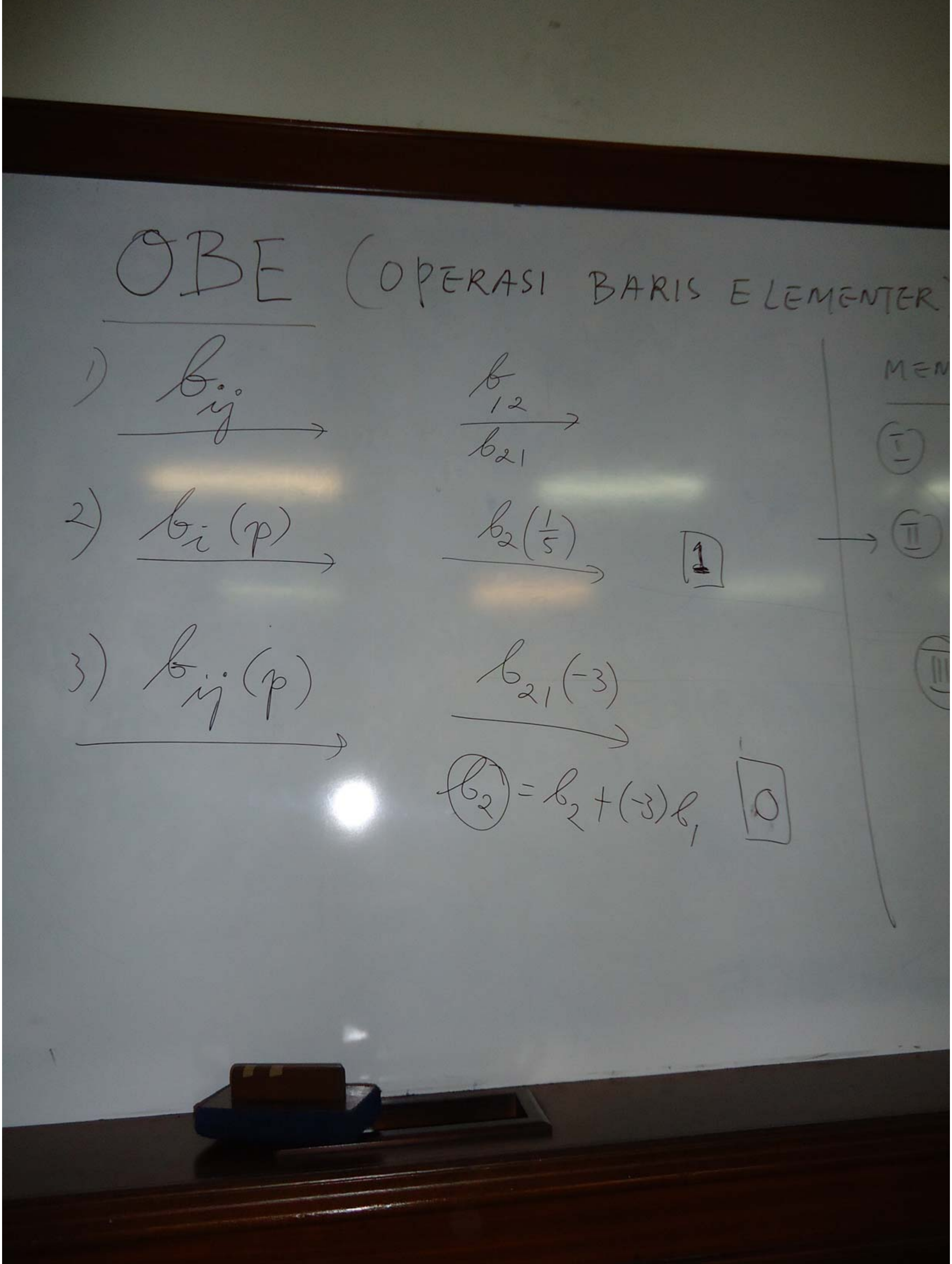


OBE (OPERASI BARIS ELEMENTER

1) $\xrightarrow{b_{ij}}$ $\qquad$ $\xrightarrow[b_{21}]{b_{12}}$

2) $\xrightarrow{b_i(p)}$ $\qquad$ $\xrightarrow{b_2(\frac{1}{5})}$ $\quad \boxed{1}$

3) $\xrightarrow{b_{ij}(p)}$ $\qquad$ $\xrightarrow{b_{21}(-3)}$

$b_2 = b_2 + (-3)b_1 \quad \boxed{0}$

MEN

I

→ II

III

ENTER)

