PD YG DAPAT DIJADIKAN PD. HOMOGEN

$$(ax + by + \underset{0}{\underset{\parallel}{c}})\,dx + (px + qy + \underset{0}{\underset{\parallel}{r}})\,dy = 0$$

$\rightarrow$ $(ax + by)\,dx + (px + qy)\,dy = 0$

(I) $\longrightarrow$ PD. HOMOGEN DERAJAT-1

(II) $(2x + 3y + 1)\,dx + (4x + 6y + 3)\,dy = 0$

$$\Big[\underbrace{(2x+3y)}_{z} + 1\Big]\,dx + \Big[2\underbrace{(2x+3y)}_{z} + 3\Big]\,dy = 0$$

MISALKAN : $2x + 3y = z$

$$3y = z - 2x$$

$$y = \frac{1}{3}z - \frac{2}{3}x$$

$$dy = \frac{1}{3}\,dz - \frac{2}{3}\,dx$$

$y = \frac{1}{3}z$ ...

diferens...

$\frac{dy}{dx} =$ ...

$dy =$ ...

PD YG DAPAT DIJADIKAN PD. HOMOGEN

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$$\rightarrow (ax+by)\,dx + (px+qy)\,dy = 0$$

(I) $\longrightarrow$ PD. HOMOGEN <u>DERAJAT</u>-1

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$y = \tfrac{1}{3}$

difer

$\frac{dy}{dx} =$

dy

...IKAN PD. HOMOGEN

$...y + 2) \, dy = 0$

$... = 0$

-1

$dy = 0$

$dy = 0$

$$y = \frac{1}{3} z - \frac{2}{3} x$$

diferensial thd. x

$$\frac{dy}{dx} = \frac{1}{3} \frac{dz}{dx} - \frac{2}{3} \frac{dx}{dx} \quad (dx)$$

$$dy = \frac{1}{3} dz - \frac{2}{3} dx$$

CONTOH :

① $(x+y+1)\,dx + (2x+2y+1)\,dy = 0$

$[(x+y)+1]\,dx + [2(x+y)+1]\,dy = 0$

MISALKAN : $\boxed{x+y=z}$

$y = z - x$

$\longrightarrow \boxed{dy = dz - dx}$

$[(x+y)+1]\,dx + [2(x+y)+1]\,dy = 0$

$(z+1)\,dx + (2z+1)(dz - dx) = 0$

$(z+1)\,dx + (2z+1)\,dz - (2z+1)\,dx = 0$

$(2z+1)\,dz = (2z+\cancel{1} - z - \cancel{1})\,dx$

$$\boxed{(2z+1)\,dz = z\,dx}$$

$$\left(\frac{2z+1}{z}\right)dz = dx$$

$$\left(2+\frac{1}{z}\right)dz - dx = 0$$

$$\int\left(2+\frac{1}{z}\right)dz - \int dx = \int 0$$

$$2\int dz + \int\frac{dz}{z} - x = C$$

$$2z + \ln z - x = C$$

$$2(x+y) + \ln(x+y) - x = C$$

$$2x + 2y + \ln(x+y) - x = C$$

$$\boxed{x + 2y + \ln(x+y) = C}$$

atau

$$\ln e^{x+2y} + \ln(x+y) = \ln C$$

$$\ln e^{x+2y}(x+y) = \ln C$$

$$\boxed{e^{x+2y}(x+y) = C}$$

$= 0$

② $$\boxed{\frac{dy}{dx} = \frac{x-y-4}{x+y-2}}$$

$$(x-y-4)\,dx = (x+y-2)\,dy$$

$$\underbrace{(x-y-4)}_{u}\,dx - \underbrace{(x+y-2)}_{v}\,dy = 0$$

11S :

$$u = x-y-4 \longrightarrow du = dx - dy$$

$$v = x+y-2 \longrightarrow dv = dx + dy$$

$$\begin{aligned} dx - dy &= du \\ dx + dy &= dv \quad + \\ \hline 2dx &= du + dv \end{aligned}$$

$$\boxed{dx = \frac{du + dv}{2}}$$

$$\begin{aligned} dx - dy &= du \\ dx + dy &= dv \quad - \\ \hline -2dy &= du - dv \end{aligned}$$

