

Soal Latihan – Aljabar Linear :

$$A = \begin{pmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix} \text{ dan } B = \begin{pmatrix} 3 & -2 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 5 \end{pmatrix}$$

1. $I + B \cdot A - B^T \cdot A^T = \dots$
2. $|A| = \det(A) = \dots$
3. $|B| = \det(B) = \dots$
4. $A^{-1} = \dots$
5. $B^{-1} = \dots$

$$A = \begin{bmatrix} 1 & 1 & -1 & 1 \\ 2 & -1 & -1 & 1 \\ 1 & -2 & 1 & 1 \\ 2 & 2 & 1 & -1 \end{bmatrix}, \text{ tentukan } \det(A) !$$

Tentukan penyelesaian SPL berikut ini dengan cara OBE (Eliminasi Gauss-Jordan) :

$$x + y + 2z = 9$$

$$2x + 4y - 3z = 1$$

$$3x + 6y - 5z = 0$$

Tentukan penyelesaian SPL di bawah ini dengan cara OBE !

$$x_1 + x_2 - x_3 + x_4 = 4$$

$$2x_1 - x_2 - x_3 + x_4 = 1$$

$$x_1 - 2x_2 + x_3 + x_4 = 4$$

$$2x_1 + 2x_2 + x_3 - x_4 = 5$$

$$D \cancel{I} + B \cdot A - B^T \cdot A^T =$$

$$B \cdot A = \begin{pmatrix} 3 & -2 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 5 \end{pmatrix} \begin{pmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 16 & -2 & 3 \\ -14 & 3 & -2 \\ -10 & 0 & 5 \end{pmatrix}$$

$$B^T \cdot A^T = \begin{pmatrix} 3 & -2 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 5 \end{pmatrix} \begin{pmatrix} 4 & -2 & -2 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 12 & -8 & -6 \\ -8 & 7 & 4 \\ 5 & 0 & 5 \end{pmatrix}$$

$$+ B \cdot A = B^T \cdot A^T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} 16 & -2 & 3 \\ -14 & 3 & -2 \\ -10 & 0 & 5 \end{pmatrix} - \begin{pmatrix} 12 & -8 & -6 \\ -8 & 7 & 4 \\ 5 & 0 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} 5 & 6 & 9 \\ -6 & -3 & -6 \\ -15 & 0 & 1 \end{pmatrix}$$

2.

$$|A| = \begin{vmatrix} 4 & 0 \\ -2 & 0 \end{vmatrix}$$

$$\begin{matrix} + & - & - & - \\ - & + & - & - \\ - & - & + & - \\ - & - & - & + \end{matrix}$$

$$= \begin{pmatrix} 4 & 0 \\ 0 & -2 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 1 & -2 \\ -2 & 0 \end{pmatrix} - \begin{pmatrix} -2 & 1 \\ 1 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + \begin{pmatrix} 1 & -2 \\ 0 & 0 \end{pmatrix}$$

$$\boxed{6} = \begin{pmatrix} 1 & -2 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} =$$

(3)

$$\frac{1}{\beta}$$

$$= 1$$

$$= 1 - \omega_2$$

$$+$$

$$+$$

$$+$$

$$+$$

$$(3.3.3) + (-2.0.0) + (0.2.0) - [(0.3.0) + (0.0.3) + (5.2.2)]$$

$$45 - 20 = 25 \\ (45 + 0 + 0) - (0 + 0 + 20)$$

Dengan cara OBE

$$\left[\begin{array}{cc|ccc} -2 & 1 & 0 & 0 & 0 \\ -2 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array} \right] \xrightarrow{\text{Step 4}}$$

A
I₃

$$\begin{array}{c|ccccc} & \textcircled{1}_2 = -2 & | & 0 & 0 & 10 \\ & 2(b_1) = & | & 2 & 0 & \cancel{b_2} \\ \hline \textcircled{1}_2 : 0 & 1 & \cancel{\frac{b_1}{2}} & \cancel{\frac{b_2}{2}} & 1 & 0 \end{array}$$

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

400
400
400
400
400

$$\begin{array}{r} \boxed{D_1 = 10 \frac{1}{4}} \\ \boxed{4B_3 = 00 - \frac{1}{4}} \\ \hline 100 \end{array}$$

$$\begin{array}{c|ccccc} & 1 & \frac{1}{2} & \frac{1}{2} & 1 & 0 \\ & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 1 \\ \hline 1 & & & & & \\ 0 & & & & & \\ \hline 1 & 0 & & & & \\ 0 & & 0 & & & \\ \hline 1 & 1 & -\frac{1}{3} & & & \\ 0 & & & & & \\ \hline 1 & 1 & -\frac{1}{3} & & & \end{array}$$

$$A = \begin{pmatrix} I_4 \\ 0 \end{pmatrix}$$

$$\begin{array}{c} \text{P}_1 \\ \parallel \\ \text{w-w-o-} \\ \text{o-o} \\ \text{w-w-o-} \end{array}$$