MENCARI MATRIKS INVERS

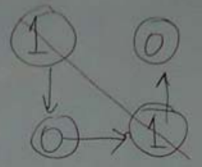
I $\quad A \cdot A^{-1} = I$

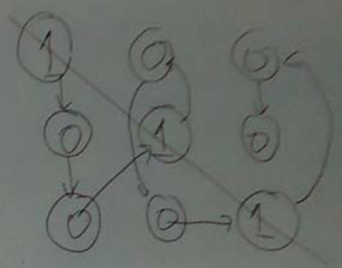
→ II $\quad (A \mid I) \xrightarrow{OBE} (I \mid A^{-1})$

III $\quad A^{-1} = \frac{1}{|A|} \cdot adj(A)$

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \longrightarrow \underline{A^{-1} = ?}$

① $(A \mid I_2) \xrightarrow{OBE} (I_2 \mid A^{-1})$





$$\left[\underbrace{\begin{matrix} \textcircled{1} & 2 \\ \textcircled{3} & 4 \end{matrix}}_{A} \middle| \underbrace{\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}}_{I_2}\right] \xrightarrow{b_{21}(-3)} \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & \textcircled{-2} & -3 & 1 \end{array}\right] \xrightarrow{b_2\left(\frac{1}{-2}\right)} \left[\begin{array}{cc|cc} 1 & \textcircled{2} & 1 & 0 \\ 0 & \textcircled{1} & 1\frac{1}{2} & -\frac{1}{2} \end{array}\right] \xrightarrow{b_{12}(-2)} \left[\underbrace{\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}}_{I_2} \middle| \underbrace{\begin{matrix} -2 & 1 \\ 1\frac{1}{2} & -\frac{1}{2} \end{matrix}}_{A^{-1}}\right]$$

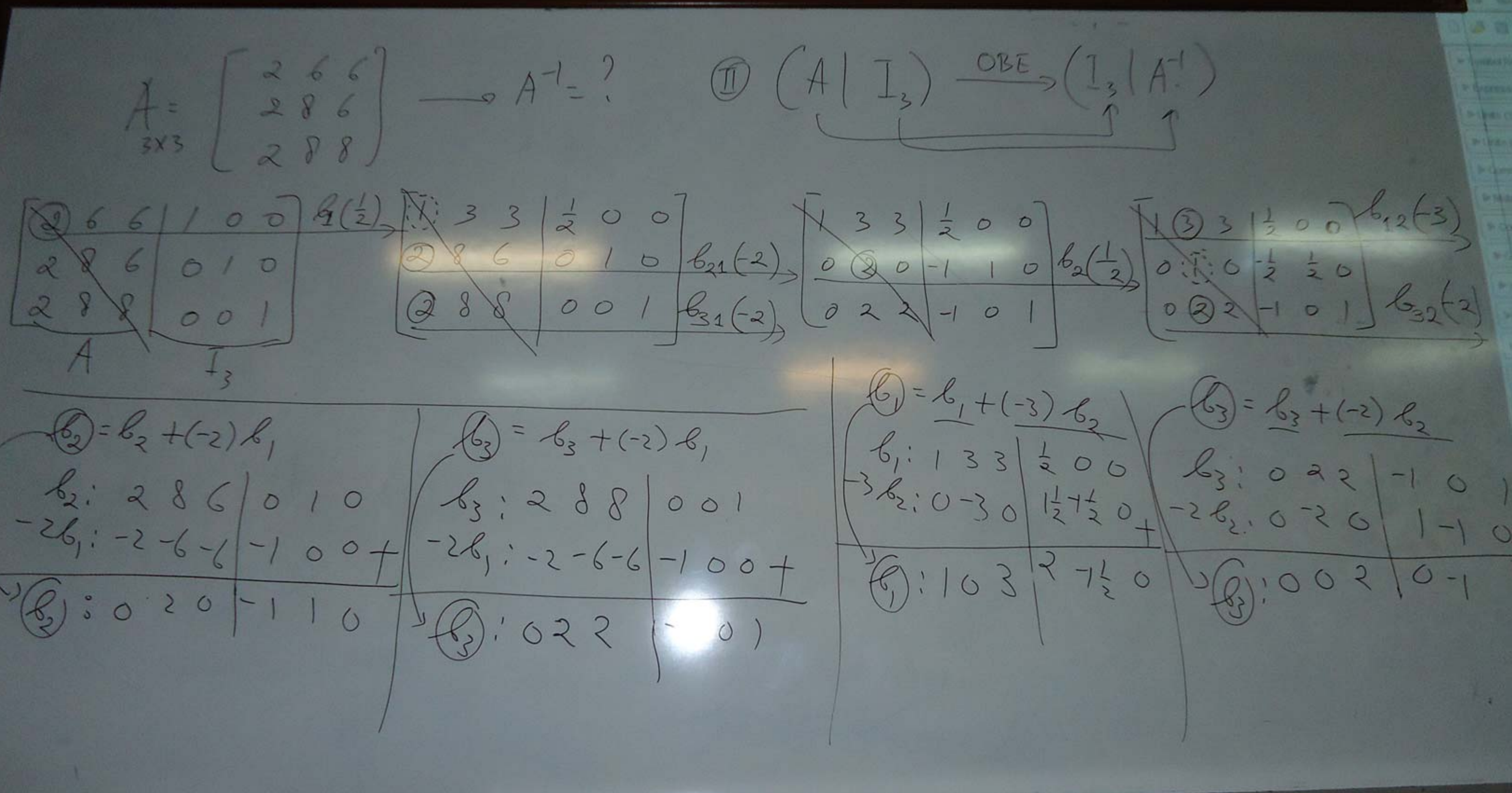
$\textcircled{b_2} = \underline{b_2} + \underline{(-3)b_1}$

