PD. ORDE-n TINGKAT-n HOMOGEN

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$$

$$D^2y - 5Dy + 6y = 0$$

$$(D^2 - 5D + 6)\,y = 0$$

PERS. KARAKTERISTIK (EIGEN)

$$\alpha^2 - 5\alpha + 6 = 0$$

$$(\alpha - 3)(\alpha - 2) = 0$$

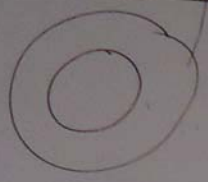
$$\alpha_1 = 3, \quad \alpha_2 = 2$$

SOLUSI UMUM HOMOGEN

$$y = C_1 e^{\alpha_1 x} + C_2 e^{\alpha_2 x}$$

$$y = C_1 e^{3x} + C_2 e^{2x}$$

HOMOGEN



0

OPERATOR D

$$\frac{dy}{dx} = Dy$$

$$\frac{d^2y}{dx^2} = D^2y$$

$$\frac{d^3y}{dx^3} = D^3y$$

$y = e^u \rightarrow y' = e^u \cdot u'$

MISALKAN :

SOLUSI PD : $y = e^{\alpha x}$

$$\frac{dy}{dx} = e^{\alpha x} \cdot (\alpha x)' = \alpha e^{\alpha x}$$

$$\frac{d^2y}{dx^2} = \alpha \cdot e^{\alpha x} \cdot (\alpha x)'$$

$$= \alpha^2 \cdot e^{\alpha x}$$

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$$

$$\alpha^2 e^{\alpha x} - 5\alpha e^{\alpha x} + 6 e^{\alpha x} = 0$$

$$(\alpha^2 - 5\alpha + 6) \cdot e^{\alpha x} = 0$$

$$\alpha^2 - 5\alpha + 6 = 0$$

$$e^{\alpha x} \neq 0$$

SOLUSI UMUM HOMOGEN

I) $\alpha_1 \neq \alpha_2 \neq \alpha_3 \longrightarrow y = c_1 e^{\alpha_1 x} + c_2 e^{\alpha_2 x} + c_3 e^{\alpha_3 x}$

II) $\alpha_1 = \alpha_2 = \alpha_3 \xrightarrow{\times 1, x, x^2} y = c_1 x^0 e^{\alpha_1 x} + c_2 x^1 e^{\alpha_2 x} + c_3 x^2 e^{\alpha_3 x} \longrightarrow y = c_1 e^{\alpha x} + c_2 x e^{\alpha x} + c_3 x^2 e^{\alpha x}$

$y = (c_1 + c_2 x + c_3 x^2) e^{\alpha x}$

III) $\alpha_{1,2} = a \pm ib \longrightarrow y = e^{ax}(A \cos bx + B \sin bx)$

$\cos(-x) = \cos x$

III) $\alpha_{1,2} = a \pm ib$

$\alpha_1 = a + ib$

$\alpha_2 = a - ib$

SOL. UMUM HOMOGEN

$y = c_1 \cdot e^{\alpha_1 x} + c_2 e^{\alpha_2 x}$

$y = c_1 e^{(a+ib)x} + c_2 e^{(a-ib)x}$

$y = c_1 e^{ax} \cdot e^{ibx} + c_2 \cdot e^{ax} \cdot e^{i(-bx)}$

$y = e^{ax}\left[c_1 e^{ibx} + c_2 e^{i(-bx)}\right]$

$y = e^{ax}\left[c_1(\cos bx + i \cdot \sin bx) + c_2(\cos bx - i \cdot \sin bx)\right]$

$y = e^{ax}\left[c_1 \cdot \cos bx + i c_1 \cdot \sin bx + c_2 \cdot \cos bx - i c_2 \cdot \sin bx\right]$

$y = e^{ax}\left[\underbrace{(c_1 + c_2)}_{A} \cos bx + \underbrace{i(c_1 - c_2)}_{B} \sin bx\right]$

