PD YANG DAPAT DIBUAT EKSAK

$$\boxed{M(x,y)\,dx + N(x,y)\,dy = 0}$$

$$\boxed{\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}} \longrightarrow \text{BUKAN PD. EKSAK}$$

MISALKAN : FAKTOR INTEGRASI : μ

1. $\mu = \mu(x)$
2. $\mu = \mu(y)$
3. $\mu = \mu(xy)$
4. $\mu = \mu(x+y)$
5. $\mu = \mu(x-y)$
6. $\mu = \mu(x^2+y^2)$

$$M(x,y)\,dx + N(x,y)\,dy = 0$$ PD lama

FAKTOR INTEGRASI

(μ)

$$\underbrace{\mu \cdot M(x,y)}_{P(x,y)}\,dx + \underbrace{\mu \cdot N(x,y)}_{Q(x,y)}\,dy = 0 \quad (\text{PD yg baru})$$

→ PD. EKSAK : $\boxed{\dfrac{\partial P}{\partial y} = \dfrac{\partial Q}{\partial x}}$

$$\frac{\partial}{\partial y}\left[\mu \cdot M(x,y)\right] = \frac{\partial}{\partial x}\left[\mu \cdot N(x,y)\right]$$

$$(\mu)' \cdot M(x,y) + \mu \cdot [M(x,y)]' = (\mu)' \cdot N(x,y) + \mu \cdot [N(x,y)]'$$

$$\frac{\partial \mu}{\partial y} \cdot M + \mu \cdot \frac{\partial M}{\partial y} = \frac{\partial \mu}{\partial x} \cdot N + \mu \cdot \frac{\partial N}{\partial x}$$

$$\mu \cdot \frac{\partial M}{\partial y} - \mu \cdot \frac{\partial N}{\partial x} = N \cdot \frac{\partial \mu}{\partial x} - M \cdot \frac{\partial \mu}{\partial y}$$

$$\boxed{\mu\left(\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}\right)=N\cdot\frac{\partial \mu}{\partial x}-M\cdot\frac{\partial \mu}{\partial y}}$$

(1) $\boxed{\mu=\mu(x)}$ $\rightarrow$ $\boxed{\frac{\partial \mu}{\partial x}=\frac{d\mu}{dx}}$

$\rightarrow \frac{\partial \mu}{\partial y}=0$

$$\mu\left(\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}\right)=N\cdot\frac{\partial \mu}{\partial x}-M\cdot\frac{\partial \mu}{\partial y}$$

$$\mu\left(\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}\right)=N\cdot\frac{d\mu}{dx}-\cancel{M\cdot 0}$$

$$\boxed{\left(\frac{\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}}{N}\right)\cdot dx=\frac{d\mu}{\mu}}$$

$$\left(\frac{\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}}{N}\right)=f(x)$$

① $$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = f(x) \longrightarrow \boxed{\mu = \mu(x)}$$

② $$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{-M} = f(y) \longrightarrow \mu = \mu(y)$$

③ $$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{y.N - x.M} = f(xy) \longrightarrow \mu = \mu(xy)$$

④ $$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N - M} = f(x+y) \longrightarrow \mu = \mu(x+y)$$

⑤ $$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N + M} = f(x-y) \longrightarrow \mu = \mu(x-y)$$

⑥ $$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{2x.N - 2y.M} = f(x^2+y^2) \longrightarrow \mu = \mu(x^2+y^2)$$

(2) $\boxed{\mu = \mu(y)}$ $\longrightarrow \dfrac{\partial \mu}{\partial x} = 0$

$\longrightarrow \dfrac{\partial \mu}{\partial y} = \dfrac{d\mu}{dy}$

$$\boxed{\mu \cdot \left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right) = N \cdot \frac{\partial \mu}{\partial x} - M \cdot \frac{\partial \mu}{\partial y}}$$

$$\mu \left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right) = \cancel{N \cdot 0} - M \cdot \frac{d\mu}{dy}$$

$$\boxed{\left(\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{-M} \right) dy = \frac{d\mu}{\mu}}$$

