

8

$$\begin{bmatrix} 1 & -1 & 1 & 4 \\ 2 & -1 & 1 & 4 \\ 1 & 1 & 1 & 4 \\ 2 & -2 & 1 & 4 \end{bmatrix} \left| \begin{array}{c} 5 \\ 4 \\ 4 \\ 5 \end{array} \right.$$

$\xrightarrow{b_{21}(-2)}$

$\xrightarrow{b_{31}(-1)}$

$\xrightarrow{b_{41}(-2)}$

$$\begin{bmatrix} 1 & -1 & 1 & 4 \\ 0 & -3 & 2 & 0 \\ 0 & -3 & 2 & 0 \\ 0 & 0 & 3 & -3 \end{bmatrix} \left| \begin{array}{c} 5 \\ 4 \\ 4 \\ 5 \end{array} \right.$$

$\xrightarrow{b_2(-\frac{1}{3})}$

$$\begin{bmatrix} 1 & -1 & 1 & 4 \\ 0 & -1 & 1 & 4 \\ 0 & -1 & 1 & 4 \\ 0 & 0 & 3 & -3 \end{bmatrix} \left| \begin{array}{c} 5 \\ 4 \\ 4 \\ 5 \end{array} \right.$$

$\xrightarrow{b_{12}(-1)}$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \left| \begin{array}{c} 5 \\ 4 \\ 4 \\ 5 \end{array} \right.$$

$\xrightarrow{b_{32}(3)}$

$$\begin{bmatrix} 1 & 1 & 4 \\ -1 & -1 & -2 \\ 0 & 0 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 2 & 3 \\ 0 & 0 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 2 & 3 \\ 3 & 2 & 3 \\ -3 \end{bmatrix}$$

$$\begin{aligned} & \xrightarrow{b_{13} \left(\frac{2}{3}\right)} \\ & \xrightarrow{b_{23} \left(\frac{1}{3}\right)} \\ & \xrightarrow{b_{43} (-3)} \end{aligned}$$

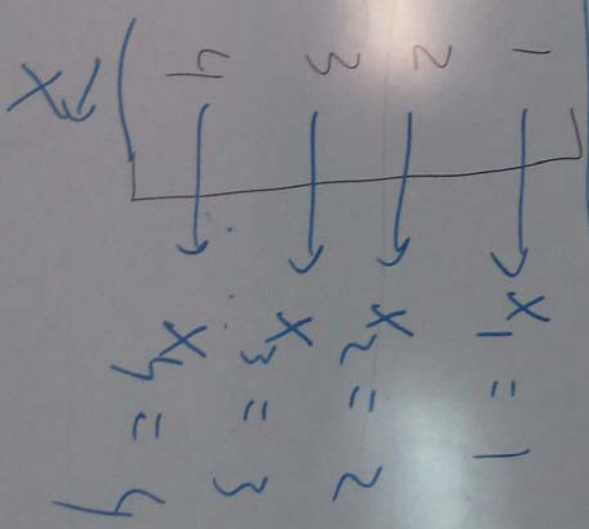
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{matrix} w/m \\ w/m \\ w/m \\ -6 \end{matrix}$$

$$\begin{aligned} & \xrightarrow{b_{14} \left(\frac{1}{3}\right)} \\ & \xrightarrow{b_{24} \left(-\frac{2}{3}\right)} \\ & \xrightarrow{b_{34} (-1)} \end{aligned}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{matrix} w/m \\ w/m \\ 1 \\ -1 \end{matrix}$$

$$\begin{aligned} & \xrightarrow{b_{14} \left(-\frac{4}{3}\right)} \\ & \xrightarrow{b_{24} \left(-\frac{2}{3}\right)} \\ & \xrightarrow{b_{34} (-1)} \end{aligned}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



$X_1=1$   
 $X_2=2$   
 $X_3=3$   
 $X_4=4$

$I_4$

# SPL HOMOGEN

$$a_{11} \cdot x_1 + a_{12} \cdot x_2 + a_{13} \cdot x_3 =$$

$$a_{21} \cdot x_1 + a_{22} \cdot x_2 + a_{23} \cdot x_3 =$$

$$a_{31} \cdot x_1 + a_{32} \cdot x_2 + a_{33} \cdot x_3 =$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} =$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A \cdot \vec{x} =$$

$$\vec{0}$$

MISAL

I) JIKA

II) JIKA

MISALKAN BANYAK PERSAMAAN =  $m$

—+— VARIABEL =  $n$

I) JIKA  $x_1 = x_2 = x_3 = 0$  → MEMUNYAI SOLUSI TRIVIAL

(  $m \geq n$  )