10

Dengan Car

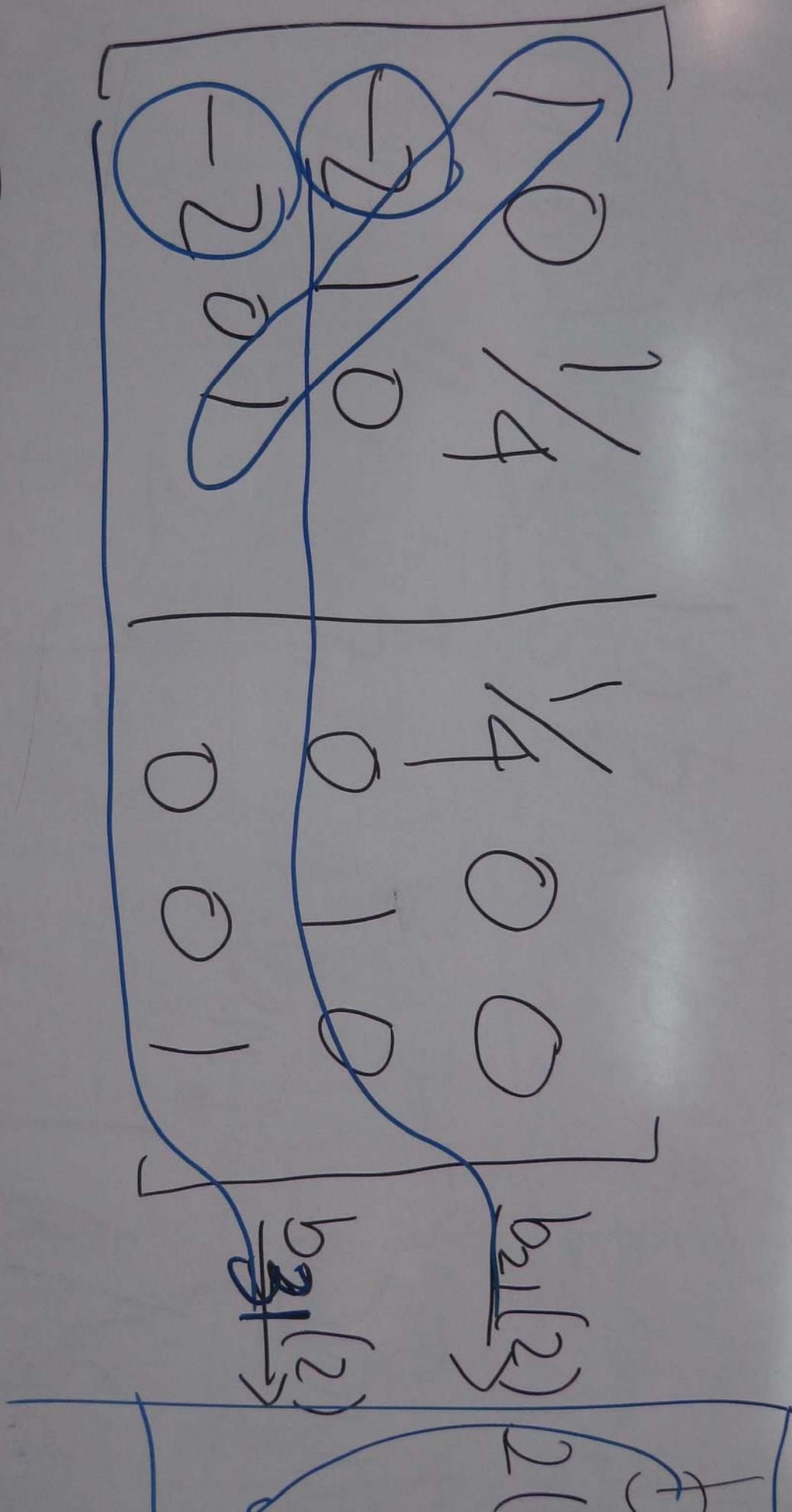
$$H_L = \dots$$

$$\begin{array}{r} A \\ \times I_3 \\ \hline -2 & -2 & -2 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ \hline 0 & 0 & 0 \end{array}$$

$\frac{b}{\sqrt{A}}$

$$a_1 \circ B E =$$

$$= 15 - 20 = 25$$



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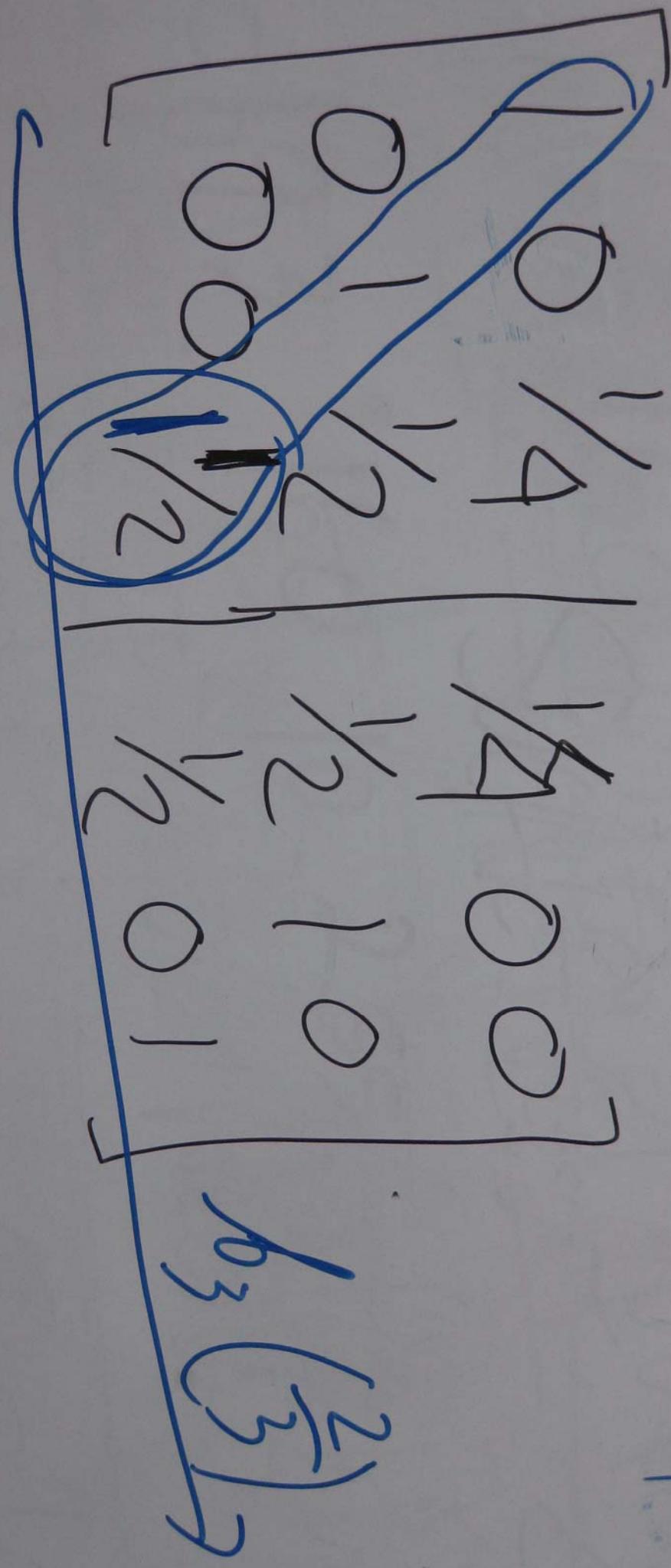
$$- (0 + 0 + 20)$$

