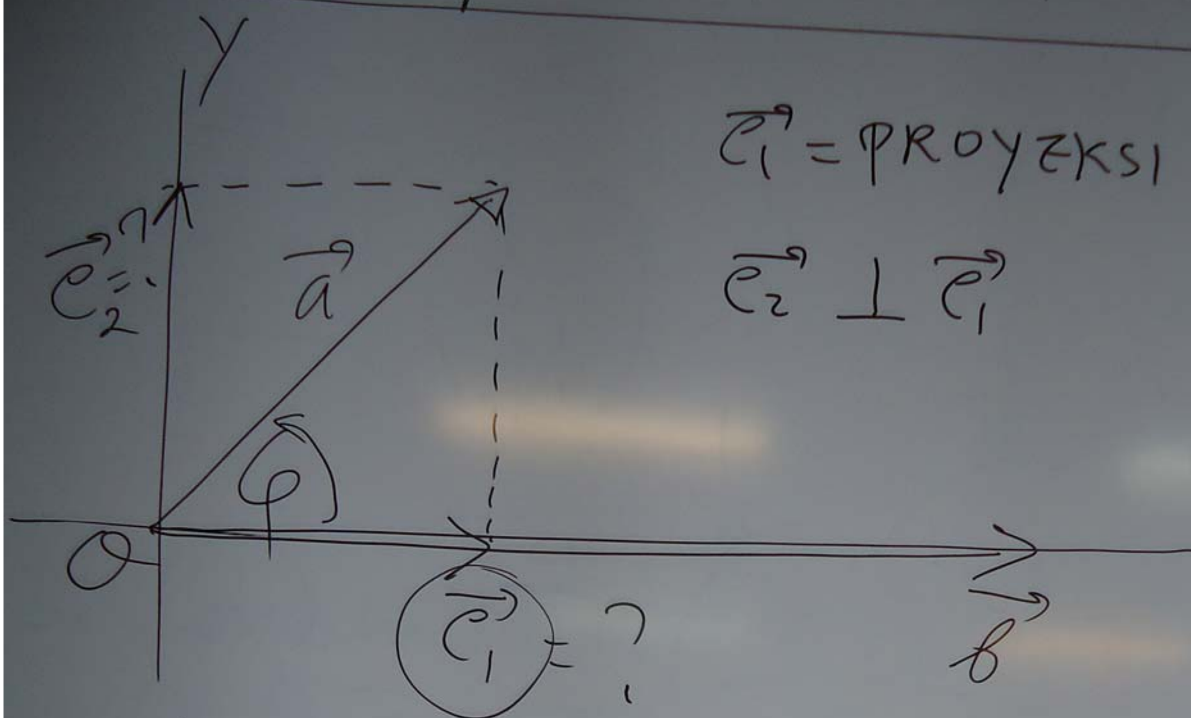
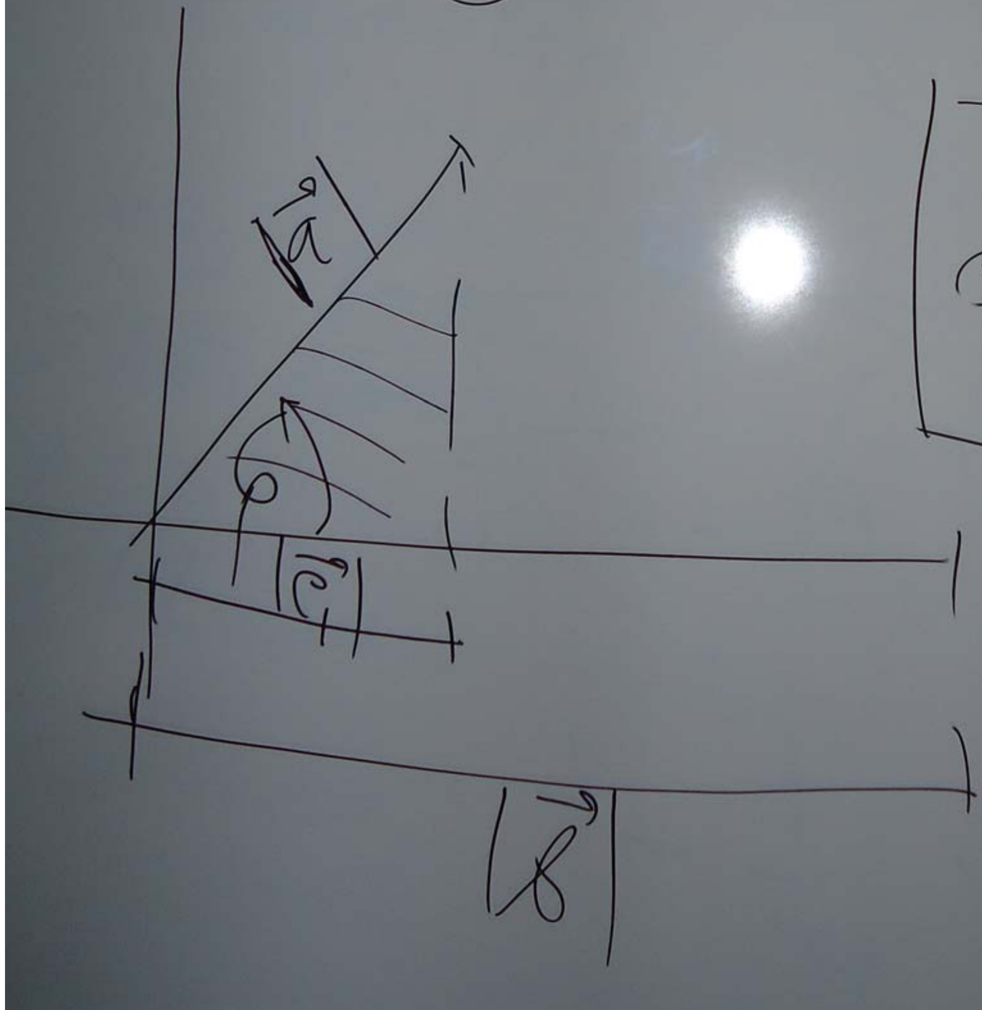
PROYEKSI $\vec{a}$ PADA $\vec{b}$