$$\boxed{dy = \frac{du - dv}{-2}}$$

$$(x-y-4)\,dx-(x+y-2)\,dy=0$$

$$u\left(\frac{du+dv}{2}\right)-v\left(\frac{du-dv}{-2}\right)=0 \qquad (2)$$

$$u\,du+u\,dv+v\,du-v\,dv=0$$

$$\boxed{(u+v)\,du+(u-v)\,dv=0}$$

M/S : $\boxed{z=\frac{v}{u}} \longrightarrow \boxed{v=uz}$

$$dv=u\,dz+z.\,du$$

$$(u+v)\,du+(u-v)\,dv=0$$

$$(u+uz)\,du+(u-uz)(u\,dz+z\,du)=0$$

$$(u+uz)\,du+(u^2-u^2z)\,dz+(uz-uz^2)\,du=0$$

$$(u^2-u^2z)\,dz=(uz^2-uz-uz-u)\,du$$

$$(u^2-u^2z)\,dz=(uz^2-2uz-u)\,du$$

$$(a+b+c)^2 = a^2+b^2+c^2+2ab+2ac+2bc$$

$$u^{\cancel{2}}(1-z)\,dz = \cancel{u}(z^2-2z-1)\,du$$

$$\boxed{u(1-z)\,dz = (z^2-2z-1)\,du}$$

$$\frac{(1-z)\,dz}{z^2-2z-1} = \frac{du}{u}$$

$$\frac{-(z-1)\,dz}{z^2-2z-1} = \frac{du}{u}$$

$$d(z^2-2z-1) = (2z-2)\,dz = 2\underline{(z-1)\,dz}$$

$$0 = \frac{du}{u} + \frac{(z-1)\,dz}{z^2-2z-1}$$

$$\int \frac{du}{u} + \int \frac{(z-1)\,dz}{z^2-2z-1} = \int 0$$

$$\ln u + \frac{1}{2}\int \frac{\overset{2(z-1)dz}{d(z^2-2z-1)}}{(z^2-2z-1)} = C$$

$$\ln u + \frac{1}{2}\ln(z^2-2z-1) = C \qquad \times 2$$

$) dz$

$1) dz$

② $\ln u + \ln(z^2 - 2z - 1) = 2C$

$\ln u^2 + \ln(z^2 - 2z - 1) = \ln C$

$\ln u^2(z^2 - 2z - 1) = \ln C$

$u^2(z^2 - 2z - 1) = C$

$u^2\left(\frac{v^2}{u^2} - 2\left(\frac{v}{u}\right) - 1\right) = C$

$$\boxed{v^2 - 2uv - u^2 = C}$$

$(x+y-2)^2 - 2(x-y-4)(x+y-2) - (x-y-4)^2 = C$

$(x^2+y^2+4+2xy-4x-4y) - 2(x^2+\cancel{xy}-2x-\cancel{xy}-y^2+2y-4x-4y+8)$

$- (x^2+y^2+16-2xy-8x+8y) = C$

$\cancel{x^2}+\cancel{y^2}+2xy-4x-\cancel{4y}+4 \; -2x^2+2y^2+12x+\cancel{4y}-16-\cancel{x^2}-\cancel{y^2}-16+2xy$

$+8x-8y = C$

$-2x^2+2y^2+4xy+16x-8y = C+28$ $\div -2$

$$\boxed{x^2 - y^2 - 2xy - 8x + 4y = C}$$

② $\boxed{\dfrac{dy}{dx} = \dfrac{x-y-4}{x+y-2}}$

II) $(x-y-4)\,dx = (x+y-2)\,dy$

$$\underbrace{(x-y-4)}_{=\,0}dx - \underbrace{(x+y-2)}_{=}dy = 0$$

MISALKAN: $\boxed{\begin{array}{l} x-y-4=0 \\ x+y-2=0 \end{array}}$

$$\begin{array}{rl} \rightarrow x - \cancel{y} &= 4 \\ \rightarrow x + \cancel{y} &= 2 \quad + \\ \hline 2x &= 6 \end{array}$$

$\boxed{x=3}$

$x - y = 4$

$3 - 4 = y$

$\boxed{-1 = y} \rightarrow$ TITIK POTONG $(\overset{x_1}{3}, \overset{y_1}{-1})$

