DETERMINAN

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 1 & 5 \\ 3 & 2 & 4 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

MATRIKS A DI...

$$|A| = a_{21} \cdot K_{21} + \dots$$
$$= (4)(-2) + \dots$$
$$= -8 \dots$$
$$= - \dots$$

$$\begin{array}{ccc} \cancel{1} & 2 & 3 \\ \cancel{4} & \cancel{1} & \cancel{5} \\ \cancel{3} & 2 & 4 \end{array} \quad K_{21} = (-1)^{2+1} \begin{vmatrix} 2 & 3 \\ 2 & 4 \end{vmatrix}$$
$$= (-1)(8-6) = -2$$

$$\begin{array}{ccc} 1 & 2 & \cancel{3} \\ \cancel{4} & \cancel{1} & \cancel{5} \\ 3 & 2 & \cancel{4} \end{array} \quad K_{23} = (-1)^{2+3} \begin{vmatrix} 1 & 2 \\ 3 & 2 \end{vmatrix}$$
$$= (-1)(2-6)$$
$$= 4$$

$$\begin{array}{ccc} 1 & \cancel{2} & 3 \\ \cancel{4} & \cancel{1} & \cancel{5} \\ 3 & \cancel{2} & 4 \end{array} \quad K_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 3 \\ 3 & 4 \end{vmatrix}$$
$$= +1(4-9) = -5$$

MATRIKS A DI EKSPANSI PADA BARIS-2

$$|A| = a_{21} \cdot K_{21} + a_{22} \cdot K_{22} + a_{23} \cdot K_{23}$$
$$= (4)(-2) + (1)(-5) + (5)(4)$$
$$= -8 - 5 + 20$$
$$= -13 + 20 = 7$$

$$\begin{vmatrix} 1 & 2 \\ 3 & 2 \end{vmatrix}$$

MATRIKS A DIEKSPANSI PADA KOLOM-3

$$|A| = a_{13} \cdot K_{13} + a_{23} \cdot K_{23} + a_{33} \cdot K_{33} = (3)(5) + (5)(4) + (4)(-7)$$

$$= 15 + 20 - 28$$

$$= 35 - 28 = 7$$

$$\begin{matrix} \cancel{1} & \cancel{2} & \cancel{3} \\ 4 & 1 & \cancel{5} \\ 3 & 2 & \cancel{4} \end{matrix} \qquad K_{13} = (-1)^{1+3} \begin{vmatrix} 4 & 1 \\ 3 & 2 \end{vmatrix} = (+1)(8-3) = 5$$

$$\begin{matrix} 1 & 2 & \cancel{3} \\ \cancel{4} & \cancel{1} & \cancel{5} \\ 3 & 2 & \cancel{4} \end{matrix} \qquad K_{23} = (-1)^{2+3} \begin{vmatrix} 1 & 2 \\ 3 & 2 \end{vmatrix} = (-1)(2-6) = 4$$

$$\begin{matrix} 1 & 2 & \cancel{3} \\ 4 & 1 & \cancel{5} \\ \cancel{3} & \cancel{2} & \cancel{4} \end{matrix} \qquad K_{33} = (-1)^{3+3} \begin{vmatrix} 1 & 2 \\ 4 & 1 \end{vmatrix} = (+1)(1-8) = -7$$

MATRIKS KOFAKTOR

$$K = \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix}$$

MATRIKS ADJOINT

$$\boxed{\operatorname{adj}(A) = K^t}$$

$$\boxed{A^{-1} = \frac{1}{|A|} \cdot \operatorname{adj}(A)}$$

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 3 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \longrightarrow A^{-1} = ?$$