$y = e^{ax}(A \cdot \cos bx + B \cdot \sin bx)$

RUMUS EULAR

$e^{i\theta} = \cos\theta + i \cdot \sin\theta$

$e^{ibx} = \cos bx + i \cdot \sin bx$

$e^{i(-bx)} = \cos(-bx) + i \sin(-bx)$

$= \cos bx - i \sin bx$

$\alpha_{1,2} = a \pm ib$, $\alpha_{3,4} = a \pm ib$

SOL. UM. HOMOGEN

$$y = e^{ax}(A_1 \cos bx + B_1 \sin bx)x^0 + e^{ax}(A_2 \cos bx + B_2 \sin bx)x$$

$$y = e^{ax}\left[A_1 \cos bx + B_1 \sin bx + A_2 x \cos bx + B_2 x \sin bx\right]$$

$$y = e^{ax}\left[(A_1 + A_2 x)\cos bx + (B_1 + B_2 x)\sin bx\right]$$

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 0$$

$$D^2(y) - 6D(y) + 9(y) = 0$$

$$(D^2 - 6D + 9).y = 0$$

Pers. EIGEN

$$\alpha^2 - 6\alpha + 9 = 0$$

$$(\alpha - 3)^2 = 0$$

$$(\alpha - 3)(\alpha - 3) = 0$$

$$\alpha_1 = 3, \quad \alpha_2 = 3$$

$$\boxed{\alpha_1 = \alpha_2 = 3}$$

SOL. UM. HOMOGEN

$$y = (c_1 + c_2 x)e^{\alpha x}$$

$$y = (c_1 + c_2 x)e^{3x}$$

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = 0$$

$$D^2(y) - 2D(y) + 5(y) = 0$$

$$(D^2 - 2D + 5) \cdot y = 0$$

↓

PERS. EIGEN

$$\alpha^2 - 2\alpha + 5 = 0$$

$$a = 1,\ b = -2,\ c = 5$$

$$\alpha_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\lambda_{1,2} = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(5)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{4 - 20}}{2}$$

$$= \frac{2 \pm \sqrt{-16}}{2}$$

$$= \frac{2 \pm \sqrt{16} \cdot \sqrt{-1}}{2}$$

$$= \frac{2 \pm 4i}{2}$$

$$\lambda_{1,2} = 1 \pm 2i$$

$a = 1$ $\quad b = 2$

SOL. UM. HOMOGEN

$$y = e^{ax}\left(A \cdot \cos bx + B \cdot \sin bx\right)$$

$$y = e^{x}\left(A \cdot \cos 2x + B \cdot \sin 2x\right)$$

PD. TINGKAT-n NON HOMOGEN

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = \boxed{f(x)} \neq 0$$

$f(x)$

- $f(x) = e^{kx} \longrightarrow \boxed{D = k}$
- $f(x) = \left\{ \begin{matrix} \sin ax \\ \cos bx \end{matrix} \right\} \longrightarrow \boxed{D^2 = -a^2}$
- $f(x) = ax^2 + bx + c$ (POLINOM)

$$(D-\alpha)e^{kx} = D(e^{kx}) - \alpha e^{kx}$$

$$= e^{kx}(kx)' - \alpha \cdot e^{kx}$$

$$= k \cdot e^{kx} - \alpha \cdot e^{kx}$$

$$\underbrace{(D-\alpha)e^{kx}}_{D=k} = (k-\alpha)\cdot e^{kx}$$

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^{5x}$$

$$D^2y - 5Dy + 6y = e^{5x}$$

$$(D^2 - 5D + 6)\cdot y = e^{5x}$$

I) $$(D^2 - 5D + 6)\,y = 0$$

Pers. EIGEN

$$\alpha^2 - 5\alpha + 6 = 0$$

$$(\alpha - 3)(\alpha - 2) = 0$$

$$\alpha_1 = 3, \quad \alpha_2 = 2$$

$$y_h = C_1 e^{\alpha_1 x} + C_2 e^{\alpha_2 x}$$

$$y_h = C_1 e^{3x} + C_2 e^{2x}$$

II) $(D^2-5D+6)y = e^{5x}$ → $K=5$

$D=K=5$

$$y_K = \frac{e^{5x}}{(D^2-5D+6)} = \frac{e^{5x}}{(D-3)(D-2)} = \frac{e^{5x}}{(5-3)(5-2)}$$

$$= \frac{e^{5x}}{6} = \frac{1}{6}e^{5x}$$

(SOLUSI KHUSUS PD)

