(8) $$\underbrace{(x^2y+2x+y^3)}_{M(x,y)}\,dx+\underbrace{(x^3+xy^2+2y)}_{N(x,y)}\,dy=0$$

$$M(x,y)=x^2y+2x+y^3$$

$$\frac{\partial M}{\partial y}=x^2(1)+0+3y^2$$

$$\boxed{\frac{\partial M}{\partial y}=x^2+3y^2}$$

$$N(x,y)=x^3+xy^2+2y$$

$$\frac{\partial N}{\partial x}=3x^2+(1)\cdot y^2+0$$

$$\boxed{\frac{\partial N}{\partial x}=3x^2+y^2}$$

$$\frac{\partial M}{\partial y}\neq\frac{\partial N}{\partial x}$$

$$\frac{\partial M}{\partial y}=x^2+3y^2$$

$$\frac{\partial N}{\partial x}=3x^2+y^2 \quad (-)$$

$$\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}=-2x^2+2y^2$$

$$yN-xM$$

$$=y(x^3+xy^2+2y)-x(x^2y+2x+y^3)$$

$$=\cancel{x^3y}+\cancel{xy^3}+2y^2-\cancel{x^3y}-2x^2-\cancel{xy^3}$$

$$=-2x^2+2y^2$$

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{yN - xM} = \frac{-2x^2+2y^2}{-2x^2+2y^2} = 1 \longrightarrow f(xy)$$

$$= (xy)^0 \Rightarrow \boxed{\mu = \mu(xy)}$$

MIS: $z = xy$

$$\boxed{\frac{\partial \mu}{\mu} = \left(\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{yN - xM}\right) \cdot \partial z}$$

$$\frac{\partial \mu}{\mu} = (1) \cdot \partial z$$

$$\int \frac{\partial \mu}{\mu} = \int \partial z$$

$$\ln \mu = z$$

$$\ln \mu = \ln e^z$$

$$\boxed{\mu = e^z = e^{xy}}$$

$$(x^2y+2x+y^3)\,dx+(x^3+xy^2+2y)\,dy=0 \qquad (e^{xy})$$

$$\underbrace{e^{xy}(x^2y+2x+y^3)\,dx}_{P(x,y)}+\underbrace{e^{xy}(x^3+xy^2+2y)}_{Q(x,y)}\,dy=0$$

PD yg baru

$$P(x,y)=e^{xy}(x^2y+2x+y^3)$$

$$\frac{\partial P}{\partial y}=(e^{xy})'(x^2y+2x+y^3)+e^{xy}(x^2y+2x+y^3)'$$

$$=e^{xy}(xy)'(x^2y+2x+y^3)+e^{xy}(x^2(1)+0+3y^2)$$

$$=e^{xy}(x^3y+2x^2+xy^3+x^2+3y^2)$$

$$=e^{xy}(x^3y+3x^2+xy^3+3y^2)$$

$$Q(x,y) = e^{xy}\left(x^3 + xy^2 + 2y\right)$$

$$\frac{\partial Q}{\partial x} = \left(e^{xy}\right)'\left(x^3 + xy^2 + 2y\right) + e^{xy}\left(x^3 + xy^2 + 2y\right)'$$

$$= e^{xy}(xy)'\left(x^3 + xy^2 + 2y\right) + e^{xy}\left(3x^2 + (1).y^2 + 0\right)$$

$$= e^{xy}\left(x^3y + xy^3 + 2y^2 + 3x^2 + y^2\right)$$

$$= e^{xy}\left(x^3y + 3x^2 + xy^3 + 3y^2\right)$$

$$\boxed{\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}} \Longrightarrow \text{PD. EKSAK}$$

$$\underbrace{e^{xy}(x^2y+2x+y^3)}_{P(x,y)}dx+\underbrace{e^{xy}(x^3+xy^2+2y)}_{Q(x,y)}dy=0$$