CARA KOFAKTOR

$$\begin{matrix} 2 & 0 & 1 \\ 3 & 1 & 0 \\ 1 & 0 & 1 \end{matrix} \quad K_{11} = (-1)^{1+1} \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = (+1)(1-0) = 1$$

$$\begin{matrix} 2 & 0 & 1 \\ 3 & 1 & 0 \\ 1 & 0 & 1 \end{matrix} \quad K_{12} = (-1)^{1+2} \begin{vmatrix} 3 & 0 \\ 1 & 1 \end{vmatrix} = (-1)(3-0) = -3$$

$$\begin{matrix} 2 & 0 & 1 \\ 3 & 1 & 0 \\ 1 & 0 & 1 \end{matrix} \quad K_{13} = (-1)^{1+3} \begin{vmatrix} 3 & 1 \\ 1 & 0 \end{vmatrix} = (+1)(0-1) = -1$$

$$\begin{matrix} 2 & 0 & 1 \\ 3 & 1 & 0 \\ 1 & 0 & 1 \end{matrix} \quad K_{21} = (-1)^{2+1} \begin{vmatrix} 0 & 1 \\ 0 & 1 \end{vmatrix} = 0$$

$$\begin{matrix} 2 & 0 & 1 \\ 3 & 1 & 0 \\ 1 & 0 & 1 \end{matrix} \quad K_{22} = (-1)^{2+2} \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} = (+1)(2-1) = 1$$

$$\begin{matrix} 2 & 0 & 1 \\ 3 & 1 & 0 \\ 1 & 0 & 1 \end{matrix} \quad K_{23} = (-1)^{2+3} \begin{vmatrix} 2 & 0 \\ 1 & 0 \end{vmatrix} = 0$$

$$\begin{matrix} 2 & 0 & 1 \\ 3 & 1 & 0 \\ 1 & 0 & 1 \end{matrix}$$

$$K_{31} = (-1)^{3+1} \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}$$

$$= (+1)(0-1) = -1$$

$$\begin{matrix} 2 & 0 & 1 \\ 3 & 1 & 0 \\ 1 & 0 & 1 \end{matrix}$$

$$K_{32} = (-1)^{3+2} \begin{vmatrix} 2 & 1 \\ 3 & 0 \end{vmatrix}$$

$$= (-1)(0-3) = 3$$

$$\begin{matrix} 2 & 0 & 1 \\ 3 & 1 & 0 \\ 1 & 0 & 1 \end{matrix}$$

$$K_{33} = (-1)^{3+3} \begin{vmatrix} 2 & 0 \\ 3 & 1 \end{vmatrix}$$

$$= (+1)(2-0) = 2$$

$$|A| = a_{11} \cdot K_{11} + a_{12} \cdot K_{12} + a_{13} \cdot K_{13}$$

$$= (2)(1) + (0)(-3) + (1)(-1)$$

$$= 2 + 0 - 1 = 1$$

$$K = \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix} = \begin{bmatrix} 1 & -3 & -1 \\ 0 & 1 & 0 \\ -1 & 3 & 2 \end{bmatrix}$$

$$adj(A) = K^t = \begin{bmatrix} 1 & 0 & -1 \\ -3 & 1 & 3 \\ -1 & 0 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \cdot adj(A) = \frac{1}{1} \begin{bmatrix} 1 & 0 & -1 \\ -3 & 1 & 3 \\ -1 & 0 & 2 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1 & 0 & -1 \\ -3 & 1 & 3 \\ -1 & 0 & 2 \end{bmatrix}$$

SISTEM PERSAMAAN LINEAR (SPL)

$$a_{11} \cdot x_1 + a_{12} \cdot x_2 + a_{13} \cdot x_3 = b_1$$
$$a_{21} \cdot x_1 + a_{22} \cdot x_2 + a_{23} \cdot x_3 = b_2$$
$$a_{31} \cdot x_1 + a_{32} \cdot x_2 + a_{33} \cdot x_3 = b_3$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$\boxed{A \cdot \vec{X} = B}$$

MENCARI SPL

1) ELIMINASI & SUBSTITUSI (SMA)