II) JIKA

$$\left\{ \begin{array}{l} x_1 = 5x_3 \\ x_2 = 2x_3 \\ x_3 = -x_3 \end{array} \right.$$

→ MEMUNYAI BANYAK SOLUSI

—+— SOLUSI NON TRIVIAL

(  $m < n$  )

$$3x_1 + x_2 + x_3 = 0$$

$$5x_1 - x_2 + x_3 = 0$$

$$\left. \begin{matrix} m=2 \\ m=3 \end{matrix} \right\} \rightarrow m < n$$

$$\left[ \begin{array}{ccc|c} 3 & 1 & 1 & 0 \\ 5 & -1 & 1 & 0 \end{array} \right] R_1 \left( \frac{1}{3} \right) \rightarrow$$

$$\left[ \begin{array}{ccc|c} 1 & \frac{1}{3} & \frac{1}{3} & 0 \\ 5 & -1 & 1 & 0 \end{array} \right] R_2 \left( -5 \right) \rightarrow$$

$$\left[ \begin{array}{ccc|c} 1 & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & -\frac{8}{3} & -\frac{2}{3} & 0 \end{array} \right] R_2 \left( -\frac{3}{8} \right) \rightarrow$$

$$\left[ \begin{array}{ccc|c} 1 & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 1 & \frac{1}{4} & 0 \end{array} \right] R_{1,2} \left( -\frac{1}{3} \right) \rightarrow$$

$$R_2 = R_2 + (-5)R_1$$

$$R_2: 5 \quad -1 \quad 1 \quad 0$$

$$-5R_1: -5 \quad -\frac{5}{3} \quad -\frac{5}{3} \quad 0$$

$$R_2: 0 \quad -\frac{5}{3} \quad -\frac{5}{3} \quad 0$$

$$R_1 = R_1 + \left(-\frac{1}{3}\right)R_2$$

$$R_1: 1 \quad 1 \quad \frac{1}{3} \quad 0$$

$$-\frac{1}{3}R_2: 0 \quad -\frac{1}{3} \quad -\frac{1}{3} \quad 0$$

$$R_1: 1 \quad 0 \quad \frac{1}{4} \quad 0$$

$$\left[ \begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \end{array} \right] \begin{array}{l} \frac{1}{4} \\ \frac{1}{4} \end{array} \left| \begin{array}{c} 0 \\ 0 \end{array} \right]$$

$$1 \cdot X_1 + 0 \cdot X_2 + \frac{1}{4} X_3 = 0 \rightarrow$$

$$0 \cdot X_1 + 1 \cdot X_2 + \frac{1}{4} X_3 = 0 \rightarrow$$

$$\boxed{\begin{array}{l} X_1 = -\frac{1}{4} X_3 \\ X_2 = -\frac{1}{4} X_3 \end{array}}$$

→ Mempunyai solusi non trivial

$$1 \cdot x_1 + 0 \cdot x_2 + \frac{1}{4} x_3 = 0$$

$$\rightarrow x_1 = -\frac{1}{4} x_3$$

$$0 \cdot x_1 + 1 \cdot x_2 + \frac{1}{4} x_3 = 0$$

$$\rightarrow x_2 = -\frac{1}{4} x_3$$

→ Mempunyai solusi non trivial

$$\left[ \begin{array}{ccc|c} 1 & 0 & \frac{1}{4} & 0 \\ 0 & 1 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

→ RANK MATRIKS = 2

$$\left[ \begin{array}{ccc|c} 1 & 0 & \frac{1}{4} & 0 \\ 0 & 1 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