$\vec{c_1}$ = PROYEKSI $\vec{a}$ PADA $\vec{b}$

$\vec{c_2} \perp \vec{c_1}$





$$\boxed{\cos\varphi = \frac{|\vec{c_1}|}{|\vec{a}|}}$$

$$\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cdot \cos\varphi$$

$$\boxed{\cos\varphi = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|}}$$

GAMBAR RUMUS $(\vec{a} \cdot \vec{b})$

$$\boxed{\cos\varphi = \cos\varphi}$$

$$\frac{|\vec{c}_1|}{\cancel{|\vec{a}|}} = \frac{\vec{a} \cdot \vec{b}}{\cancel{|\vec{a}|} \cdot |\vec{b}|}$$

$$\boxed{|\vec{c}_1| = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}}$$

$\vec{c_1}$ & $\vec{b}$ SEGARIS & SEARAH

$$\boxed{\vec{\mu}_{\vec{c_1}} = \vec{\mu}_{\vec{b}}}$$

$$\frac{\vec{c_1}}{|\vec{c_1}|} = \frac{\vec{b}}{|\vec{b}|}$$

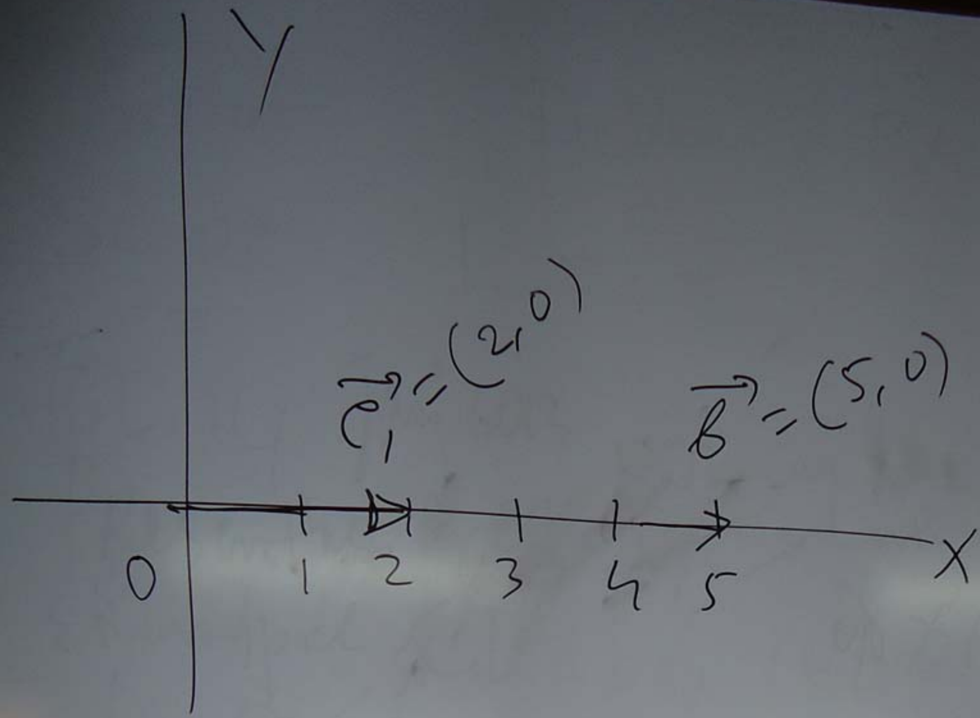
$$\vec{c_1} = |\vec{c_1}| \, \frac{\vec{b}}{|\vec{b}|}$$

$$\vec{c_1} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} \cdot \frac{\vec{b}}{|\vec{b}|}$$

$$\boxed{\vec{c_1} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|^2} \cdot \vec{b}}$$

$$\vec{c_1} + \vec{c_2} = \vec{a}$$

$$\boxed{\vec{c_2} = \vec{a} - \vec{c_1}}$$



$\vec{e_1}$ & $\vec{b}$ Segaris & Searah

$$\vec{e_1} = (2,0) = 2\vec{i} + 0.\vec{j} = 2\vec{i}$$

$$|\vec{e_1}| = \sqrt{(2)^2 + (0)^2} = 2$$

$$\vec{\mu}_{\vec{e_1}} = \frac{\vec{e_1}}{|\vec{e_1}|} = \frac{\cancel{2}\vec{i}}{\cancel{2}} = \vec{i}$$