MISALKAN: $p = x - x_1$

$\boxed{p = x - 3}$ $\rightarrow$ $\boxed{p + 3 = x}$

$\dfrac{dp}{dx} = 1$

$\boxed{dp = dx}$

$q = y - y_1$

$q = y - (-1) = y + 1$ $\rightarrow$ $\boxed{y = q - 1}$

$\dfrac{dq}{dy} = 1 \rightarrow \boxed{dq = dy}$

$$(x-y-4)\,dx-(x+y-2)\,dy=0$$

$$[(p+3)-(q-1)-4]\,dp-[(p+3)+(q-1)-2]\,dq=0$$

$$(p-q)\,dp-(p+q)\,dq=0$$

MIS: $\boxed{z=\frac{q}{p}} \longrightarrow \boxed{q=p.z}$

$$dq=p.dz+z.dp$$

$$(p-q)\,dp-(p+q)\,dq=0$$

$$(p-pz)\,dp-(p+pz)(p.dz+z.dp)=0$$

$$(p-pz)\,dp-(p^2+p^2z)\,dz-(pz+pz^2)\,dp=0$$

$$-(p^2+p^2z)\,dz=(pz+pz^2-p+pz)\,dp$$

$$-p^2(1+z)\,dz=(pz^2+2pz-p)\,dp$$

$$-p^2(z+1)\,dz=p(z^2+2z-1)\,dp$$

$$-p(z+1)\,dz=(z^2+2z-1)\,dp$$

$=q-1$

$$-\frac{(z+1)\,dz}{z^2+2z-1} = \frac{dp}{p}$$

$$d(z^2+2z-1) = (2z+2)\,dz$$
$$= 2(z+1)\,dz$$

$$0 = \frac{dp}{p} + \frac{(z+1)\,dz}{z^2+2z-1}$$

$$\int \frac{dp}{p} + \int \frac{(z+1)\,dz}{(z^2+2z-1)} = \int 0$$

$$\int \frac{dp}{p} + \frac{1}{2}\int \frac{\overset{2(z-1)dz}{d(z^2+2z-1)}}{(z^2+2z-1)} = C$$

$$\ln p + \frac{1}{2}\ln(z^2+2z-1) = C \qquad \times 2$$

$$2\ln p + \ln(z^2+2z-1) = 2C$$

$$\ln p^2 + \ln(z^2+2z-1) = \ln C$$

$$\ln p^2(z^2+2z-1) = \ln C$$

$$p^2(z^2+2z-1) = C$$

$2z+2)dz$
$(z+1)dz$

$$p^2\left(\frac{q^2}{p^2}+2\cdot\frac{q}{p}-1\right)=c$$

$$\boxed{q^2+2pq-p^2=c}$$

$$(y+1)^2+2(x-3)(y+1)-(x-3)^2=c$$

$$y^2+2y+1+2(xy+x-3y-3)-(x^2-6x+9)=c$$

$$y^2+2y+1+2xy+2x-6y-6-x^2+6x-9=c$$

$$-x^2+y^2+2xy+8x-4y=c-14 \quad x-1$$

$$x^2-y^2-2xy-8x+4y=c$$

$$\cancel{x^2}+\cancel{y^2}+2xy-4x-\cancel{4y}+4-2x^2+2y^2+12x+\cancel{4y}-16-\cancel{x^2}-\cancel{y^2}$$
$$+8x-8y$$

$$-2x^2+2y^2+4xy+16x-8y=c+28 \quad \div -2$$

$$\boxed{x^2-y^2-2xy-8x+4y=c}$$

PD. EKSAK

$$M(x,y)\,dx + N(x,y)\,dy = 0 \quad \ldots (1)$$

TOTAL DIFERENSIAL

$$z = f(x,y)$$

$$dz = \frac{\partial f}{\partial x} \cdot dx + \frac{\partial f}{\partial y} \cdot dy$$

PD. EKSAK : $F(x,y) = C$

$$\rightarrow \boxed{dF = 0}$$

$$\frac{\partial F}{\partial x} \cdot dx + \frac{\partial F}{\partial y} \cdot dy = 0 \quad \ldots (2)$$