SOLUSI UMUM PD

$$y = y_h + y_K$$

$$y = c_1 e^{3x} + c_2 e^{2x} + \frac{1}{6}e^{5x}$$

$$(D-a)\,e^{ax}.V = D(e^{ax}.V) - a.e^{ax}V$$

$$= D(e^{ax})V + e^{ax}.D(V) - a e^{ax}.V$$

$$= a e^{ax}.V + e^{ax}.D(V) - a e^{ax}V$$

$$\boxed{(D-a)\,e^{ax}.V = e^{ax}.D(V)}$$

$$(D-a)^{-1}\,e^{ax}.V = e^{ax}.D^{-1}(V)$$

(4) $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^{3x}$

$D^2y - 5Dy + 6y = e^{3x}$

(5) $(D^2 - 5D + 6) \cdot y = e^{3x}$

I) $(D^2 - 5D + 6)\, y = 0$

Pers. EIGEN

$\alpha^2 - 5\alpha + 6 = 0$

$(\alpha - 3)(\alpha - 2) = 0$

(6) $\alpha_1 = 3, \quad \alpha_2 = 2$

$y_h = C_1 e^{\alpha_1 x} + C_2 e^{\alpha_2 x}$

$y_h = C_1 e^{3x} + C_2 e^{2x}$

II) $(D^2-5D+6)y = e^{3x}$ → $K=3$

$D=K=3$

$$y_K = \frac{e^{3x}}{(D^2-5D+6)} = \frac{e^{3x}}{(D-3)(D-2)} = \frac{e^{3x}}{(D-2)} \cdot (D-3)^{-1}(1$$

$$= \frac{e^{3x}}{(3)-2} \cdot D^{-1}(1) = e^{3x} \int \frac{(1)\,dx}{x} = x e^{3x}$$

Solusi umum PD

$$y = y_h + y_K$$

$$y = c_1 e^{3x} + c_2 e^{2x} + x e^{3x}$$

II) $(D^2-5D+6)y = e^{3x}$ → $K=3$

$D=K=3$

$$y_K = \frac{e^{3x}}{(D^2-5D+6)} = \frac{e^{3x}}{(D-3)(D-2)} = \frac{e^{3x}}{(D-2)} \cdot (D-3)^{-1}(1)$$

$$= \frac{e^{3x}}{(3)-2} \cdot D^{-1}(1) = e^{3x} \int (1)\,dx = x e^{3x}$$

Solusi umum PD

$$y = y_h + y_K$$

$$y = c_1 e^{3x} + c_2 e^{2x} + x e^{3x}$$

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = e^{3x}$$

$$D^2y - 6Dy + 9y = e^{3x}$$

$$(D^2 - 6D + 9).y = e^{3x}$$

I) $(D^2 - 6D + 9).y = 0$

PERS. EIGEN

$$\alpha^2 - 6\alpha + 9 = 0$$

$$(\alpha - 3)^2 = 0$$

$$\alpha_1 = \alpha_2 = 3$$

$$y_h = (c_1 + c_2 x)e^{\alpha x}$$

$$y_h = (c_1 + c_2 x)e^{3x}$$

$$(D-a)^{-1} e^{ax} v = e^{ax} \cdot D^{-1}(v)$$

II)

II) $(D^2-6D+9)(y) = e^{3x}$, $K=3$

$D=K=3$

$$y_K = \frac{e^{3x}}{D^2-6D+9} = \frac{e^{3x}}{(D-3)^2}$$

$$= (D-3)^{-2} e^{3x}(1)$$

$$= e^{3x} \cdot D^{-2}(1)$$

$$= e^{3x} \cdot D^{-1} \cdot D^{-1}(1)$$

$$= e^{3x} \int\int (1)\,dx\,dx$$

$$= e^{3x} \int x\,dx$$

$$= e^{3x} \left(\frac{1}{2}x^2\right) = \boxed{\frac{1}{2}x^2 e^{3x}}$$

(SOL. KHUS PD)