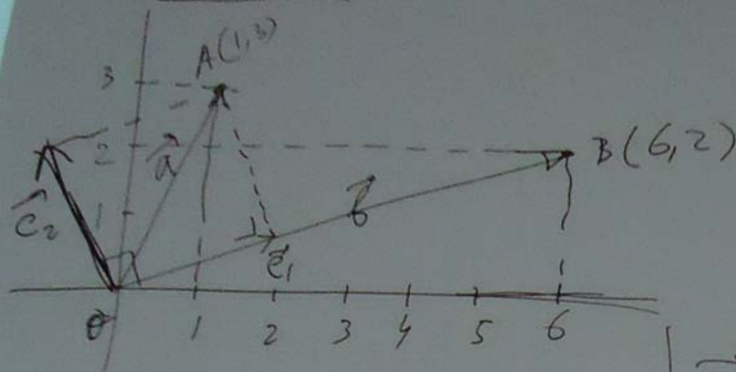
$$\vec{b} = (5,0) = 5\vec{i} + 0\vec{j} = 5\vec{i}$$

$$|\vec{b}| = \sqrt{(5)^2 + (0)^2} = 5$$

$$\vec{\mu}_{\vec{b}} = \frac{\cancel{5}\vec{i}}{\cancel{5}} = \vec{i}$$

$$\longrightarrow \boxed{\vec{\mu}_{\vec{e_1}} = \vec{\mu}_{\vec{b}}}$$

CONTOH



PROYEKSI $\vec{a}$ PADA $\vec{b} = \vec{c_1} = ?$

$\vec{c_2} \perp \vec{c_1} \longrightarrow \vec{c_2} = ?$

$\vec{OA} = \vec{a} = (1,3)$

$\vec{OB} = \vec{b} = (6,2)$

$\vec{a} \cdot \vec{b} = (1,3) \cdot (6,2) = (1)(6) + (3)(2)$
$= 6 + 6 = 12.$

$|\vec{b}| = \sqrt{6^2 + 2^2} = \sqrt{36+4} = \sqrt{40}$

$|\vec{b}|^2 = (\sqrt{40})^2 = 40.$

$$\boxed{\vec{c_1} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|^2} \cdot \vec{b}}$$

$$\vec{c_1} = \frac{\cancel{12}^{3}}{\cancel{40}_{10}} \cdot (6,2)$$

$$= \left(\frac{18}{10}, \frac{6}{10}\right) = \left(\frac{9}{5}, \frac{3}{5}\right)$$

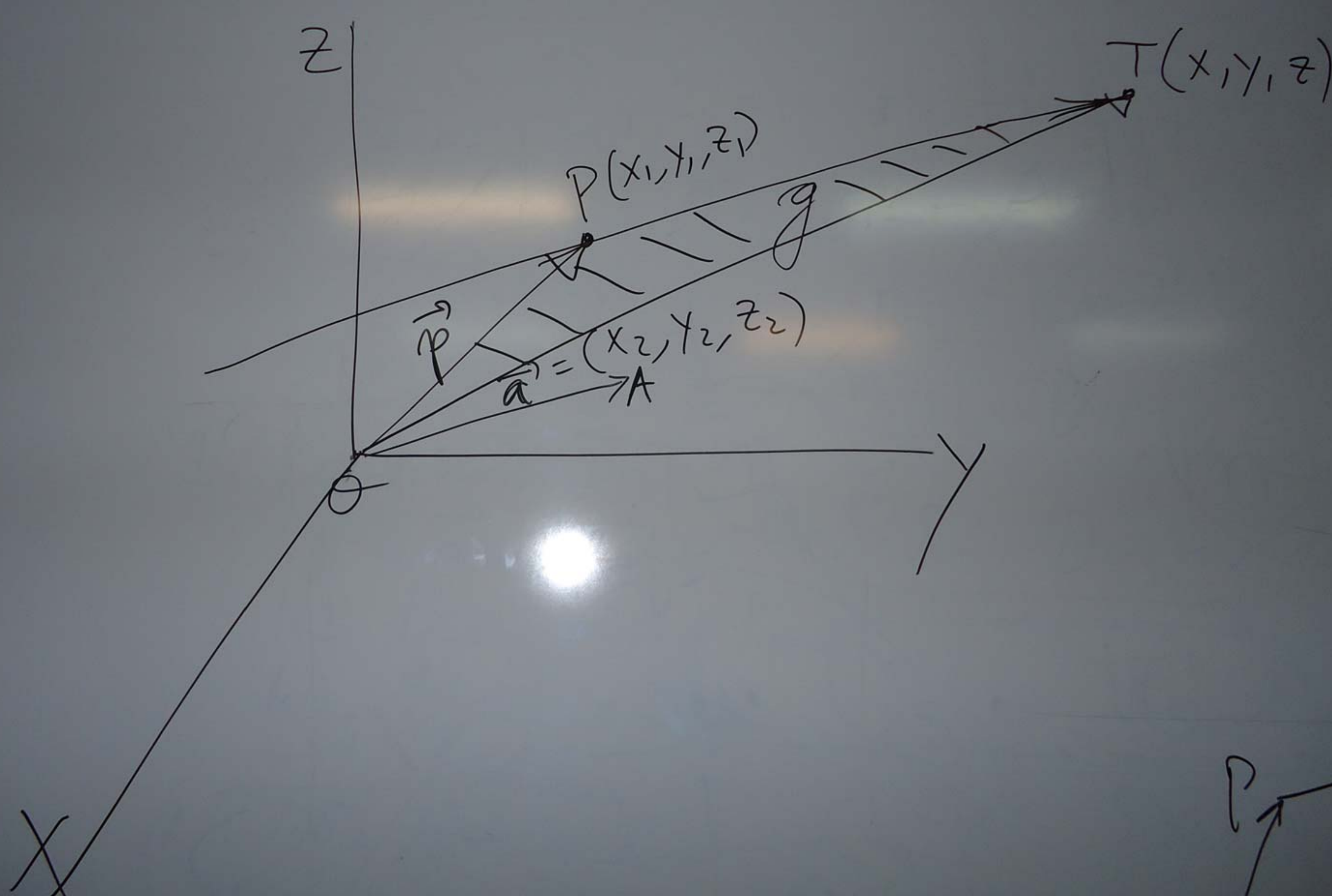
$$\boxed{\vec{c_1} + \vec{c_2} = \vec{a}}$$