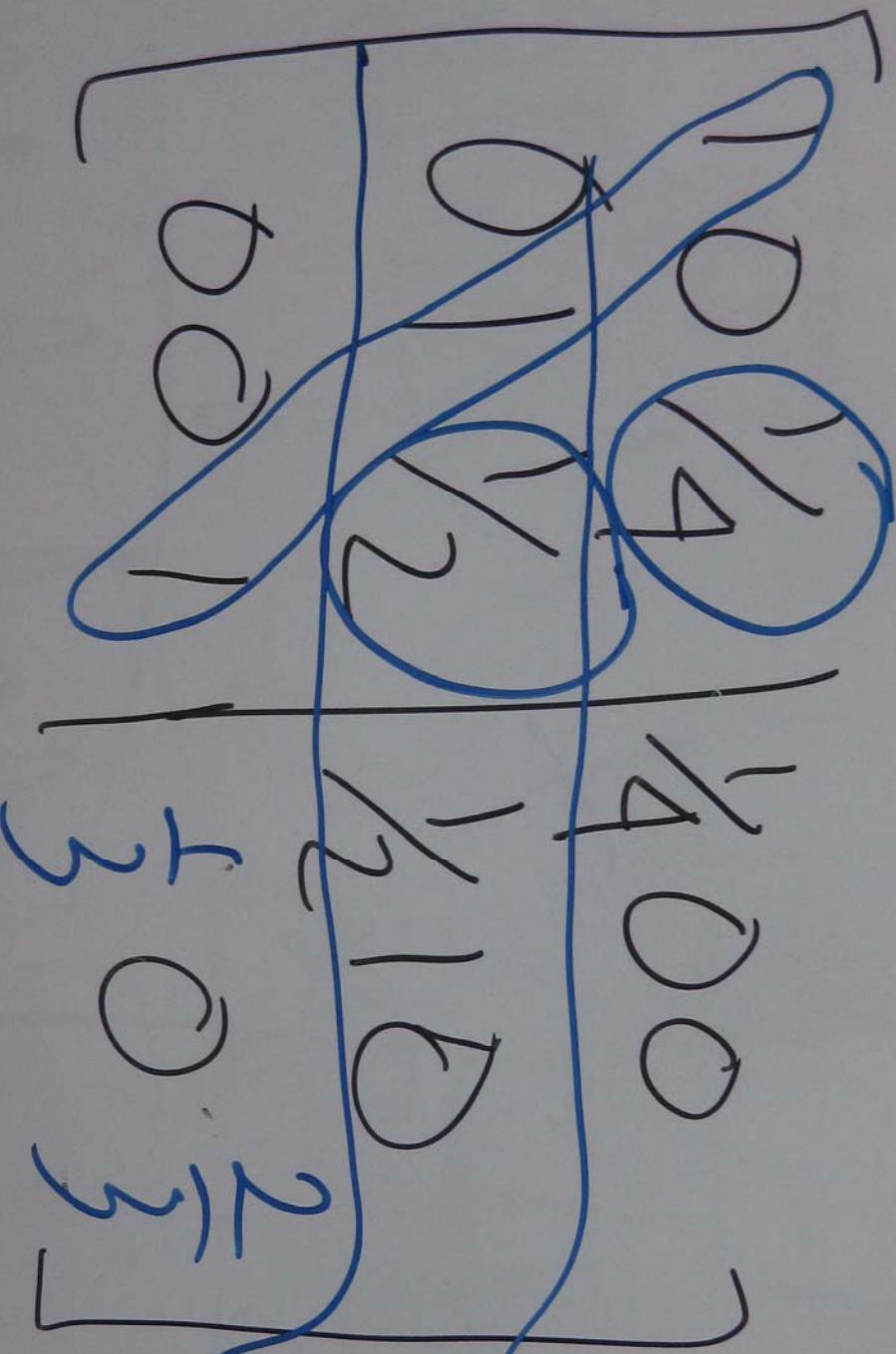
$$[(0 \cdot 5 \cdot 0) + (0 \cdot 0 \cdot 3) + (5 \cdot 2 \cdot -2)]$$