→ RANK MATRIKS = 2

$$3x_1 + x_2 + x_3 = 0$$

$$5x_1 - x_2 + x_3 = 0 +$$

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$$8x_1 + 2x_3 = 0$$

(2)

$$4x_1 + x_3 = 0$$

$$4x_1 = -x_3$$

$$x_1 = -\frac{1}{4}x_3$$

$$3x_1 + x_2 + x_3 = 0$$

$$3\left(-\frac{1}{4}x_3\right) + x_2 = -x_3$$

$$x_2 = -x_3 + \frac{3}{4}x_3$$

$$x_2 = -\frac{1}{4}x_3$$



$$\begin{cases} x_1 = -\frac{1}{4}x_3 \\ x_2 = -\frac{1}{4}x_3 \end{cases}$$

Memiliki solusi non trivial

$$3x_1 + x_2 = 0$$

$$x_1 - x_2 = 0$$

$$4x_1 = 0$$

$$x_1 = 0$$

$$x_1 - x_2 = 0$$

$$x_1 = x_2$$

$$x_2 = x_1 = 0$$

$$x_1 = x_2 = 0$$

Memiliki solusi trivial

$$x_2 = -\frac{1}{4}x_1$$

$$\begin{aligned} 3x_1 + x_2 &= 0 \\ x_1 - x_2 &= 0 \end{aligned} \quad +$$

$$4x_1 = 0$$

$$x_1 = 0$$

$$x_1 - x_2 = 0$$

$$x_1 = x_2$$

$$x_2 = x_1 = 0$$

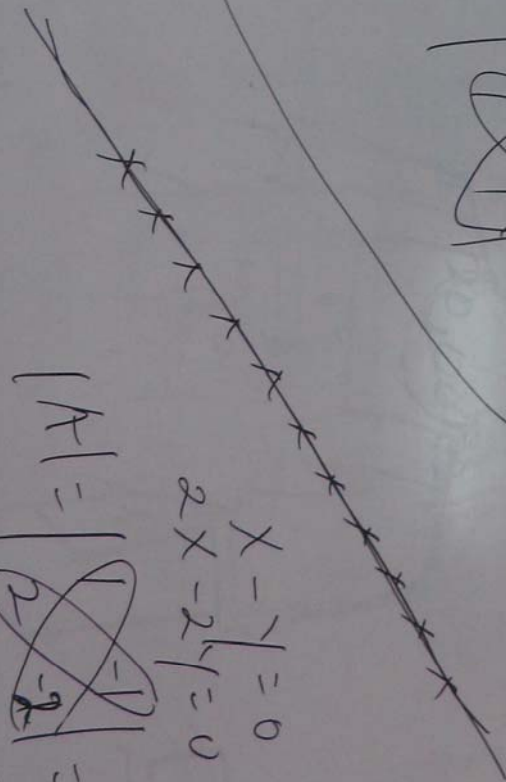
$$x_1 = x_2 = 0$$

MEMPERNYAI SOLUSI TRIVIAL

$$\begin{bmatrix} 3 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$A \cdot \vec{x} = \vec{0}$$

$$|A| = \begin{vmatrix} 3 & 1 \\ 1 & -1 \end{vmatrix} = -3 - 1 = -4 \neq 0$$



$$\begin{aligned} x - y &= 0 \\ 2x - 2y &= 0 \end{aligned}$$

$$|A| = \begin{vmatrix} 1 & -1 \\ 2 & -2 \end{vmatrix} = -2 - (-2) = -2 + 2 = 0$$

```
> A := matrix([[2, -1, 1, -4], [1, -2, 1, 2], [3, 1, -2, 2], [4, 2, -3, 2]]);
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$$A := \begin{bmatrix} 2 & -1 & 1 & -4 \\ 1 & -2 & 1 & 2 \\ 3 & 1 & -2 & 2 \\ 4 & 2 & -3 & 2 \end{bmatrix}$$

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> det(A);
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4

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> restart;
```

$$|A| = \begin{vmatrix} 2 & -1 & 1 & -4 \\ 1 & -2 & 1 & 2 \\ 3 & 1 & -2 & 2 \\ 4 & 2 & -3 & 2 \end{vmatrix}$$

$R_2 \rightarrow R_2 - R_1$   
 $R_3 \rightarrow R_3 - R_1$   
 $R_4 \rightarrow R_4 - R_1$

$$\begin{vmatrix} 2 & -1 & 1 & -4 \\ 0 & -\frac{3}{2} & 0 & 6 \\ 0 & \frac{5}{2} & -\frac{3}{2} & 6 \\ 0 & 4 & -5 & 10 \end{vmatrix}$$

$R_2 \rightarrow R_2 \cdot \frac{2}{-3}$   
 $R_3 \rightarrow R_3 \cdot \frac{2}{-3}$

$$\begin{vmatrix} 2 & -1 & 1 & -4 \\ 0 & 1 & 0 & -4 \\ 0 & \frac{5}{3} & -1 & 4 \\ 0 & 4 & -5 & 10 \end{vmatrix}$$