2) $(A \mid B) \xrightarrow{OBE} (I \mid \vec{X})$

3) $A \cdot \vec{X} = B$

$$\underbrace{A^{-1} \cdot A}_{I} \cdot \vec{X} = A^{-1} \cdot B$$

$$\boxed{\vec{X} = \frac{1}{|A|} \cdot \operatorname{adj}(A) \cdot B}$$

4) ATURAN CRAMER

$$x_j = \frac{|A_j|}{|A|}$$

$$\boxed{\begin{aligned} 2x_1 - x_2 &= 3 \\ x_1 + 3x_2 &= 5 \end{aligned}}$$

I)
$$\begin{aligned} 2x_1 - x_2 &= 3 \\ x_1 + 3x_2 &= 5 \end{aligned} \quad \begin{vmatrix} \times 1 \\ \times 2 \end{vmatrix}$$

$$\begin{aligned} \cancel{2x_1} - x_2 &= 3 \\ \cancel{2x_1} + 6x_2 &= 10 \quad - \\ \hline -7x_2 &= -7 \end{aligned}$$

$$\boxed{x_2 = 1}$$

$$x_1 + 3x_2 = 5$$
$$x_1 + 3(1) = 5$$
$$x_1 = 5 - 3$$

$$\boxed{x_1 = 2}$$

II)

II) $\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$

$$A \cdot \vec{x} = B$$

$$(A \mid B) \xrightarrow{OBE} (I \mid \vec{x})$$

$$\left[\underbrace{\begin{matrix} 2 & -1 \\ 1 & 3 \end{matrix}}_{A} \;\middle|\; \underbrace{\begin{matrix} 3 \\ 5 \end{matrix}}_{B}\right] \xrightarrow[b_{21}]{b_{12}} \left[\begin{array}{cc|c} 1 & 3 & 5 \\ 2 & -1 & 3 \end{array}\right] \xrightarrow{b_{21}(-2)} \left[\begin{array}{cc|c} 1 & 3 & 5 \\ 0 & -7 & -7 \end{array}\right] \xrightarrow{b_2\left(\frac{1}{-7}\right)} \left[\begin{array}{cc|c} 1 & 3 & 5 \\ 0 & 1 & 1 \end{array}\right] \xrightarrow{b_{12}(-3)}$$

$b_2 = \underline{b_2} + \underline{(-2)\,b_1}$

$$\begin{array}{rrr|r} b_2: & 2 & -1 & 3 \\ -2b_1: & -2 & -6 & -10 \\ \hline b_2: & 0 & -7 & -7 \end{array} +$$

$b_1 = \underline{b_1} + \underline{(-3)\,b_2}$

$$\begin{array}{rrr|r} b_1: & 1 & 3 & 5 \\ -3b_2: & 0 & -3 & -3 \\ \hline b_1: & 1 & 0 & 2 \end{array} +$$

$$\left[\underbrace{\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}}_{I_2} \;\middle|\; \underbrace{\begin{matrix} 2 \\ 1 \end{matrix}}_{\vec{x}}\right] \to \boxed{\begin{matrix} x_1 = 2 \\ x_2 = 1 \end{matrix}}$$

III

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} \quad \vec{X} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad B = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & -1 \\ 1 & 3 \end{vmatrix} = 6 - (-1) = 6 + 1 = 7$$

$$\vec{X} = A^{-1} \cdot B$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \frac{1}{|A|} \cdot adj(A) \cdot B$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \frac{1}{7} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \frac{1}{7} \begin{bmatrix} 9+5 \\ -3+10 \end{bmatrix} = \frac{1}{7} \begin{bmatrix} 14 \\ 7 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \begin{array}{l} \rightarrow x_1 = 2 \\ \rightarrow x_2 = 1 \end{array}$$

IV ATURAN CRAMER

$$X_1 = \frac{|A_1|}{|A|} = \frac{\begin{vmatrix} 3 & -1 \\ 5 & 3 \end{vmatrix}}{\begin{vmatrix} 2 & -1 \\ 1 & 3 \end{vmatrix}} = \frac{9-(-5)}{7} = \frac{14}{7} = 2$$

$$X_2 = \frac{|A_2|}{|A|} = \frac{\begin{vmatrix} 2 & 3 \\ 1 & 5 \end{vmatrix}}{\begin{vmatrix} 2 & -1 \\ 1 & 3 \end{vmatrix}} = \frac{10-3}{7} = \frac{7}{7} = 1$$