$$\vec{c_2} = \vec{a} - \vec{c_1}$$

$$= (1,3) - \left(\frac{9}{5}, \frac{3}{5}\right)$$

$$= \left(-\frac{4}{5}, 2\frac{2}{5}\right)$$

PERS. GARIS LURUS DI R^3



R^3

Pers. garis g melalui $P(x_1, y_1, z_1)$ DAN SEJAJAR $\vec{a} = (x_2, y_2, z_2)$.

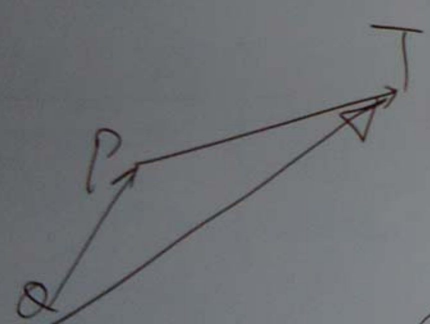
$T(x, y, z)$ $\quad \overrightarrow{OA} = \vec{a} = (x_2, y_2, z_2)$, $\overrightarrow{OP} = \vec{p} = (x_1, y_1, z_1)$

$\overrightarrow{OT} = (x, y, z)$

$\overrightarrow{PT}$ & $\overrightarrow{OA}(\vec{a})$ SEJAJAR DAN SEARAH

$\longrightarrow \overrightarrow{PT} = \lambda . \vec{a}$

PERHATIKAN $\triangle OPT$



$$\overrightarrow{OT} = \overrightarrow{OP} + \overrightarrow{PT}$$

$$(x, y, z) = \vec{p} + \lambda . \vec{a}$$

$$(x, y, z) = (x_1, y_1, z_1) + \lambda . (x_2, y_2, z_2)$$

$$(x, y, z) = (x_1 + \lambda . x_2 , y_1 + \lambda . y_2 , z_1 + \lambda . z_2)$$

PERS VEKTOR DARI GARIS g

$$\boxed{\begin{aligned} x &= x_1 + \lambda . x_2 \\ y &= y_1 + \lambda . y_2 \\ z &= z_1 + \lambda . z_2 \end{aligned}}$$

$\rightarrow x - x_1 = \lambda . x_2 \rightarrow \boxed{\lambda = \frac{x - x_1}{x_2}}$

$\rightarrow y - y_1 = \lambda . y_2 \rightarrow \boxed{\lambda = \frac{y - y_1}{y_2}}$

$\rightarrow z - z_1 = \lambda . z_2 \rightarrow \boxed{\lambda = \frac{z - z_1}{z_2}}$

PERS. PARAMETER DARI GARIS g.