(I) $F(x,y)=\int \underline{P(x,y)dx}+C(y) \longrightarrow \boxed{\frac{\partial F}{\partial y}=Q(x,y)}$

$$F(x,y)=\int e^{xy}(x^2y+2x+y^3)dx+C(y)$$

$$F(x,y)=\int e^{xy}(x^2y)dx+\int e^{xy}(2x)dx+\int e^{xy}\cdot y^3dx+C(y)$$

$$F(x,y)=\int x^2\cdot e^{xy}\cdot(y\,dx)+2\int x\cdot e^{xy}dx+y^3\int e^{xy}dx+C(y)$$

$$F(x,y)=\int x^2\cdot\underline{e^{xy}\cdot\overbrace{d(xy)}^{y\,dx}}+2\left(\frac{1}{y}\right)\int x\cdot\underline{e^{xy}\,\overbrace{d(xy)}^{y\,dx}}+y^3\left(\frac{1}{y}\right)\int e^{xy}\,\overbrace{d(xy)}^{y\,dx}+C(y)$$

$$F(x,y)=\int\underbrace{x^2}_{u}\,d(\underbrace{e^{xy}}_{v})+\frac{2}{y}\int x\cdot d(e^{xy})+y^2e^{xy}+C(y)$$

$$\boxed{d(e^u) = e^u \cdot d(u)}$$

$$F(x,y) = x^2 \cdot e^{xy} - \int e^{xy}\, d(x^2) + \frac{2}{y}\int x \cdot d(e^{xy}) + y^2 e^{xy} + c(y)$$

$$F(x,y) = x^2 \cdot e^{xy} - \int e^{xy} (2x\, dx) + \frac{2}{y}\int x \cdot d(e^{xy}) + y^2 e^{xy} + c(y)$$

$$F(x,y) = x^2 \cdot e^{xy} - 2\left(\frac{1}{y}\right)\int x \cdot e^{xy} \cdot \overset{y\cdot dx}{d(xy)} + \frac{2}{y}\int x \cdot d(e^{xy}) + y^2 e^{xy} + c(y)$$

$$F(x,y) = x^2 \cdot e^{xy} \cancel{- \frac{2}{y}\int x \cdot d(e^{xy})} \cancel{+ \frac{2}{y}\int x \cdot d(e^{xy})} + y^2 e^{xy} + c(y)$$

$$\boxed{F(x,y) = x^2 e^{xy} + y^2 \cdot e^{xy} + c(y)}$$

$$\frac{\partial F}{\partial y} = x^2 e^{xy} \cdot (xy)' + (y^2)' e^{xy} + y^2 (e^{xy})' + c'(y) = Q(x,y)$$

$$x^3 e^{xy} + 2y e^{xy} + y^2 e^{xy} (xy)' + c'(y) = Q(x,y)$$

$$\cancel{e^{xy}(x^3 + 2y + xy^2)} + c'(y) = \cancel{e^{xy}(x^3 + xy^2 + 2y)}$$

$$\boxed{c'(y) = 0}$$

$$c(y) = \int c'(y)\,dy = \int 0 \cdot dy = c$$

SOLUSI UMUM PD

$$\boxed{F(x,y) = 0}$$

$$x^2 e^{xy} + y^2 e^{xy} + \underline{c(y)} = 0$$

$$e^{xy}(x^2+y^2) + C = 0$$

$$e^{xy}(x^2+y^2) = -C$$

$$= C$$

PD. LINEAR

$$\boxed{\frac{dy}{dx} + P(x) \cdot y = Q(x)}$$

SOL. UM. PD.

$$\boxed{y = e^{-\int P(x)\,dx}\left[\int e^{\int P(x)\,dx} \cdot Q(x)\,dx + C\right]}$$

$$y = \frac{1}{e^{\int P(x)\,dx}}\left[\int e^{\int P(x)\,dx} \cdot Q(x)\,dx + C\right]$$