$$\begin{array}{r} \textcircled{b}_2 = -2 \quad 0 \quad 0 \quad 1 \quad 0 \\ \textcircled{b}_1 = 2 \quad 0 \quad \frac{1}{2} \quad \frac{1}{2} \quad 0 \quad 0 \\ \hline 0 \quad 1 \quad \frac{1}{2} \quad \frac{1}{2} \quad 1 \quad 0 \end{array}$$

$$\begin{array}{r} \textcircled{b}_3 = -2 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \\ \textcircled{b}_1 = 2 \quad 0 \quad \frac{1}{2} \quad \frac{1}{2} \quad 0 \quad 0 \\ \hline 0 \quad 0 \quad \frac{1}{2} \quad \frac{1}{2} \quad 0 \quad 1 \end{array}$$

$$\begin{array}{r} \textcircled{b}_1 = 1 \quad 0 \quad \frac{1}{4} \quad \frac{1}{4} \quad 0 \quad 0 \\ \textcircled{b}_3 = 0 \quad 0 \quad -\frac{1}{4} \quad -\frac{1}{2} \quad 0 \quad -\frac{1}{6} \\ \hline 0 \quad 0 \quad \frac{1}{4} \quad 0 \quad 0 \quad -\frac{1}{2} \end{array}$$