$$\lambda = \lambda = \lambda$$

$$\boxed{\frac{x - x_1}{x_2} = \frac{y - y_1}{y_2} = \frac{z - z_1}{z_2}}$$

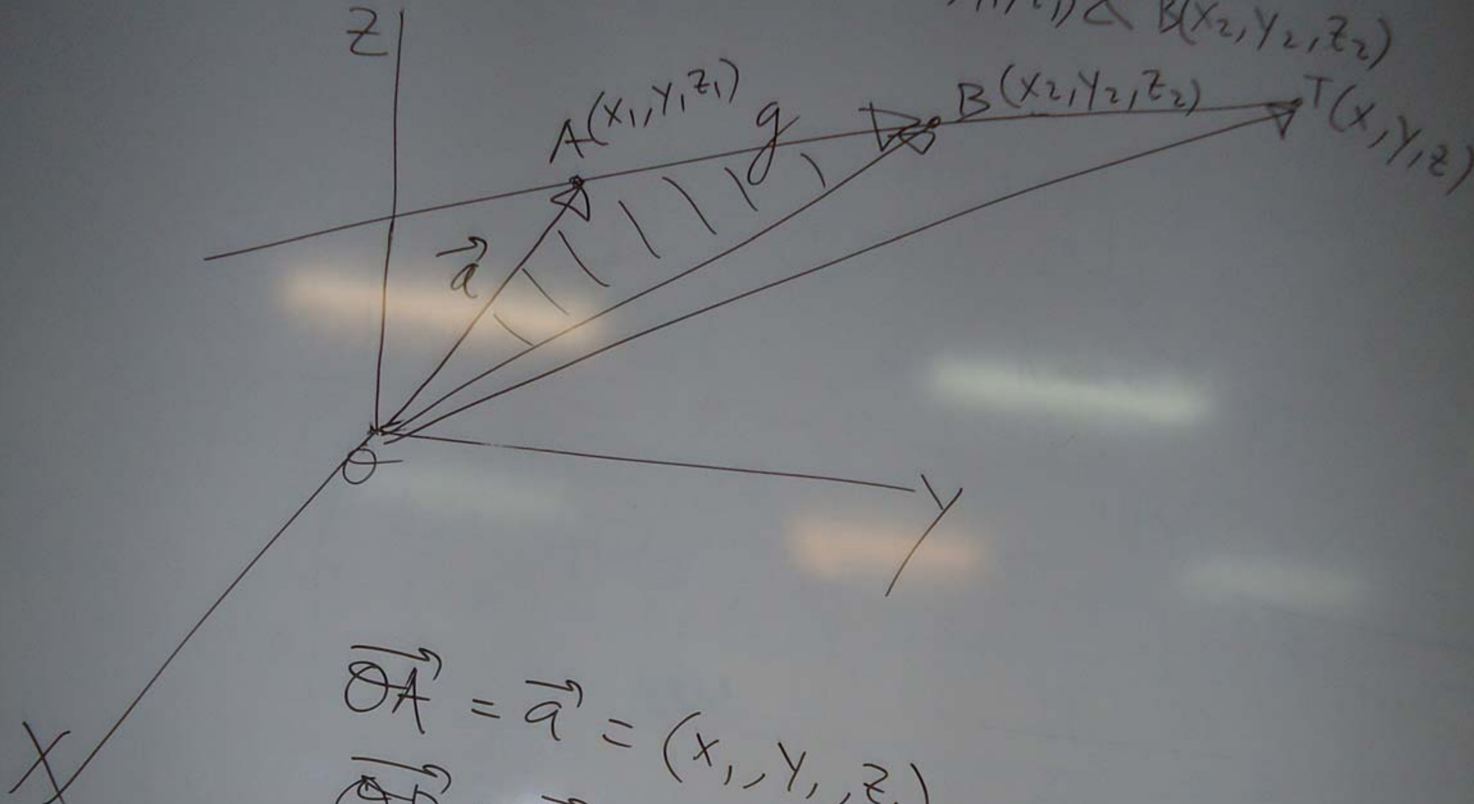
PERS. CARTESIUS DARI garis g

Pers. grs g

melalui $P(x_1, y_1, z_1)$

$// \vec{a} = (x_2, y_2, z_2)$

PERS. GARIS g MELALUI $A(x_1, y_1, z_1)$ & $B(x_2, y_2, z_2)$



$$\overrightarrow{OA} = \vec{a} = (x_1, y_1, z_1)$$

$$\overrightarrow{OB} = \vec{b} = (x_2, y_2, z_2)$$

$$\overrightarrow{OT} = (x, y, z)$$

PERHATIKAN $\triangle OAB$

$$\vec{OA} + \vec{AB} = \vec{OB}$$

$$\vec{a} + \vec{AB} = \vec{b}$$

$$\boxed{\vec{AB} = \vec{b} - \vec{a}}$$

$$\vec{AB} = (x_2, y_2, z_2) - (x_1, y_1, z_1)$$

$$= (x_2 - x_1, y_2 - y_1, z_2 - z_1)$$

$\vec{AT}$ & $\vec{AB}$ SEGARIS & SEARAH

$$\vec{AT} = \lambda . \vec{AB}$$

PERHATIKAN $\triangle OAT$

$$\vec{OT} = \vec{OA} + \vec{AT}$$

$$\boxed{(x, y, z) = \vec{a} + \lambda . \vec{AB}}$$

$$(x, y, z) = (x_1, y_1, z_1) + \lambda (x_2 - x_1, y_2 - y_1, z_2 - z_1)$$

$$(x, y, z) = [x_1 + \lambda(x_2 - x_1), y_1 + \lambda(y_2 - y_1), z_1 +$$

PERS. VEKTOR DARI GARIS g

$$\boxed{\begin{aligned} x &= x_1 + \lambda(x_2 - x_1) \\ y &= y_1 + \lambda(y_2 - y_1) \\ z &= z_1 + \lambda(z_2 - z_1) \end{aligned}}$$