$$\begin{array}{rcc|cc} b_2: & 3 & 4 & 0 & 1 \\ -3b_1: & -3 & -6 & -3 & 0 \; + \\ \hline \textcircled{b_2}: & 0 & -2 & -3 & 1 \end{array}$$

$\textcircled{b_1} = \underline{b_1} + \underline{(-2)b_2}$

$$\begin{array}{rcc|cc} b_1: & 1 & 2 & 1 & 0 \\ -2b_2: & 0 & -2 & -3 & 1 \; + \\ \hline \textcircled{b_1}: & 1 & 0 & -2 & 1 \end{array}$$

$$A^{-1} = \begin{bmatrix} -2 & 1 \\ 1\frac{1}{2} & -\frac{1}{2} \end{bmatrix}$$



$$\underset{3\times3}{A} = \begin{bmatrix} 2 & 6 & 6 \\ 2 & 8 & 6 \\ 2 & 8 & 8 \end{bmatrix} \longrightarrow A^{-1} = ?$$

Ⅱ $(A \mid I_3) \xrightarrow{OBE} (I_3 \mid A^{-1})$

$$\underbrace{\begin{bmatrix} \textcircled{2} & 6 & 6 \\ 2 & 8 & 6 \\ 2 & 8 & 8 \end{bmatrix}}_{A} \left| \underbrace{\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}}_{I_3} \right] \xrightarrow{b_1(\frac{1}{2})} \left[\begin{array}{ccc|ccc} 1 & 3 & 3 & \frac{1}{2} & 0 & 0 \\ \textcircled{2} & 8 & 6 & 0 & 1 & 0 \\ \textcircled{2} & 8 & 8 & 0 & 0 & 1 \end{array}\right] \begin{matrix} \xrightarrow{b_{21}(-2)} \\ \xrightarrow{b_{31}(-2)} \end{matrix} \left[\begin{array}{ccc|ccc} 1 & 3 & 3 & \frac{1}{2} & 0 & 0 \\ 0 & \textcircled{2} & 0 & -1 & 1 & 0 \\ 0 & 2 & 2 & -1 & 0 & 1 \end{array}\right] \xrightarrow{b_2(\frac{1}{2})} \left[\begin{array}{ccc|ccc} 1 & \textcircled{3} & 3 & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \textcircled{2} & 2 & -1 & 0 & 1 \end{array}\right] \begin{matrix} \xrightarrow{b_{12}(-3)} \\ \xrightarrow{b_{32}(-2)} \end{matrix}$$

$\textcircled{b_2} = b_2 + (-2)b_1$

$$\begin{array}{rccc|ccc} b_2: & 2 & 8 & 6 & 0 & 1 & 0 \\ -2b_1: & -2 & -6 & -6 & -1 & 0 & 0 \\ \hline \textcircled{b_2}: & 0 & 2 & 0 & -1 & 1 & 0 \end{array} +$$

$\textcircled{b_3} = b_3 + (-2)b_1$

$$\begin{array}{rccc|ccc} b_3: & 2 & 8 & 8 & 0 & 0 & 1 \\ -2b_1: & -2 & -6 & -6 & -1 & 0 & 0 \\ \hline \textcircled{b_3}: & 0 & 2 & 2 & -1 & 0 & 1 \end{array} +$$

$\textcircled{b_1} = \underline{b_1} + (-3)\underline{b_2}$

$$\begin{array}{rccc|ccc} b_1: & 1 & 3 & 3 & \frac{1}{2} & 0 & 0 \\ -3b_2: & 0 & -3 & 0 & 1\frac{1}{2} & -1\frac{1}{2} & 0 \\ \hline \textcircled{b_1}: & 1 & 0 & 3 & 2 & -1\frac{1}{2} & 0 \end{array} +$$