$$D_{13}(-\frac{1}{4}) b_{23}(-\frac{1}{2})$$

$$\frac{1}{4} \text{ st}$$

$$-\frac{g}{f} = 0 \quad \frac{g}{f} = 0$$

$$\frac{g}{f} - 0 = \frac{1}{2} - 0 = \frac{1}{2}$$

$$\frac{1}{2} - 0 = \frac{1}{4}$$

$$\zeta_1 = 1 + \frac{1}{T}$$

$$\zeta_2 = 0 + \frac{1}{T}$$

+

$$\zeta_3 = -\frac{1}{T}$$

$$\zeta_4 = 0 + \frac{1}{T}$$

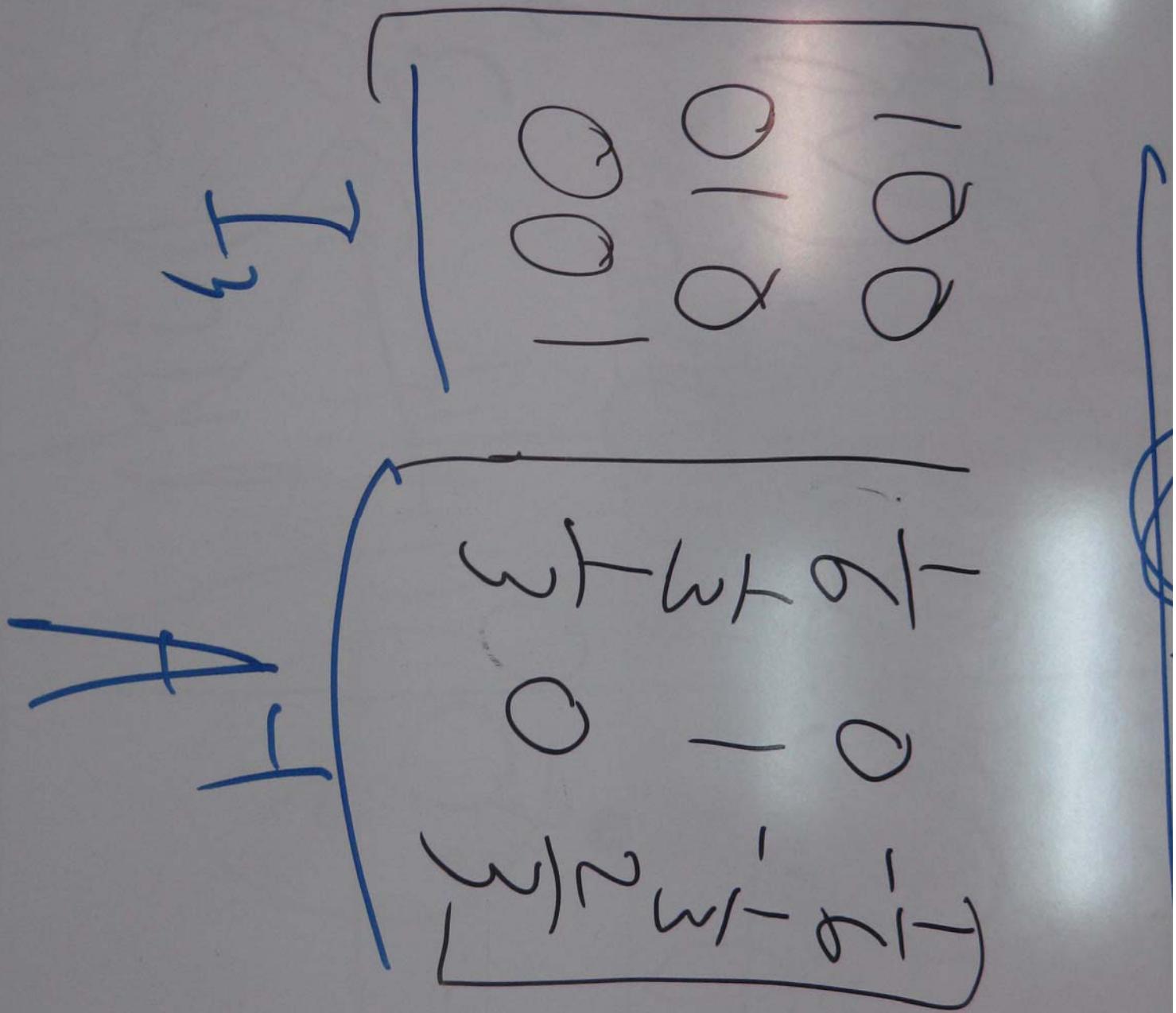
$$2\zeta_1 - 2\zeta_3 = 0$$

$$2\zeta_2 - 2\zeta_4 = 0$$

$$\phi_2 = 0$$

$$2\zeta_1 - 1 = 0$$

$$2\zeta_2 - 1 = 0$$



$$F = \frac{1}{r}$$

$$\left(\omega + \omega - \sigma \right) \frac{1}{r}$$

$$0 - 0$$

$$\left(\omega / 2 \quad \omega - \sigma \right) \frac{1}{r}$$

$$\left[\begin{array}{c} 0 \\ 0 \\ \hline 3 \\ 0 \\ \hline 3 \\ b_{23}(-j)^{1/2} \end{array} \right]$$

5)

$$B = \begin{pmatrix} 3 & -2 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 5 \end{pmatrix}$$

$$K_{11} = (-1)^{1+1} \begin{vmatrix} 3 & 0 \\ 0 & 5 \end{vmatrix} = + (15 - 0) = 15$$

$$K_{12} = (-1)^{1+2} \begin{vmatrix} -2 & 0 \\ 0 & 5 \end{vmatrix} = -(-10 - 0) = 10$$

$$K_{13} = (-1)^{1+3} \begin{vmatrix} -2 & 3 \\ 0 & 0 \end{vmatrix} = 0$$

$$K_{21} = (-1)^{2+1} \begin{vmatrix} -2 & 0 \\ 0 & 5 \end{vmatrix} = -(-10 - 0) = 10$$

$$K_{22} = (-1)^{2+2} \begin{vmatrix} 3 & 0 \\ 0 & 5 \end{vmatrix} = + (15 - 0) = 15$$

$$K_{23} = (-1)^{2+3} \begin{vmatrix} 3 & -2 \\ 0 & 0 \end{vmatrix} = 0$$

$$K_{31} = (-1)^{3+1} \begin{vmatrix} -2 & 0 \\ 3 & 0 \end{vmatrix} = 0$$

$$K_{32} = (-1)^{3+2} \begin{vmatrix} 3 & 0 \\ -2 & 0 \end{vmatrix} = 0$$

$$K_{33} = (-1)^{3+3} \begin{vmatrix} 3 & -2 \\ -2 & 3 \end{vmatrix} = + (9 - 4) = 5$$

$$|B| = b_{11} \cdot K_{11} + b_{12} \cdot K_{12} + b_{13} \cdot K_{13}$$

$$= 3 \cdot 15 + (-2) \cdot 10 + 0 \cdot 0$$

$$= 45 - 20 = \textcircled{25}$$

$$K = \begin{bmatrix} 15 & 10 & 0 \\ 10 & 15 & 0 \\ 0 & 0 & 5 \end{bmatrix} \longrightarrow \text{adj}(B) = K^t = \begin{bmatrix} 15 & 10 & 0 \\ 10 & 15 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$B^{-1} = \frac{1}{25} \begin{bmatrix} 15 & 10 & 0 \\ 10 & 15 & 0 \\ 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} 3/5 & 2/5 & 0 \\ 2/5 & 3/5 & 0 \\ 0 & 0 & 1/5 \end{bmatrix}$$

$$B = \begin{bmatrix} 3 & -2 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$K_{11} = (-1)^{1+1} \begin{vmatrix} 3 & 0 \\ 0 & 5 \end{vmatrix} = + (15 - 0) = 15$$

$$K_{12} = (-1)^{1+2} \begin{vmatrix} -2 & 0 \\ 0 & 5 \end{vmatrix} = -(-10 - 0) = 10$$

$$K_{13} = (-1)^{1+3} \begin{vmatrix} -2 & 3 \\ 0 & 0 \end{vmatrix} = 0$$

$$K_{21} = (-1)^{2+1} \begin{vmatrix} -2 & 0 \\ 0 & 5 \end{vmatrix} = -(-10 - 0) = 10$$

$$K_{22} = (-1)^{2+2} \begin{vmatrix} 3 & 0 \\ 0 & 5 \end{vmatrix} = + (15 - 0) = 15$$

$$K_{23} = (-1)^{2+3} \begin{vmatrix} 3 & -2 \\ 0 & 0 \end{vmatrix} = 0$$

$$\textcircled{5} = 45 - 20 = 25 = 3 \cdot 15 + (-2) \cdot 10 + 0 \cdot 0$$

$$|\beta| = \beta_{11} \cdot K_{11} + \beta_{12} \cdot K_{12} + \beta_{13} \cdot K_{13}$$