PERS. PARAMETER DARI GARIS g

$$(x, y, z) = \left[x_1 + \lambda(x_2 - x_1),\ y_1 + \lambda(y_2 - y_1),\ z_1 + \lambda(z_2 - z_1)\right]$$

PERS. VEKTOR DARI GARIS g

$$\boxed{\begin{aligned} x &= x_1 + \lambda(x_2 - x_1) \\ y &= y_1 + \lambda(y_2 - y_1) \\ z &= z_1 + \lambda(z_2 - z_1) \end{aligned}}$$

PERS. PARAMETER DARI GARIS g

$$x - x_1 = \lambda(x_2 - x_1) \longrightarrow \boxed{\lambda = \frac{x - x_1}{x_2 - x_1}}$$

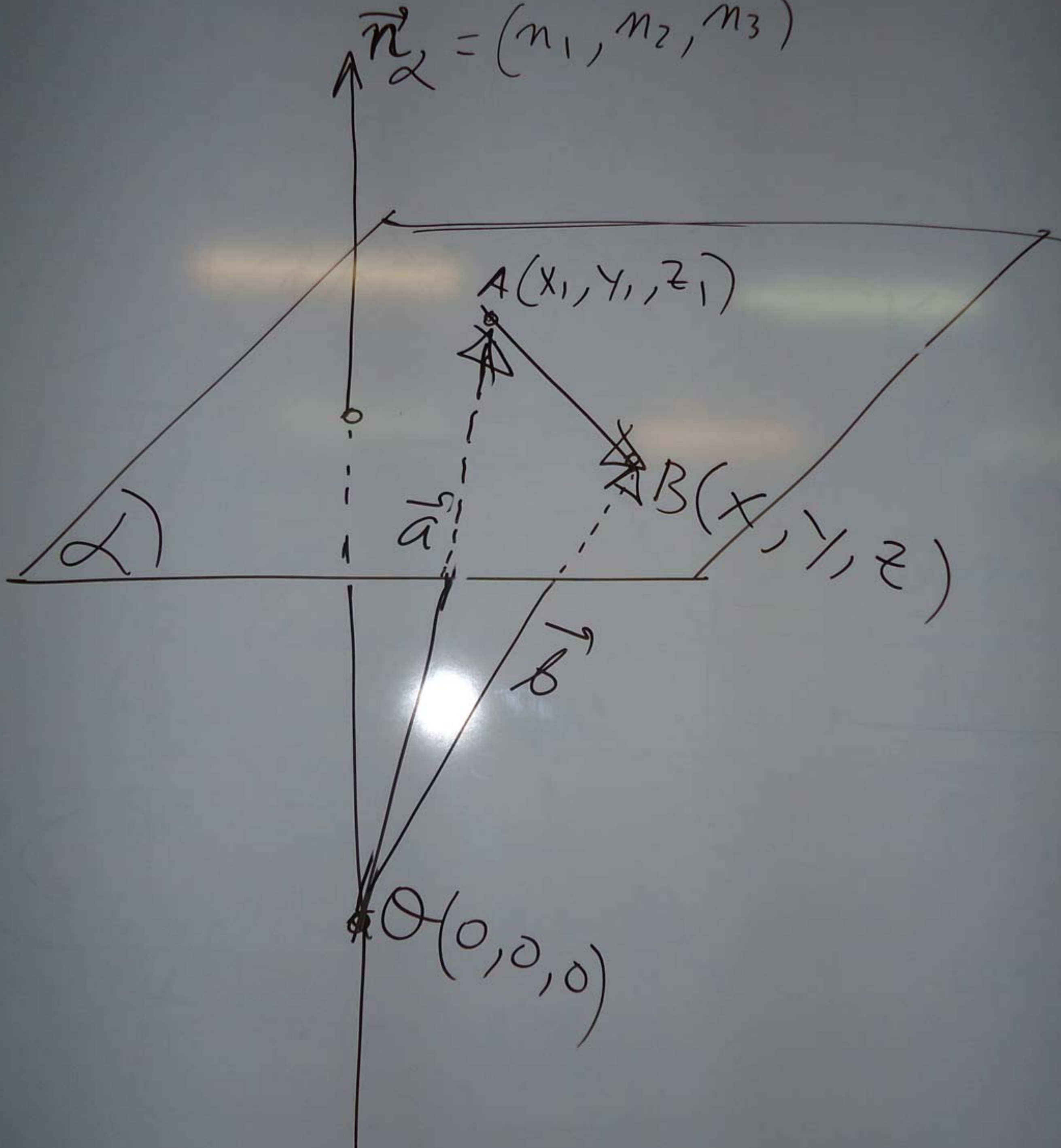
$$y - y_1 = \lambda(y_2 - y_1) \longrightarrow \boxed{\lambda = \frac{y - y_1}{y_2 - y_1}}$$

$$z - z_1 = \lambda(z_2 - z_1) \longrightarrow \boxed{\lambda = \frac{z - z_1}{z_2 - z_1}}$$

$$\lambda = \lambda = \lambda$$

$$\boxed{\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}}$$

PERS. BIDANG DI R^3



$$\overrightarrow{OA} = \vec{a} = (x_1, y_1, z_1)$$
$$\overrightarrow{OB} = \vec{b} = (x, y, z)$$