③ $\mu = \mu(\underset{z}{xy})$, MIS: $\boxed{z = xy}$, $\boxed{z = xy}$

$$\frac{\partial z}{\partial x} = (1)\cdot y = y \quad , \quad \frac{\partial z}{\partial y} = x(1) = x$$

$$\frac{\partial \mu}{\partial x} = \frac{\partial \mu}{\partial z}\cdot\frac{\partial z}{\partial x} = \frac{\partial \mu}{\partial z}\cdot y = y\cdot\frac{\partial \mu}{\partial z}$$

$$\frac{\partial \mu}{\partial y} = \frac{\partial \mu}{\partial z}\cdot\frac{\partial z}{\partial y} = \frac{\partial \mu}{\partial z}\cdot x = x\cdot\frac{\partial \mu}{\partial z}$$

$$\boxed{\mu\cdot\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = N\cdot\frac{\partial \mu}{\partial x} - M\cdot\frac{\partial \mu}{\partial y}}$$

$$\mu\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = N\cdot y\cdot\frac{\partial \mu}{\partial z} - M\cdot x\cdot\frac{\partial \mu}{\partial z}$$

$$= (y\cdot N - xM)\frac{\partial \mu}{\partial z}$$

$$\boxed{\left(\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{y\cdot N - x\cdot M}\right)\partial z = \frac{\partial \mu}{\mu}}$$

(4) $\boxed{\mu = \mu(x+y)}$, MIS : $z = x + y$

$$\frac{\partial z}{\partial x} = 1 + 0 = 1$$

$z = x + y$

$$\frac{\partial z}{\partial y} = 0 + (1) = 1$$

$$\frac{\partial \mu}{\partial x} = \frac{\partial \mu}{\partial z} \cdot \frac{\partial z}{\partial x} = \frac{\partial \mu}{\partial z} \cdot (1) = \frac{\partial \mu}{\partial z}$$

$$\frac{\partial \mu}{\partial y} = \frac{\partial \mu}{\partial z} \cdot \frac{\partial z}{\partial y} = \frac{\partial \mu}{\partial z}(1) = \frac{\partial \mu}{\partial z}$$

$$\mu \cdot \left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = N \cdot \frac{\partial \mu}{\partial x} - M \cdot \frac{\partial \mu}{\partial y}$$

$$\mu \left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = N \cdot \frac{\partial \mu}{\partial z} - M \cdot \frac{\partial \mu}{\partial z} = (N - M) \cdot \frac{\partial \mu}{\partial z}$$

$$\left(\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N - M}\right) \partial z = \frac{\partial \mu}{\mu}$$

(5) $\mu = \mu(x-y)$, MIS: $\boxed{z = x - y}$ | $\boxed{z = x - y}$

$\frac{\partial z}{\partial x} = 1 - 0 = 1$ | $\frac{\partial z}{\partial y} = 0 - (1) = -1$

$$\frac{\partial \mu}{\partial x} = \frac{\partial \mu}{\partial z} \cdot \frac{\partial z}{\partial x} = \frac{\partial \mu}{\partial z} \cdot (1) = \frac{\partial \mu}{\partial z}$$

$$\frac{\partial \mu}{\partial y} = \frac{\partial \mu}{\partial z} \cdot \frac{\partial z}{\partial y} = \frac{\partial \mu}{\partial z} \cdot (-1) = -\frac{\partial \mu}{\partial z}$$

$$\boxed{\mu \cdot \left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = N \cdot \frac{\partial \mu}{\partial x} - M \cdot \frac{\partial \mu}{\partial y}}$$

$$\mu \left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = N \cdot \frac{\partial \mu}{\partial z} - M \cdot \left(-\frac{\partial \mu}{\partial z}\right)$$

$$= (N + M) \cdot \frac{\partial \mu}{\partial z}$$

$$\left(\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N + M}\right) \partial z = \frac{\partial \mu}{\mu}$$