$R_3 \rightarrow R_3 - 5R_2$   
 $R_4 \rightarrow R_4 - 4R_2$

$$\begin{vmatrix} 2 & -1 & 1 & -4 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & -1 & 14 \\ 0 & 0 & -5 & 26 \end{vmatrix}$$

$R_4 \rightarrow R_4 + 5R_3$

$$\begin{vmatrix} 2 & -1 & 1 & -4 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & -1 & 14 \\ 0 & 0 & 0 & 98 \end{vmatrix}$$

$R_3 \rightarrow R_3 \cdot (-1)$

$$\begin{vmatrix} 2 & -1 & 1 & -4 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & -14 \\ 0 & 0 & 0 & 98 \end{vmatrix}$$

$R_1 \rightarrow R_1 + R_2 - R_3$

$$\begin{vmatrix} 2 & 0 & 0 & 10 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & -14 \\ 0 & 0 & 0 & 98 \end{vmatrix}$$

$R_1 \rightarrow R_1 \cdot \frac{1}{2}$

$$\begin{vmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & -14 \\ 0 & 0 & 0 & 98 \end{vmatrix}$$

$$R_2 = R_2 + (-\frac{1}{2})R_1$$

$$\begin{matrix} R_2: & 1 & -2 & 1 & 2 \\ -\frac{1}{2}R_1: & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -1 \end{matrix}$$


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$$R_2: 0 \quad -1\frac{1}{2} \quad \frac{1}{2} \quad 1$$

$$R_4 = R_4 + (-2)R_1$$

$$\begin{matrix} R_4: & 4 & 2 & -3 & 2 \\ -2R_1: & -4 & -2 & 2 & -2 \end{matrix}$$


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$$R_4: 0 \quad 4 \quad -5 \quad 10$$

$$R_3 = R_3 + (-\frac{3}{2})R_1$$

$$\begin{matrix} R_3: & 3 & 1 & -2 & 2 \\ -\frac{3}{2}R_1: & -\frac{3}{2} & \frac{3}{2} & \frac{3}{2} & -3 \end{matrix}$$


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$$R_3: 0 \quad \frac{5}{2} \quad -\frac{7}{2} \quad -1$$

$$R_3 = R_3 + (\frac{5}{2})R_2$$

$$\begin{matrix} R_3: & 0 & \frac{5}{2} & -\frac{7}{2} & -1 \\ \frac{5}{2}R_2: & 0 & -\frac{5}{2} & \frac{5}{2} & \frac{5}{2} \end{matrix}$$


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$$R_3: 0 \quad 0 \quad 0 \quad \frac{1}{2}$$

m

$$R_{22} \begin{pmatrix} -5/2 \\ -4 \\ -3/2 \end{pmatrix}$$

$$\begin{array}{ccc|ccc} 2 & -1 & 1 & -4 & & \\ 0 & -3/2 & 1/2 & 4 & & \\ 0 & 0 & -1/3 & 4/3 & & \\ 0 & 0 & -11/3 & 6/3 & & \end{array}$$

$$R_{43} \begin{pmatrix} 11/3 \\ -8/3 \end{pmatrix}$$

$$\begin{array}{ccc|ccc} 2 & -1 & 1 & -4 & & \\ 0 & -3/2 & 1/2 & 4 & & \\ 0 & 0 & -8/3 & 4/3 & & \\ 0 & 0 & 0 & 1/2 & & \end{array}$$

$$R_4 = R_4 + \left(\frac{8}{3}\right)R_2$$

$$R_4: 0 \quad 4 \quad -5 \quad 10$$

$$\frac{8}{3}R_2: 0 \quad -4 \quad \frac{4}{3} \quad \frac{32}{3} +$$

$$R_4: 0 \quad 0 \quad -11/3 \quad 62/3$$

$$= (2) \begin{pmatrix} -3/2 \\ -8/3 \end{pmatrix} \begin{pmatrix} -8/3 \end{pmatrix} \begin{pmatrix} 1/2 \end{pmatrix}$$

$$= \boxed{4}$$

$$R_4 = R_4 + \left(-\frac{11}{8}\right)R_3$$

$$R_4: 0 \quad 0 \quad 0 \quad -11/3 \quad 62/3 = 124$$

$$-\frac{11}{8}R_3: 0 \quad 0 \quad 0 \quad -11/3 \quad -121/6 +$$

$$R_4: 0 \quad 0 \quad 0 \quad 0 \quad 1$$