PERHATIKAN $\triangle OAB$

$$\overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OB}$$
$$\vec{a} + \overrightarrow{AB} = \vec{b}$$
$$\overrightarrow{AB} = \vec{b} - \vec{a}$$
$$\overrightarrow{AB} = (x, y, z) - (x_1, y_1, z_1)$$
$$\overrightarrow{AB} = (x - x_1, y - y_1, z - z_1)$$

$$\left.\begin{array}{l} \vec{n}_\alpha \perp \text{bid. } \alpha \\ \overrightarrow{AB} \text{ PADA bid. } \alpha \end{array}\right\} \longrightarrow \vec{n}_\alpha \perp \overrightarrow{AB}$$

$$\Longrightarrow \varphi = 90^\circ$$

$$\vec{n_\alpha} \cdot \vec{AB} = |\vec{n_\alpha}| \, . \, |\vec{AB}| \, . \, \underbrace{\cos 90^\circ}_{= 0}$$

$$\boxed{\vec{n_\alpha} \cdot \vec{AB} = 0}$$

$$\boxed{(n_1, n_2, n_3) \cdot (x - x_1, y - y_1, z - z_1) = 0}$$

PERS. VEKTOR DARI BIDANG α

$$\boxed{n_1 (x - x_1) + n_2 (y - y_1) + n_3 (z - z_1) = 0}$$

PERS. CARTESIUS DARI BIDANG α

$$\text{Bid. } \alpha \begin{cases} \text{melalui } P(x_1, y_1, z_1) \\ \perp \vec{n_\alpha} = (n_1, n_2, n_3) \end{cases}$$

<u>PERS. Bid α</u> :

$$\boxed{n_1(x - x_1) + n_2(y - y_1) + n_3(z - z_1) = 0}$$

Bid. α $\begin{cases} \text{melalui } P(x_1, y_1, z_1) \\ \perp \vec{n_\alpha} = (n_1, n_2, n_3) \end{cases}$

PERS. Bid α :

$$\boxed{n_1(x - x_1) + n_2(y - y_1) + n_3(z - z_1) = 0}$$

CONTOH:

Bid. α $\begin{cases} \text{melalui } A(\overset{x_1}{1}, \overset{y_1}{2}, \overset{z_1}{3}) \\ \perp \vec{n_\alpha} = 4\vec{i} + 7\vec{j} + 2\vec{k} = (\overset{n_1}{4}, \overset{n_2}{7}, \overset{n_3}{2}) \end{cases}$

Bid. α :

$$n_1(x - x_1) + n_2(y - y_1) + n_3(z - z_1) =$$

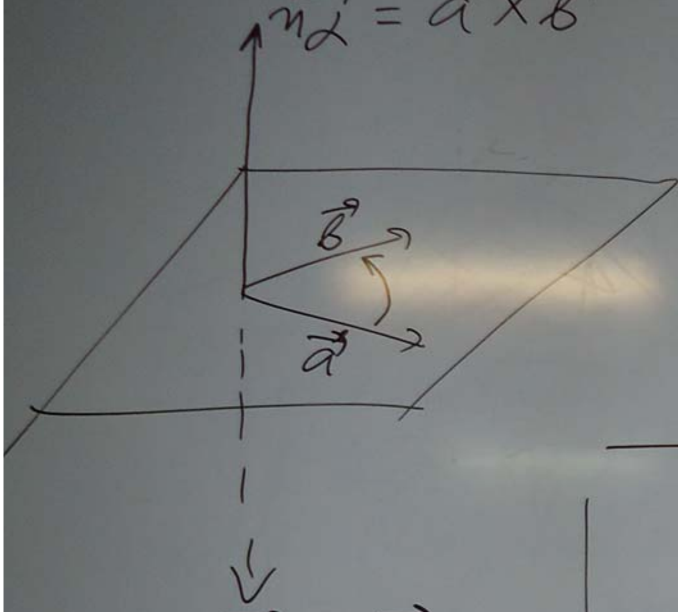
$$4(x - 1) + 7(y - 2) + 2(z - 3) = 0$$

$$4x - 4 + 7y - 14 + 2z - 6 = 0$$

$$\boxed{4x + 7y + 2z - 24 = 0}$$

PERKALIAN SILANG (CROSS VECTOR)