$$K_{32} = (-1)^{3+3} \begin{vmatrix} -2 & 3 \\ 0 & 0 \end{vmatrix} = 0$$

$$K_{33} = (-1)^{3+3} \begin{vmatrix} -2 & 3 \\ 0 & 0 \end{vmatrix} = 0$$

$$K_{13} = (-1)^{3+1} = 1$$

$$S = (h-g) = 5$$

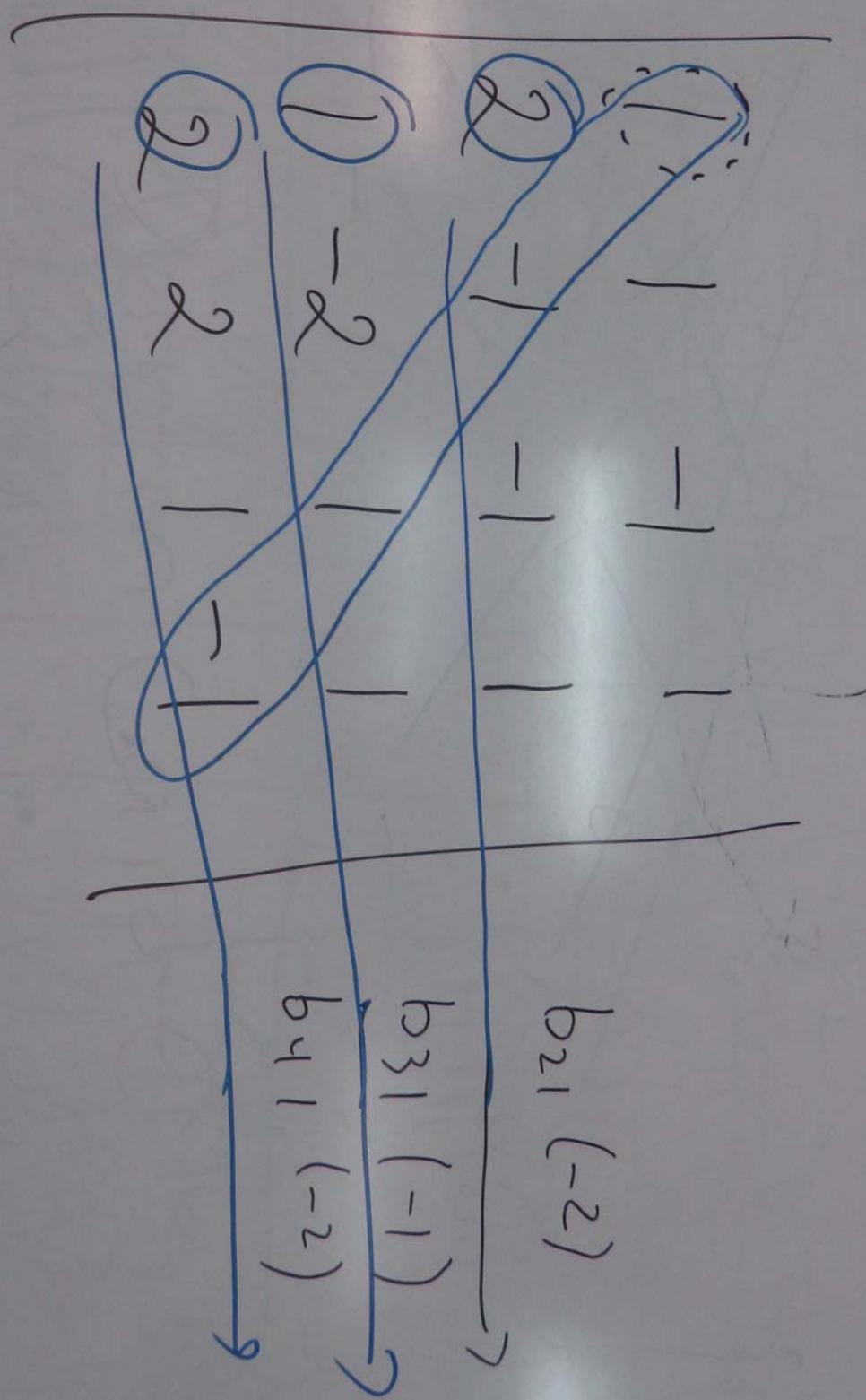
$$D = D_{11} \cdot K_{11} + D_{12} \cdot K_{12} + D_{13} \cdot K_{13} \\ = 3 \cdot 15 + (-2) \cdot 10 + 0 \cdot 0 \\ = 45 - 20 = \textcircled{25}$$

$$K = \begin{bmatrix} 15 & 10 & 0 \\ 10 & 15 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