MISALKAN: $\boxed{S(x) = e^{\int P(x)\,dx}}$

SOL. UM. PD

$$y = \frac{1}{S(x)}\left[\int S(x) \cdot Q(x)\,dx + C\right]$$

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PD. LINEAR

$$\boxed{\frac{dy}{dx} + P(x) \cdot y = Q(x)}$$

MISALKAN : SOLUSI UM. PD :

$$\boxed{y = U \cdot V}$$

$$\rightarrow \frac{dy}{dx} = U' \cdot V + U \cdot V'$$

$$\boxed{\frac{dy}{dx} + P(x) \cdot y = Q(x)}$$

$$U' \cdot V + U \cdot V' + P(x) \cdot U \cdot V = Q(x)$$

$$V\underbrace{\left[U' + P(x) \cdot U\right]}_{=0} + U \cdot V' = Q(x)$$

$$U' + P(x) \cdot U = 0$$

$$\boxed{\frac{dU}{dx} = -P(x) \cdot U}$$

$$\frac{dU}{U} = -P(x) \cdot dx$$

$$\int \frac{dU}{U} = -\int P(x)\,dx$$

$$\ln U = \ln e^{-\int P(x)dx}$$

$$\boxed{U = e^{-\int P(x)\,dx}}$$

$$V \cdot 0 + U \cdot V' = Q(x)$$

$$V' = \frac{Q(x)}{U}$$

$$\frac{dV}{dx} = \frac{Q(x)}{e^{-\int P(x)dx}}$$

$$\frac{dV}{dx} = e^{\int P(x)dx} \cdot Q(x)$$

$$dV = e^{\int P(x)dx} \cdot Q(x)dx$$

$$\int dV = \int e^{\int P(x)dx} \cdot Q(x)dx + C$$

$$V = \int e^{\int P(x)dx} \cdot Q(x)dx + C$$

<u>Sol. UM. PD</u>

$$Y = U \cdot V$$

$$Y = e^{-\int P(x)dx} \left[\int e^{\int P(x)dx} \cdot Q(x)dx + C \right]$$

PD. LINEAR

$$\boxed{\frac{dy}{dx} + P(x) \cdot y = Q(x)}$$

SOL. UM. PD.

$$\boxed{y = e^{-\int P(x)dx} \left[\int e^{\int P(x)dx} \cdot Q(x)dx + C \right]}$$

$$y = \frac{1}{e^{\int P(x)dx}} \left[\int e^{\int P(x)dx} \cdot Q(x)dx + C \right]$$

MISALKAN: $\boxed{S(x) = e^{\int P(x)dx}}$

SOL. UM. PD

$$y = \frac{1}{S(x)} \left[\int S(x) \cdot Q(x) dx + C \right]$$

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PD. BERNOULLY

$$\boxed{\frac{dy}{dx} + P(x).\,y = y^{n}.\,Q(x)}$$

SOL. UM. PD.

$$\boxed{y^{1-n} = (1-n)\,e^{(n-1)\int P(x)dx}\cdot\left[\int e^{(1-n)\int P(x)\,dx}.\,Q(x)\,dx + C\right]}$$

$$y^{1-n} = \frac{(1-n)}{e^{(1-n)\int P(x)dx}}\left[\int e^{(1-n)\int P(x)\,dx}.\,Q(x)\,dx + C\right]$$

MISALKAN: $\boxed{S(x) = e^{(1-n)\int P(x)\,dx}}$

SOL. UM. PD

$$y^{1-n} = \frac{1-n}{S(x)}\left[\int S(x).\,Q(x)\,dx + C\right]$$

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PD. BERNOULLY

$$\boxed{\frac{dy}{dx} + P(x) \cdot y = y^n Q(x)}$$

MISALKAN : SOLUSI UM. PD :

$$\boxed{y = U \cdot V} \longrightarrow y^n = (U \cdot V)^n = U^n \cdot V^n$$

$$\longrightarrow \frac{dy}{dx} = U' \cdot V + U \cdot V'$$

$$\boxed{\frac{dy}{dx} + P(x) \cdot y = y^n \cdot Q(x)}$$

$$U' \cdot V + U \cdot V' + P(x) \cdot U \cdot V = y^n \cdot Q(x)$$

$$V \underbrace{\left[U' + P(x) \cdot U\right]}_{= 0} + U \cdot V' = y^n \cdot Q(x)$$