Sol. um. PD

$$y = y_c + y_K$$

$$y = (C_1 + C_2 x)e^{3x} + \frac{1}{2}x^2 e^{3x}$$

$K = n$ AKAR PERS. EIGEN

$$\boxed{y_K = \frac{x^n}{n!} e^{ax}}$$

$n = 1 \rightarrow y_K = \frac{x}{1!} e^{3x} = \underline{x e^{3x}}$

$n = 2 \rightarrow y_K = \frac{x^2}{2!} e^{3x} = \underline{\frac{1}{2} x^2 e^{3x}}$

$K = n$ AKAR PERS. EIGEN

$$y_k = \frac{x^n}{n!} e^{ax}$$

$$n = 1 \rightarrow y_k = \frac{x}{1!} e^{3x} = \underline{x e^{3x}}$$

$$n = 2 \rightarrow y_k = \frac{x^2}{2!} e^{3x} = \underline{\frac{1}{2} x^2 e^{3x}}$$

II) $f(x) = \cos ax$

$$D(\cos ax) = -a.\sin ax$$

$$D^2(\cos ax) = -a^2.(\cos ax)$$

$$D^2 = -a^2$$

$f(x) = \sin ax$

$$D(\sin ax) = a.\cos ax$$

$$D^2(\sin ax) = -a^2.(\sin ax)$$

$$D^2 = -a^2$$

$$\frac{d^2y}{dx^2} + y = 2\cos 2x$$

$$D^2(y) + (y) = 2\cos 2x$$

$$(D^2+1)\,y = 2\cos 2x$$

(I) $(D^2+1)\cdot y = 0$

↓

Pers. EIGEN

$$\alpha^2 + 1 = 0$$

$$\alpha^2 = -1$$

$$\alpha_{1,2} = \pm\sqrt{-1}$$

$$\alpha_{1,2} = 0 \pm i$$

$a = 0$ $\quad b = 1$

$$y_h = e^{ax}(A\cdot\cos bx + B\cdot\sin bx)$$

$$y_h = e^{0x}(A\cdot\cos x + B\cdot\sin x)$$

$$y_h = A\cos x + B\sin x$$

(II) $(D^2+1).(y) = 2\cos 2x$

$a = 2$

$D^2 = -a^2$

$D^2 = -(2)^2 = -4$

$$y_k = \frac{2\cos 2x}{D^2+1}$$

$$y_k = \frac{2\cos 2x}{\frac{-4+1}{-3}} = -\frac{2}{3}\cos 2x$$

(Sol KHUS PD)

Sol. UM. PD

$$y = y_h + y_k$$

$$y = A\cos x + B.\sin x - \frac{2}{3}\cos 2x$$

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = \sin 5x$$

$$D^2y - 5Dy + 6y = \sin 5x$$

$$(D^2 - 5D + 6) \cdot y = \sin 5x$$

I) $(D^2 - 5D + 6) \cdot y = 0$

↓

Pers. EIGEN

$$\alpha^2 - 5\alpha + 6 = 0$$

$$(\alpha - 3)(\alpha - 2) = 0$$

$$\alpha_1 = 3, \quad \alpha_2 = 2$$

$$y_h = c_1 \cdot e^{\alpha_1 x} + c_2 \cdot e^{\alpha_2 x}$$

$$y_h = c_1 \cdot e^{3x} + c_2 e^{2x}$$

II) $(D^2-5D+6)y = \sin 5x$ $\rightarrow a=5$

$D^2 = -a^2 = -(5)^2 = -25$

$$y_K = \frac{\sin 5x}{D^2-5D+6} = \frac{\sin 5x}{-25-5D+6} = \frac{\sin 5x}{-5D-19} = \frac{\sin 5x}{-(5D+19)}$$

$$= -\frac{\sin 5x}{5D+19}\cdot\frac{5D-19}{5D-19}$$

$$= -\frac{5D(\sin 5x)-19\sin 5x}{(5D)^2-(19)^2} = -\frac{5(\cos 5x)(5)-19\sin 5x}{25D^2-361}$$

$$= -\frac{25\cos 5x-19\sin 5x}{25(-25)-361} = -\frac{25\cos 5x-19\sin 5x}{-625-361}$$

$$= \frac{25\cos 5x-19\sin 5x}{986} = \frac{25}{986}\cos 5x - \frac{19}{986}\sin 5x$$