DARI (1) & (2) $\longrightarrow$

$$M(x,y) = \frac{\partial F}{\partial x}$$

$$\boxed{\frac{\partial M}{\partial y} = \frac{\partial^2 F}{\partial x . \partial y}}$$

$$N(x,y) = \frac{\partial F}{\partial y}$$

$$\boxed{\frac{\partial N}{\partial x} = \frac{\partial^2 F}{\partial y . \partial x}}$$

$$\longrightarrow \boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$

Syarat PD. EKSAK

$$\underline{M(x,y)}\,dx + \underline{N(x,y)}\,dy = 0$$

$$\boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}} \Longrightarrow \text{PD. EKSAK}$$

I $\quad F(x,y) = \int \underline{M(x,y)}\,dx + c(y) \longrightarrow \boxed{\frac{\partial F}{\partial y} = N(x,y)}$

II $\quad F(x,y) = \int \underline{N(x,y)}\,dy + c(x) \longrightarrow \boxed{\frac{\partial F}{\partial x} = M(x,y)}$

CONTOH

$$\underbrace{(3x+5y)}_{M(x,y)}\,dx + \underbrace{(5x+3y)}_{N(x,y)}\,dy = 0$$

$$M(x,y) = 3x + 5y \qquad\qquad N(x,y) = 5x + 3y$$

$$\frac{\partial M}{\partial y} = 0 + 5(1) = 5 \qquad\qquad \frac{\partial N}{\partial x} = 5(1) + 0 = 5$$

$$\boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$

$\Longrightarrow$ PD. EKSAK

(I) $F(x,y) = \int M(x,y)\,dx + c(y) \rightarrow \boxed{\frac{\partial F}{\partial y} = N(x,y)}$

$$F(x,y) = \int (3x + 5y)\,dx + c(y)$$

$$F(x,y) = 3\int x\,dx + 5y\int dx + c(y)$$

$$\boxed{F(x,y) = \frac{3}{2}x^2 + 5yx + c(y)}$$

$$\frac{\partial F}{\partial y} = 0 + 5x(1) + c'(y) = N(x,y)$$

$$\cancel{5x} + c'(y) = \cancel{5x} + 3y$$

$$c'(y) = 3y$$

$$c(y) = \int c'(y)\,dy = \int 3y\,dy = \frac{3}{2}y^2 + c$$

Sol. um. PD

$$F(x,y) = 0$$

$$\frac{3}{2}x^2 + 5xy + c(y) = 0$$

$$\frac{3}{2}x^2 + 5xy + \frac{3}{2}y^2 + c = 0 \quad \times 2$$

$$3x^2 + 10xy + 3y^2 + c = 0$$

II $F(x,y) = \int N(x,y)\,dy + C(x) \to \boxed{\frac{\partial F}{\partial x} = M(x,y)}$

$F(x,y) = \int (5x+3y)\,dy + C(x)$

$F(x,y) = 5x\int dy + 3\int y\,dy + C(x)$

$$\boxed{F(x,y) = 5xy + \frac{3}{2}y^2 + C(x)}$$

$\frac{\partial F}{\partial x} = 5y(1) + 0 + C'(x) = M(x,y)$

$\cancel{5y} + C'(x) = 3x + \cancel{5y}$

$C'(x) = 3x$

$C(x) = \int C'(x)\,dx = \int 3x\,dx = \frac{3}{2}x^2 + C$

Sol. um. PD

$F(x,y) = 0$

$5xy + \frac{3}{2}y^2 + C(x) = 0$

$5xy + \frac{3}{2}y^2 + \frac{3}{2}x^2 + C = 0$ $\times 2$

$10xy + 3y^2 + 3x^2 + C = 0$

Soal:

1. $(2x+3y)\,dx+(3x+4y)\,dy=0$

   $x^2+3xy+2y^2+D=0$

2. $(15x^2y^2-y^4)\,dx+(10x^3y-4xy^3+5y^4)\,dy=0$

   $5x^3y^2-xy^4+y^5+D=0$

3. $e^{x^2y}(1+2x^2y)\,dx+x^3e^{x^2y}\,dy=0$

   $x\,e^{x^2y}+D=0$

4. $3xy\sqrt{1+x^2}\,dx+\left[\sqrt{(1+x^2)^3}+\sin y\right]dy=0$