$$U' + P(x) \cdot U = 0$$

$$\boxed{\frac{dU}{dx} = -P(x) \cdot U}$$

$$\frac{dU}{U} = -P(x) \cdot dx$$

$$\int \frac{dU}{U} = -\int P(x)\,dx$$

$$\ln U = \ln e^{-\int P(x)dx}$$

$$\boxed{U = e^{-\int P(x)dx}}$$

$$V \cdot 0 + U \cdot V' = y^n \cdot Q(x)$$

$$U \frac{dV}{dx} = U^n \cdot V^n \cdot Q(x)$$

$$\frac{dV}{dx} = \frac{U^n}{U} V^n \cdot Q(x)$$

$$(a^2)^3 = a^6$$

$$U^{n-1} = \left[e^{-\int P(x)dx}\right]^{(n-1)}$$

$$\boxed{U^{n-1} = e^{(1-n)\int P(x)dx}}$$

$$U^{1-n} = \left[e^{-\int P(x)dx}\right]^{(1-n)}$$

$$\boxed{U^{1-n} = e^{-(1-n)\int P(x)dx}}$$

$$\frac{dV}{V^n} = U^{n-1} \cdot Q(x)\,dx$$

$$\int V^{-n}\,dV = \int U^{n-1} \cdot Q(x)\,dx + C$$

$$\frac{1}{-n+1} V^{-n+1} = \int e^{(1-n)\int P(x)dx} \cdot Q(x)\,dx + C$$

$$\boxed{V^{1-n} = (1-n)\left[\int e^{(1-n)\int P(x)dx} \cdot Q(x)\,dx + C\right]}$$

L. UM. PD : $\boxed{y = U \cdot V}$

$$y^{1-n} = (U \cdot V)^{(1-n)}$$

$$y^{1-n} = U^{1-n} \cdot V^{1-n}$$

$$y^{1-n} = e^{-(1-n)\int P(x)dx} \; (1-n)\left[\int e^{(1-n)\int P(x)dx} \cdot Q(x)\,dx + C\right]$$

$$y^{1-n} = (1-n)\, e^{(n-1)\int P(x)dx} \left[\int e^{(1-n)\int P(x)dx} \cdot Q(x)\,dx + C\right]$$

PD. BERNOULLY

$$\boxed{\frac{dy}{dx} + P(x).\, y = y^{n} Q(x)}$$

SOL. UM. PD.

$$\boxed{y^{1-n} = (1-n)\, e^{(n-1)\int P(x)dx} . \left[\int e^{(1-n)\int P(x)\,dx} . Q(x)\,dx + C \right]}$$

$$y^{1-m} = \frac{(1-n)}{e^{(1-m)\int p(x)\,dx}} \left[\int e^{(1-m)\int P(x)\,dx} . Q(x)\,dx + C \right]$$

MISALKAN: $\boxed{S(x) = e^{(1-n)\int P(x)\,dx}}$

SOL. UM. PD

$$y^{1-m} = \frac{1-m}{S(x)} \left[\int S(x) . Q(x)\,dx + C \right]$$

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Hal 8

(1)

$$(x+y)\,dx + x\,dy = 0$$

$$x\,dy = -(x+y)\,dx$$

$$\frac{dy}{dx} = \frac{-x-y}{x}$$

$$\frac{dy}{dx} = -1 - \frac{y}{x}$$

$$\boxed{\frac{dy}{dx} + \frac{y}{x} = -1}$$

$$\boxed{\frac{dy}{dx} + P(x)\cdot y = Q(x)}$$

$$P(x) = \frac{1}{x}$$

$$Q(x) = -1$$

$$\boxed{a^{{}^{a}\log b} = b}$$

$$e^{\ln u} = e^{{}^{e}\log u} = u$$

$$S(x) = e^{\int P(x)\,dx} = e^{\int \frac{1}{x}dx} = e^{\int \frac{dx}{x}} = e^{\ln x} = x$$

SOL. UM. PD.