⑥ $\boxed{\mu = \mu(x^2+y^2)}$, MIS: $\boxed{z = x^2+y^2}$

$$\frac{\partial z}{\partial x} = 2x + 0 = 2x$$

$\boxed{z = x^2+y^2}$

$$\frac{\partial z}{\partial y} = 0 + 2y = 2y$$

$$\frac{\partial \mu}{\partial x} = \frac{\partial \mu}{\partial z} \cdot \frac{\partial z}{\partial x} = \frac{\partial \mu}{\partial z} \cdot (2x) = 2x \cdot \frac{\partial \mu}{\partial z}$$

$$\frac{\partial \mu}{\partial y} = \frac{\partial \mu}{\partial z} \cdot \frac{\partial z}{\partial y} = \frac{\partial \mu}{\partial z} \cdot (2y) = 2y \cdot \frac{\partial \mu}{\partial z}$$

$$\boxed{\mu \cdot \left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = N \cdot \frac{\partial \mu}{\partial x} - M \cdot \frac{\partial \mu}{\partial y}}$$

$$\mu \left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = N \cdot \left(2x \cdot \frac{\partial \mu}{\partial z}\right) - M \left(2y \cdot \frac{\partial \mu}{\partial z}\right)$$

$$= (2x \cdot N - 2y \cdot M) \cdot \frac{\partial \mu}{\partial z}$$

$$\left(\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{2x \cdot N - 2y M}\right) \cdot \partial z = \frac{\partial \mu}{\mu}$$

$$\underbrace{e^x(x+y)}_{P(x,y)}\,dx + \underbrace{e^x}_{Q(x,y)}\,dy = 0$$

(II) $$F(x,y) = \int Q(x,y)\,dy + c(x) \longrightarrow \boxed{\frac{\partial F}{\partial x} = P(x,y)}$$

$$F(x,y) = \int e^x\,dy + c(x)$$

$$F(x,y) = e^x \int dy + c(x)$$

$$\boxed{F(x,y) = e^x \cdot y + c(x)}$$

$$\to \frac{\partial F}{\partial x} = (e^x)\,y + c'(x) = P(x,y)$$

$$\cancel{e^x \cdot y} + c'(x) = e^x \cdot x + \cancel{e^x \cdot y}$$

$$\underline{c'(x) = e^x \cdot x}$$

$$C(x) = \int c'(x)\,dx = \int e^x \cdot x\,dx$$

$$= \int \underbrace{x}_{u} \cdot \underbrace{e^x\,dx}_{dv}$$

$$= \int \underbrace{x}_{u} \cdot d(\underbrace{e^x}_{v})$$

$dv = e^x dx$

$\int dv = \int e^x dx$

$\boxed{v = e^x}$

$$= x \cdot e^x - \int e^x d(x)$$

$$= x \cdot e^x - \int e^x \cdot dx$$

$$C(x) = x \cdot e^x - e^x + C$$

Sol. um. PD

$$F(x,y) = 0$$

$$e^x \cdot y + C(x) = 0$$

$$e^x \cdot y + x e^x - e^x + C = 0$$

④ $\underbrace{(x^2+y^2+x)}_{M(x,y)}\,dx + \underbrace{y}_{N(x,y)}\,dy = 0$

$M(x,y) = x^2 + y^2 + x$ | $N(x,y) = y$

$\frac{\partial M}{\partial y} = 0 + 2y + 0 = 2y$ | $\frac{\partial N}{\partial x} = 0$

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{2x\cdot N - 2y\cdot M} = \frac{2y - 0}{-2y(x^2+y^2)} = \frac{-1}{x^2+y^2} = f(x^2+y^2)$$

$$2x.N = 2x(y) = \cancel{2xy}$$

$$2y.M = 2y(x^2+y^2+x) = 2x^2y + 2y^3 + \cancel{2xy} \quad \ominus$$

$$2x.N - 2y.M = -2x^2y - 2y^3$$

$$= -2y(x^2+y^2)$$

$\rightarrow \mu = \mu(x^2+y^2)$

MIS: $\boxed{z = x^2+y^2}$

$$\boxed{\frac{\partial \mu}{\mu} = \left(\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{2x \cdot N - 2y \cdot M}\right) \cdot \partial z}$$