(SOL. KHOS. PD)

SOL. UM. PD

$y = y_h + y_K$

$y = c_1 e^{3x} + c_2 e^{2x} + \frac{25}{986}\cos 5x - \frac{19}{986}\sin 5x$

POLINOM

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 3x^3 + 4x^2 - 2x + 1$$

$$D^2y + 3Dy + 2y = 3x^3 + 4x^2 - 2x + 1$$

$$(D^2 + 3D + 2)\cdot y = 3x^3 + 4x^2 - 2x + 1$$

I) $(D^2 + 3D + 2)\cdot y = 0$

↓

PERS. EIGEN

$$\alpha^2 + 3\alpha + 2 = 0$$

$$(\alpha + 2)(\alpha + 1) = 0$$

$$\alpha_1 = -2, \quad \alpha_2 = -1$$

$$y_h = C_1 e^{\alpha_1 x} + C_2 e^{\alpha_2 x}$$

$$y_h = C_1 e^{-2x} + C_2 e^{-x}$$

(II) $(D^2+3D+2).y = 3x^3+4x^2-2x+1$

$$y_k = \frac{3x^3+4x^2-2x+1}{D^2+3D+2}$$

$$y_k = \frac{1}{2+3D+D^2}\cdot(3x^3+4x^2-2x+1)$$

$$y_k = \left(\frac{1}{2}-\frac{3}{4}D+\frac{7}{8}D^2-\frac{15}{16}D^3\right)(3x^3+4x^2-2x+1)$$

$$y_k = \frac{1}{2}(3x^3+4x^2-2x+1) - \frac{3}{4}D(3x^3+4x^2-2x+1) + \frac{7}{8}D^2(3x^3+4x^2-2x+1) - \frac{15}{16}D^3(3x^3+4x^2-2x+1)$$

$$y_k = \frac{3}{2}x^3+2x^2-x+\frac{1}{2} - \frac{3}{4}(9x^2+8x-2)+\frac{7}{8}(18x+8) - \frac{15}{16}(18)$$

$$y_k = \frac{3}{2}x^3+2x^2-x+\frac{1}{2}-\frac{27}{4}x^2-6x+\frac{3}{2}+\frac{63}{4}x+7-\frac{135}{8}$$

$$\boxed{y_k = \frac{3}{2}x^3-\frac{19}{4}x^2+\frac{35}{4}x-\frac{63}{8}}$$

$-7x+\frac{63}{4}$

$\frac{-28x+63x}{4}$

$\frac{35x}{4}$

$D(3x^3+4x^2-2x+1) = 9x^2+8x-2$

$D^2(3x^3+4x^2-2x+1) = 18x+8$

$D^3(3x^3+4x^2-2x+1) = 18$

$D^4(\text{——}) = 0$ ×

$2+3D+D^2$) $\frac{1}{2} - \frac{3}{4}D + \frac{7}{8}D^2 - \frac{15}{16}D^3$

1

$1 + 1\frac{1}{2}D + \frac{1}{2}D^2$ —

$-1\frac{1}{2}D - \frac{1}{2}D^2$

$-1\frac{1}{2}D - \frac{9}{4}D^2 - \frac{3}{4}D^3$ —

$\frac{7}{4}D^2 + \frac{3}{4}D^3$

$\frac{7}{4}D^2 + \frac{21}{8}D^3 + \frac{7}{8}D^4$ —

$-\frac{15}{8}D^3 - \frac{7}{8}D^4$

Solusi umum PD

$$y = y_h + y_k$$

$$y = c_1 e^{-2x} + c_2 e^{-x} + \frac{3}{2}x^3 - \frac{19}{4}x^2 + \frac{35}{8}x - \frac{63}{8}$$