$$\vec{n}_\alpha = \vec{a} \times \vec{b}$$



$$\vec{n}_\alpha = \vec{b} \times \vec{a}$$

$$\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cdot \cos\varphi$$

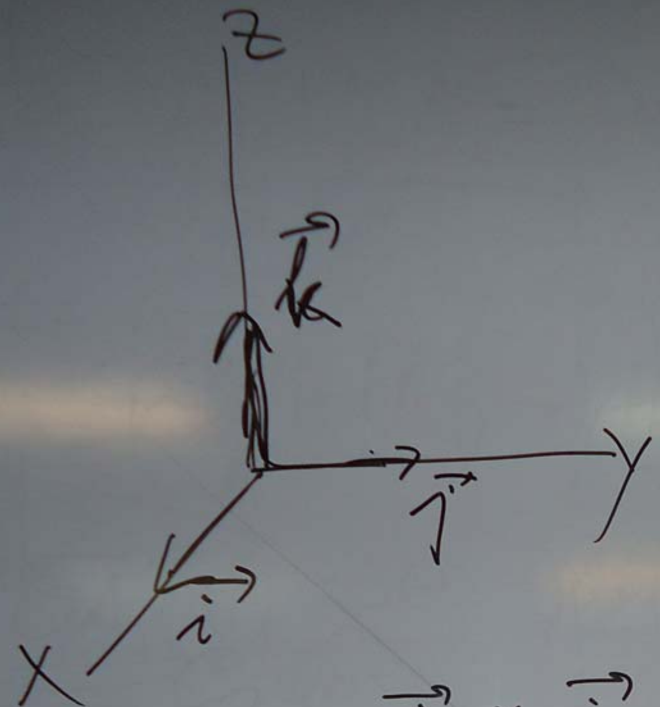
$$\vec{a} \times \vec{b} = |\vec{a}| \cdot |\vec{b}| \cdot \sin\varphi \cdot \vec{u}$$

$$|\vec{a} \times \vec{b}| = |\vec{a}| \cdot |\vec{b}| \cdot \sin\varphi$$

$$\vec{a} = (x_1, y_1, z_1)$$

$$\vec{b} = (x_2, y_2, z_2)$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix}$$



$$\vec{i} = (1,0,0) \rightarrow |\vec{i}| = \sqrt{1^2+0^2+0^2} = 1$$

$$\vec{j} = (0,1,0) \rightarrow |\vec{j}| = \sqrt{0^2+1^2+0^2} = 1$$

$$\vec{k} = (0,0,1) \rightarrow |\vec{k}| = \sqrt{0^2+0^2+1^2} = 1$$

$$\vec{i} \times \vec{i} = \underbrace{|\vec{i}|}_{1} \cdot \underbrace{|\vec{i}|}_{1} \cdot \underbrace{\sin 0}_{0} \cdot \vec{u} = \vec{0}$$

$$\vec{j} \times \vec{j} = |\vec{j}| \cdot |\vec{j}| \cdot \underbrace{\sin 0}_{0} \cdot \vec{u} = \vec{0}$$

$$\vec{k} \times \vec{k} = |\vec{k}| \cdot |\vec{k}| \cdot \underbrace{\sin 0}_{0} \cdot \vec{u} = \vec{0}$$

$\vec{i} \times \vec{j} = \vec{k}$	$\vec{i} \times \vec{k} = -\vec{j}$	$\vec{j} \times \vec{k} = \vec{i}$
$\vec{j} \times \vec{i} = -\vec{k}$	$\vec{k} \times \vec{i} = \vec{j}$	$\vec{k} \times \vec{j} = -\vec{i}$

$$\vec{a} = (x_1, y_1, z_1) = x_1\vec{i} + y_1\vec{j} + z_1\vec{k}$$

$$\vec{b} = (x_2, y_2, z_2) = x_2\vec{i} + y_2\vec{j} + z_2\vec{k}$$

$$\vec{a}\times\vec{b} = (x_1\vec{i} + y_1\vec{j} + z_1\vec{k}) \times (x_2\vec{i} + y_2\vec{j} + z_2\vec{k})$$

$$= \cancel{x_1x_2\underbrace{(\vec{i}\times\vec{i})}_{0}} + x_1y_2\underbrace{(\vec{i}\times\vec{j})}_{\vec{k}} + x_1z_2\underbrace{(\vec{i}\times\vec{k})}_{-\vec{j}} + y_1x_2\underbrace{(\vec{j}\times\vec{i})}_{-\vec{k}} + \cancel{y_1y_2\underbrace{(\vec{j}\times\vec{j})}_{0}} + y_1z_2\underbrace{(\vec{j}\times\vec{k})}_{\vec{i}} + z_1x_2\underbrace{(\vec{k}\times\vec{i})}_{\vec{j}} + z_1y_2\underbrace{(\vec{k}\times\vec{j})}_{-\vec{i}} + \cancel{z_1z_2\underbrace{(\vec{k}\times\vec{k})}_{0}}$$

$$= \vec{i}(y_1z_2 - z_1y_2) - \vec{j}(x_1z_2 - z_1x_2) + \vec{k}(x_1y_2 - y_1x_2)$$

$$= \underset{a_{11}}{\vec{i}}\begin{vmatrix} y_1 & z_1 \\ y_2 & z_2 \end{vmatrix} - \underset{a_{12}}{\vec{j}}\begin{vmatrix} x_1 & z_1 \\ x_2 & z_2 \end{vmatrix} + \underset{a_{13}}{\vec{k}}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix}$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix}$$