$f(x^2+y^2)$

$$\frac{\partial \mu}{\mu} = \frac{-1}{x^2+y^2} \cdot \partial z$$

$$\frac{\partial \mu}{\mu} = -\frac{\partial z}{z}$$

$$\int \frac{\partial \mu}{\mu} = -\int \frac{\partial z}{z}$$

$$\ln \mu = -\ln z$$

$$\ln \mu = \ln z^{-1}$$

$$\mu = z^{-1} = \frac{1}{z}$$

$$\boxed{\mu = \frac{1}{x^2+y^2}}$$

$$\underline{(x^2+y^2+x)\,dx + y\,dy = 0}\left(\frac{1}{x^2+y^2}\right)$$

$$\frac{(x^2+y^2)+x}{(x^2+y^2)}dx + \frac{y}{x^2+y^2}dy = 0$$

$$\left(\cancel{\frac{x^2+y^2}{x^2+y^2}} + \frac{x}{x^2+y^2}\right)dx + \frac{y}{x^2+y^2}dy = 0$$

$$\underbrace{\left(1 + \frac{x}{x^2+y^2}\right)}_{P(x,y)}dx + \underbrace{\frac{y}{x^2+y^2}}_{Q(x,y)}dy = 0$$

$$P(x,y) = 1 + \frac{x}{x^2+y^2} = 1 + x(x^2+y^2)^{-1}$$

$$\frac{\partial P}{\partial y} = 0 + x.(-1).(x^2+y^2)^{-1-1}.(x^2+y^2)'$$

$$= -x(x^2+y^2)^{-2}.(2y)$$

$$= \boxed{\frac{-2xy}{(x^2+y^2)^2}}$$

$$Q(x,y) = \frac{y}{x^2+y^2}$$

$$\frac{\partial Q}{\partial x} = \frac{y' \cdot (x^2+y^2) - (x^2+y^2)' \cdot y}{(x^2+y^2)^2}$$

$$= \frac{\cancel{0 \cdot (x^2+y^2)} - (2x+0) \cdot y}{(x^2+y^2)^2}$$

$$= \frac{-2xy}{(x^2+y^2)^2}$$

$$\longrightarrow \boxed{\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}} \longrightarrow \text{PD. EKSAK}$$

$$\underbrace{\left(1+\frac{x}{x^2+y^2}\right)}_{P(x,y)} dx + \underbrace{\frac{y}{x^2+y^2}}_{Q(x,y)} dy = 0$$

II $F(x,y) = \int Q(x,y)\,dy + c(x)$ $\boxed{\frac{\partial F}{\partial x} = P(x,y)}$

$$F(x,y) = \int \frac{y\,dy}{x^2+y^2} + c(x)$$

$$F(x,y) = \frac{1}{2}\int \frac{\overset{2y\,dy}{\cancel{d(x^2+y^2)}}}{(x^2+y^2)} + c(x)$$

$$\boxed{F(x,y) = \frac{1}{2}\ln(x^2+y^2) + c(x)}$$

$$\longrightarrow \frac{\partial F}{\partial x} = \frac{1}{2}\cdot\frac{1}{x^2+y^2}\left((x^2)+y^2\right)' + c'(x) = P(x,y)$$

$$\cancel{\frac{2x}{2(x^2+y^2)}} + c'(x) = 1 + \cancel{\frac{x}{x^2+y^2}}$$

$c'(x$

$c(x)=$

Sol

$$C'(x) = 1$$

$$C(x) = \int C'(x)\,dx = \int (1)\,dx = \int dx = \underline{x + C}$$

SOL. UM. PD

$$F(x,y) = 0$$

$$\frac{1}{2}\ln(x^2+y^2) + C(x) = 0$$

$$\boxed{\frac{1}{2}\ln(x^2+y^2) + x + C = 0} \quad \times 2$$

$$\ln(x^2+y^2) + 2x + 2C = 0$$

$$\ln(x^2+y^2) + 2x = -2C$$

$$\ln(x^2+y^2) + \ln e^{2x} = \ln C$$

$$\ln e^{2x}(x^2+y^2) = \ln C$$

$$\boxed{e^{2x}(x^2+y^2) = C}$$