$$y = \frac{1}{S(x)}\left[\int S(x)\cdot Q(x)\,dx + C\right]$$

$$y = \frac{1}{x}\left[\int x\cdot(-1)\,dx + c\right]$$

$$y = \frac{1}{x}\left(-\int x\,dx + c\right)$$

$$\boxed{y = \frac{1}{x}\left(-\frac{1}{2}x^2 + c\right)}$$

$$xy = -\frac{1}{2}x^2 + c \quad \times 2$$

$$2xy = -x^2 + 2c$$

$$x^2 + 2xy = c$$

(7) $dy + (\underline{y - 2\sin x})\cos x \, dx = 0$

$$dy = (2\sin x - y)\cdot \cos x \, dx$$

$$\frac{dy}{dx} = 2\sin x \cdot \cos x - \underline{y \cdot \cos x}$$

$$\boxed{\frac{dy}{dx} + y\cdot\cos x = 2\sin x\cdot\cos x}$$

$$\frac{dy}{dx} + P(x)\cdot y = Q(x)$$

$P(x) = \cos x$

$Q(x) = 2\sin x \cdot \cos x$

$$S(x) = e^{\int P(x)\,dx} = e^{\int \cos x\,dx} = e^{\sin x}$$

<u>SOL. UM. PD</u>

$$y = \frac{1}{S(x)}\left[\int S(x)\cdot Q(x)\,dx + C\right]$$

$$y = \frac{1}{e^{\sin x}}\left[\int e^{\sin x}(2\sin x\cdot\cos x)\,dx + C\right]$$

$$y = e^{-\sin x}\left[2\int \sin x\cdot e^{\sin x}\cdot\underline{(\cos x\,dx)} + C\right]$$

$$e^{u}\,du = d(e^{u})$$

$$y = e^{-\sin x}\left[2\int \sin x \cdot \underline{e^{\sin x}\, d(\sin x)} + c\right]$$

$$y = e^{-\sin x}\left[2\int \underbrace{\sin x}_{u} \cdot d(\underbrace{e^{\sin x}}_{v}) + c\right]$$

$$y = e^{-\sin x}\left[2\left\{\underline{(\sin x)\cdot e^{\sin x} - \underline{\int e^{\sin x}\, d(\sin x)}}\right\}\right]$$

$$y = e^{-\sin x}\left[2\,e^{\sin x}\cdot \sin x - 2e^{\sin x} + c\right]$$

$$\boxed{y = 2\sin x - 2 + c \cdot e^{-\sin x}}$$

$$e^{u}\,du = d(e^{u})$$

$$y = e^{-\sin x}\left[2\int \sin x \cdot \underline{e^{\sin x}\,d(\sin x)} + c\right]$$

$$y = e^{-\sin x}\left[2\int \underbrace{\sin x}_{U} \cdot d(\underbrace{e^{\sin x}}_{V}) + c\right]$$

$$y = e^{-\sin x}\left[2\left\{(\sin x)\,e^{\sin x} - \int e^{\sin x}\,d(\sin x)\right\} + c\right]$$

$$y = e^{-\sin x}\left[2\,e^{\sin x}\sin x - 2e^{\sin x} + c\right]$$

$$\boxed{y = 2\sin x - 2 + c \cdot e^{-\sin x}}$$

CONTOH PD. BERNOULLY

(1)

$$X \quad \frac{dy}{dx} + y = x y^3 \quad \left(\frac{1}{x}\right)$$

$$\boxed{\begin{aligned} \frac{dy}{dx} + \frac{y}{x} &= y^3 . (1) \\ \frac{dy}{dx} + P(x) y &= y^n . Q(x) \end{aligned}}$$

$$P(x) = \frac{1}{x} \quad , Q(x) = 1 \quad , n = 3$$

$$S(x) = e^{(1-n)\int P(x)\,dx} = e^{(1-3)\int \frac{1}{x}\,dx}$$

$$= e^{-2\int \frac{dx}{x}} = e^{-2 \ln x} = e^{\ln x^{-2}} = x^{-2}$$

SOL. UM. PD.

$$y^{1-n} = \frac{1-n}{s(x)}\left[\int s(x).Q(x)\,dx + C\right]$$

$$y^{1-3} = \frac{1-3}{x^{-2}}\left[\int x^{-2}.(1)\,dx + c\right]$$

$$y^{-2} = -2x^{2}\left(\int x^{-2}\,dx + c\right)$$

$$\frac{1}{y^2} = -2x^2\left(\frac{1}{-2+1}x^{-2+1} + c\right)$$

$$\frac{1}{y^2} = -2x^2\left(-\frac{1}{x} + c\right)$$

$$\boxed{\frac{1}{y^2} = 2x - 2cx^2}$$

$$y^2 = \frac{1}{2x-2cx^2} \rightarrow y = \pm\sqrt{\frac{1}{2x-2cx^2}}$$