$a=5$

$D^2=-a^2$

$D^2=-(5)^2$

$D^2=-25$

$$= e^{3x}\left[\frac{1}{-25+2D+1}(\sin 5x)\right]$$

$$= e^{3x}\left(\frac{1}{2D-24}(\sin 5x)\right)$$

$$Y_K = e^{3x}\left[\frac{1}{2(D-12)}(\sin 5x)\right]$$

$$= \frac{1}{2}e^{3x}\left[\frac{\sin 5x}{D-12}\cdot\frac{D+12}{D+12}\right]$$

$$= \frac{1}{2}e^{3x}\left[\frac{D(\sin 5x) + 12\sin 5x}{D^2-(12)^2}\right]$$

$$= \frac{1}{2}e^{3x}\left[\frac{5.\cos 5x + 12\sin 5x}{-25-144}\right]$$

$$= \frac{1}{2}e^{3x}\left[\frac{5\cos 5x + 12\sin 5x}{-169}\right]$$

$$Y_K = -\frac{1}{338}e^{3x}(5\cos 5x + 12\sin 5x)$$

SOL. UM. PD : $Y = Y_h + Y_K$

$$Y = (C_1 + C_2x)e^{2x} - \frac{1}{338}e^{3x}(5\cos 5x + 12\sin 5x)$$

OPERATOR INVERS THD. $\boxed{x \cdot F(x)}$

$$\frac{1}{f(D)}[x \cdot F] = x \cdot \frac{1}{f(D)} \cdot F - \frac{f'(D)}{[f(D)]^2} \cdot F$$

MISALKAN : $\boxed{y = x \cdot F}$

$$Dy = D(xF)$$

$$Dy = D(x) \cdot F + x\, D(F)$$

$$Dy = F + x\, DF$$

$$Dy = x\, DF + F$$

$$\boxed{Dy = x\, DF + \left(\frac{d}{dD} D\right) F}$$

$$Dy = F + xDF$$

$$\frac{d}{dx}(x) = \frac{dx}{dx} = 1$$

$$\frac{d}{dD}(D) = \frac{dD}{dD} = 1$$

$$D(Dy) = D(F + xDF)$$

$$D^2y = DF + D(x.DF)$$

$$D^2y = DF + \frac{D(x)}{1}.DF + x.\underline{D(DF)}$$

$$D^2y = \underline{DF + DF} + x.D^2F$$

$$\underline{D^2y = 2DF + x.D^2F}$$

$$D^2y = x.D^2F + 2DF$$

$$\boxed{D^2y = x.D^2F + \left(\frac{d}{dD}D^2\right)F}$$

$$\boxed{D^n y = x \cdot D^n F + \left(\frac{d}{dD} D^n\right) F}$$

MIS: $\boxed{D^n = f(D)}$

$$\left(\frac{d}{dD} D^n\right) F = f'(D) \cdot F$$

$$f(D) \cdot y = x \cdot f(D) \cdot F + f'(D) \cdot F$$

$\boxed{y = xF} \longrightarrow \boxed{f(D) \cdot xF = x \cdot f(D) \cdot F + f'(D) F}$

$$f(D) \cdot [x F_1] = x \cdot \underbrace{f(D) \cdot F_1}_{F} + f'(D) \cdot F_1$$

MIS: $f(D) \cdot F_1 = F$

$$\boxed{F_1 = \frac{1}{f(D)} \cdot F}$$

$$f(D)\, x \cdot \frac{1}{f(D)} \cdot F = [x \cdot F] + f'(D) \cdot \frac{1}{f(D)} \cdot F$$

$$\frac{1}{f(D)}\, \cancel{f(D)}\, x \cdot \frac{1}{f(D)} \cdot F = \frac{1}{f(D)} [x \cdot F] + \frac{f'(D)}{[f(D)]^2} \cdot F$$

$$x \cdot \frac{1}{f(D)} \cdot F - \frac{f'(D)}{[f(D)]^2} \cdot F = \frac{1}{f(D)} [xF]$$

$$\boxed{x = \frac{1}{f(D)} [xF] = x \cdot \frac{1}{f(D)} \cdot F - \frac{f'(D)}{[f(D)]^2} F}$$

$$+ 12\sin(5x))$$

```
> restart:
> ode2 := diff(y(x),x,x)-4*diff(y(x),x)+4*y(x)=x*exp(3
;
```

$$ode2 := \frac{d^2}{dx^2}y(x) - 4\left(\frac{d}{dx}y(x)\right) + 4\,y(x) = x\,e^{(3x)}$$

```
odeadvisor(ode2);
```

$$odeadvisor\left(\frac{d^2}{dx^2}y(x) - 4\left(\frac{d}{dx}y(x)\right) + 4\,y(x) = x\,e^{(3x)}\right)$$

```
dsolve(ode2);'
```

$$y(x) = e^{(2x)}\,\_C2 + e^{(2x)}\,x\,\_C1 + (-2 + x)\,e^{(3x)}$$

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = x \cdot e^{3x}$$

$$D^2y - 4Dy + 4y = x \cdot e^{3x}$$

$$\boxed{(D^2 - 4D + 4)y = xe^{3x}}$$

$$(D^2 - 4D + 4)y = 0$$

PERS. EIGEN

$$\alpha^2 - 4\alpha + 4 = 0$$

$$(\alpha - 2)^2 = 0$$

$$\alpha_1 = \alpha_2 = 2$$

$$y_h = (c_1 + c_2 x)e^{\alpha x}$$

$$\boxed{y_h = (c_1 + c_2 x)e^{2x}}$$

I) $(D^2 - 4D + 4)\,y = x \cdot e^{3x}$

$f(D) = D^2 - 4D + 4$

$f'(D) = 2D - 4$

$$y_K = \frac{1}{D^2 - 4D + 4} \cdot (x \cdot e^{3x})$$

$$= x \cdot \frac{1}{f(D)} \cdot F - \frac{f'(D)}{[f(D)]^2} \cdot F$$

$$= x \left\{ \frac{1}{D^2 - 4D + 4} (e^{3x}) \right\} - \frac{2D - 4}{(D^2 - 4D + 4)^2} \cdot e^{3x}$$

$k = 3$

$D = k = 3$

$$= x \left\{ \frac{1}{3^2 - 4(3) + 4} e^{3x} \right\} - \frac{2(3) - 4}{[3^2 - 4(3) + 4]^2} \cdot e^{3x}$$

$$= x e^{3x} - \frac{2}{1} e^{3x}$$

$$y_K = (x - 2) e^{3x}$$

Sol. um. PD: $y = y_h + y_K$

$$y = (c_1 + c_2 x) e^{2x} + (x - 2) e^{3x}$$