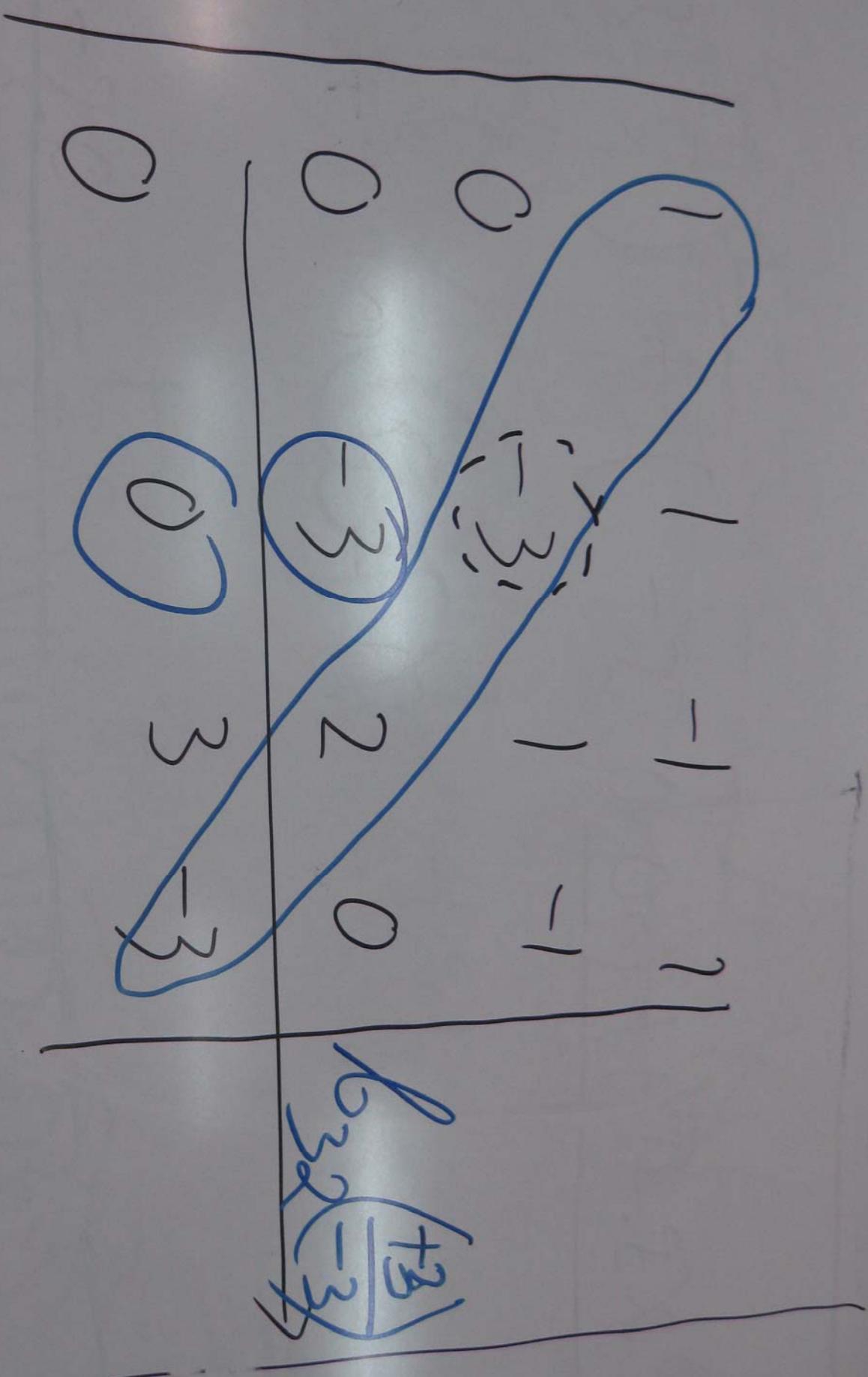
$$\rightarrow \text{adj}(B) = K^t = \begin{bmatrix} 15 & 10 & 0 \\ 10 & 15 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

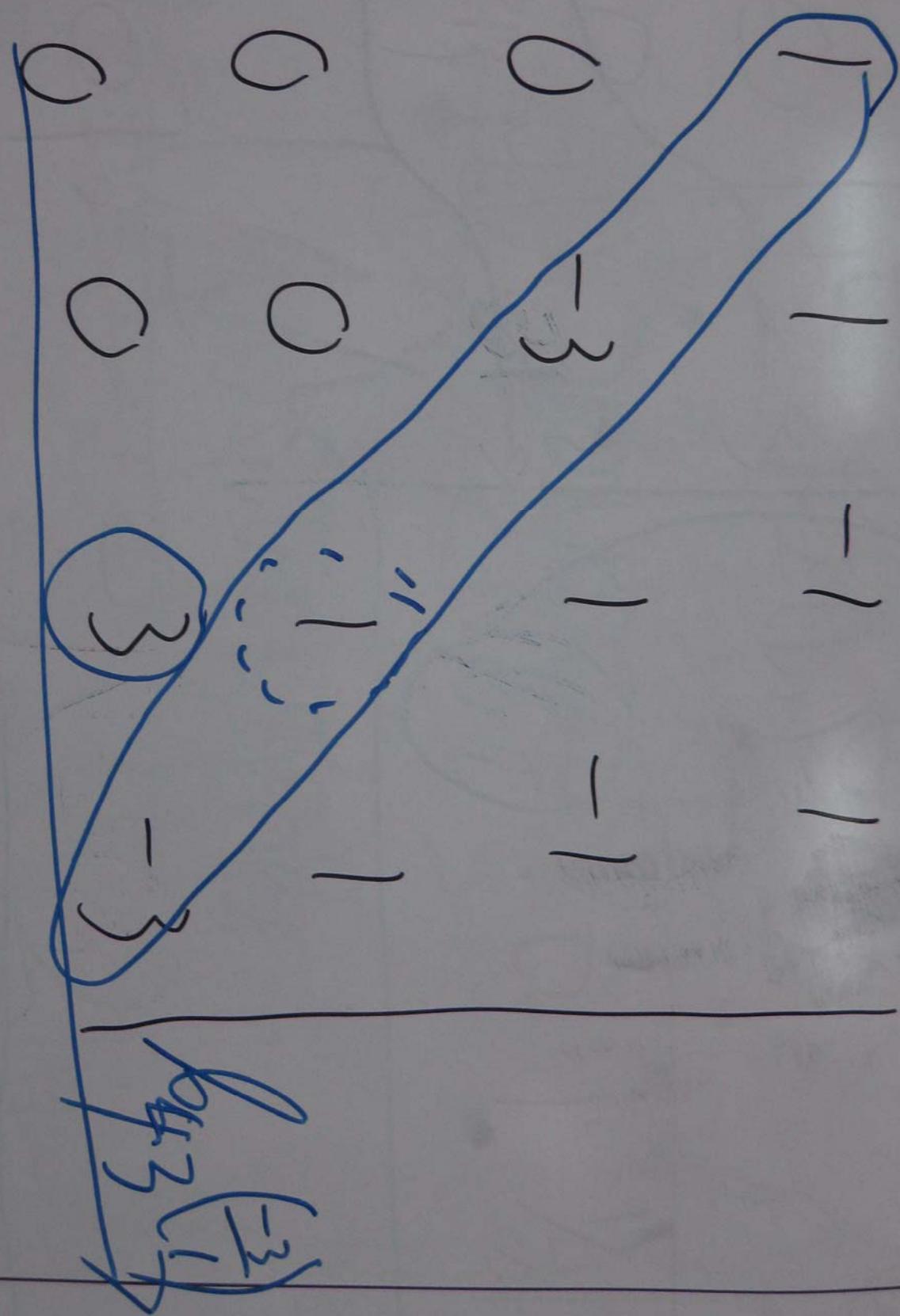
$$B^{-1} = \frac{1}{25} \begin{bmatrix} 15 & 10 & 0 \\ 10 & 15 & 0 \\ 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} 3/5 & 2/5 & 0 \\ 2/5 & 3/5 & 0 \\ 0 & 0 & 1/5 \end{bmatrix}$$