Warning, the protected names norm and trace have been redefined and unprotected

```
> A:=matrix([[5,6,7,-1,2],[-3,2,1,0,3],[0,3,-2,0,4],[0,1,0,0,0],[0,0,1,0,-1]]);
```

$$A := \begin{bmatrix} 5 & 6 & 7 & -1 & 2 \\ -3 & 2 & 1 & 0 & 3 \\ 0 & 3 & -2 & 0 & 4 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 \end{bmatrix} \quad (13)$$

```
> 
```

```
> det(A);
```

$$-6$$

$$\begin{vmatrix} 5 & 6 & 7 & -1 & 2 \\ -3 & 2 & 1 & 0 & 3 \\ 0 & 3 & -2 & 0 & 4 \\ \boxed{0 \quad 1 \quad 0 \quad 0 \quad 0} \\ 0 & 0 & 1 & 0 & -1 \end{vmatrix} = a_{41}.K_{41} + a_{42}.K_{42} + a_{43}.K_{43} + a_{44}.K_{43} + a_{45}.K_{45}$$

$$= \cancel{0.K_{41}} + (1).K_{42} + \cancel{0.K_{43}} + \cancel{0.K_{43}} + \cancel{0.K_{45}}$$

$$= K_{42} = \underbrace{(-1)^{4+2}}_{+1} \begin{vmatrix} 5 & 7 & -1 & 2 \\ -3 & 1 & 0 & 3 \\ 0 & -2 & 0 & 4 \\ 0 & 1 & 0 & -1 \end{vmatrix} = a_{13}.K_{13} + a_{23}.K_{23} + a_{33}.K_{33} + a_{43}.K_{43}$$

$$= (-1).K_{13} + \cancel{0.K_{23}} + \cancel{0.K_{33}} + \cancel{0.K_{43}}$$

$$= (-1).\underbrace{(-1)^{1+3}}_{+1} \begin{vmatrix} -3 & 1 & 3 \\ 0 & -2 & 4 \\ 0 & 1 & -1 \end{vmatrix} \begin{matrix} -3 & 1 \\ 0 & -2 \\ 0 & 1 \end{matrix}$$

$$= (-1)\left[(-6+0+0) - (0-12-0)\right]$$

$$= -\left(\underbrace{-6+12}_{6}\right) = -6$$

$$\begin{vmatrix} 5 & 6 & 7 & -1 & 2 \\ -3 & 2 & 1 & 0 & 3 \\ 0 & 3 & -2 & 0 & 4 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 \end{vmatrix}$$

$= a_{41} \cdot$

$= 0$

$=$

$a_{41} \cdot K_{41} + a_{42} \cdot K_{42} + a_{43} \cdot K_{43} + a_{44} \cdot K_{43} + a_{45} \cdot K_{45}$

$\cancel{0 \cdot K_{41}} + (1) \cdot K_{42} + \cancel{0 \cdot K_{43}} + \cancel{0 \cdot K_{43}} + \cancel{0 \cdot K_{45}}$

$= K_{42} = \underbrace{(-1)^{4+2}}_{+1} \begin{vmatrix} 5 & 7 & -1 & 2 \\ -3 & 1 & 0 & 3 \\ 0 & -2 & 0 & 4 \\ 0 & 1 & 0 & -1 \end{vmatrix} = a_{13} \cdot K_{13} + a_{23} \cdot K_{23} + a_{33} \cdot K_{33} + a_{43} \cdot K_{43}$

$= (-1) \cdot K_{13} + \cancel{0 \cdot K_{23}} + \cancel{0 \cdot K_{33}} + \cancel{0 \cdot K_{43}}$

$= (-1) \cdot \underbrace{(-1)^{1+3}}_{+1} \begin{vmatrix} -3 & 1 & 3 \\ 0 & -2 & 4 \\ 0 & 1 & -1 \end{vmatrix} \begin{matrix} -3 & 1 \\ 0 & -2 \\ 0 & 1 \end{matrix}$

$= (-1)[(-6 + 0 + 0) - (0 - 12 - 0)]$

$= -(-6 + 12) = -6$