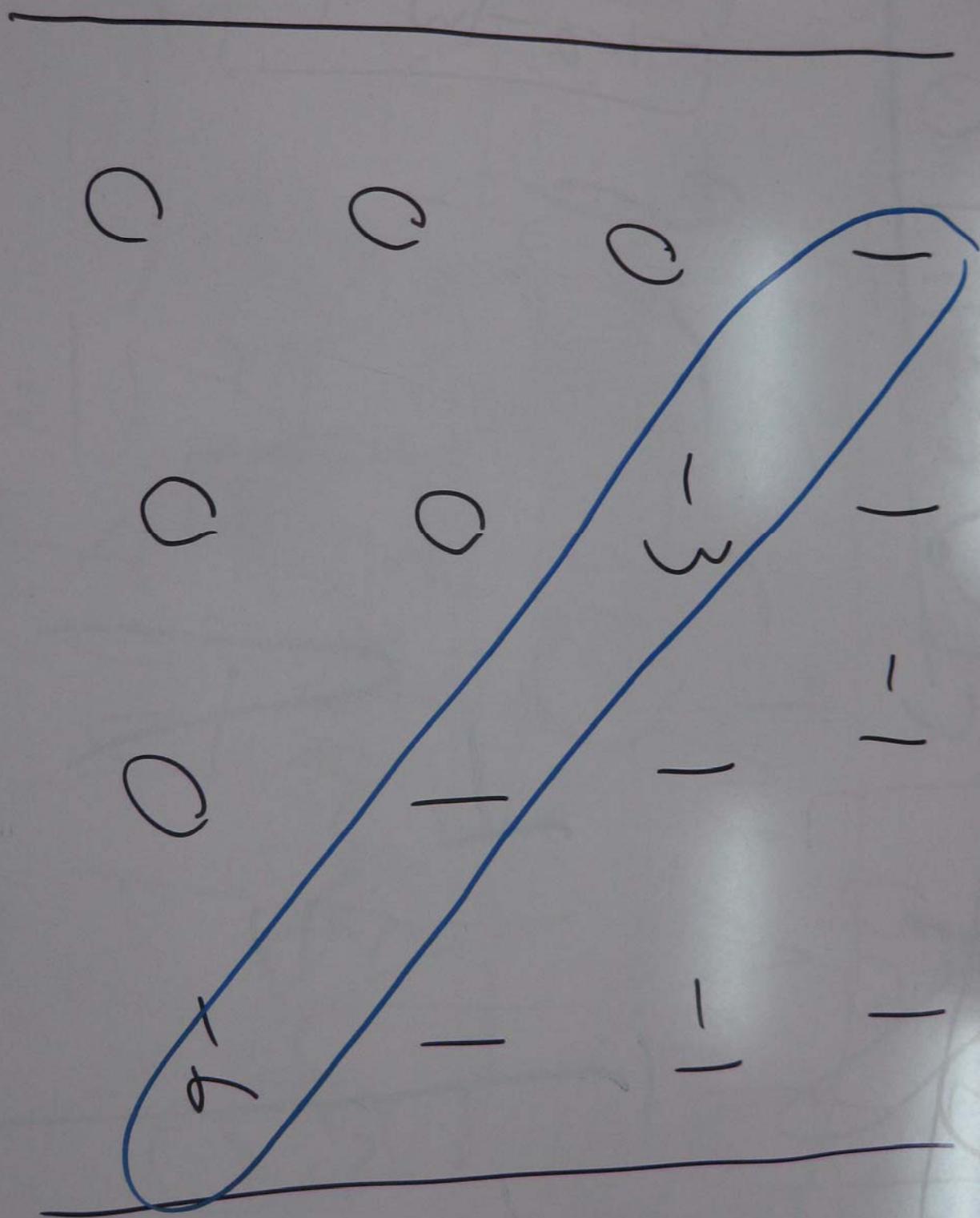
$$|A| =$$



b₃₂(-1)







$$18 = |A| = (-1) \cdot (-3) \cdot (1) \cdot (-6)$$