$\textcircled{b_3} = \underline{b_3} + (-2)\underline{b_2}$

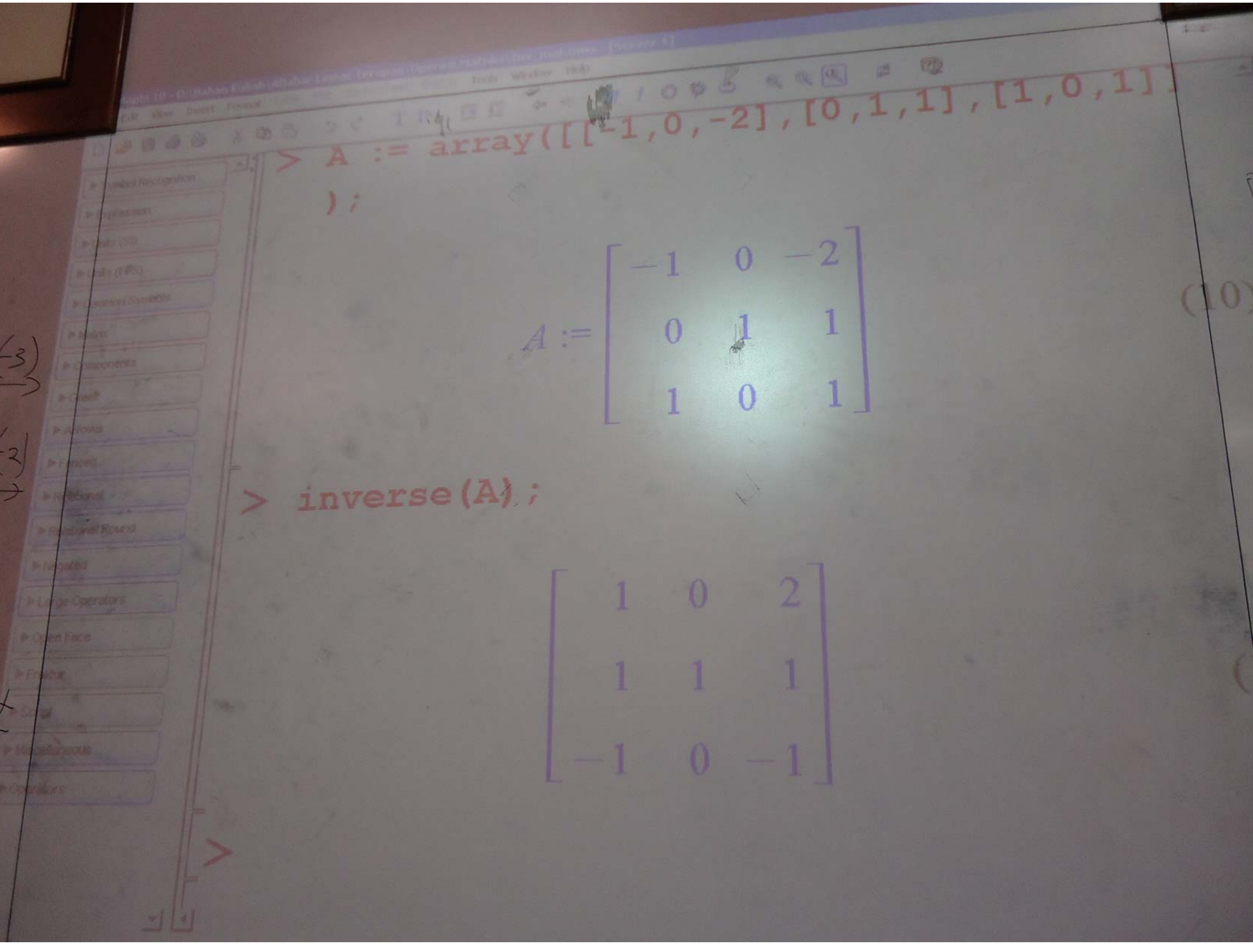
$$\begin{array}{rccc|ccc} b_3: & 0 & 2 & 2 & -1 & 0 & 1 \\ -2b_2: & 0 & -2 & 0 & 1 & -1 & 0 \\ \hline \textcircled{b_3}: & 0 & 0 & 2 & 0 & -1 & \end{array}$$

$$\left[\begin{array}{ccc|ccc} & 0 & 3 & 2 & -1\frac{1}{2} & 0\\ 0 & 1 & 0 & -\frac{1}{2} & \frac{1}{2} & 0\\ 0 & 0 & \textcircled{2} & 0 & -1 & 1\end{array}\right] \xrightarrow{b_3(\frac{1}{2})} \left[\begin{array}{ccc|ccc} 1 & 0 & \textcircled{3} & 2 & -1\frac{1}{2} & 0\\ 0 & 1 & 0 & -\frac{1}{2} & \frac{1}{2} & 0\\ 0 & 0 & \textcircled{1} & 0 & -\frac{1}{2} & \frac{1}{2}\end{array}\right] \xrightarrow{b_{13}(-3)} \left[\underbrace{\begin{matrix} 1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{matrix}}_{I_3}\middle|\underbrace{\begin{matrix} 2 & 0 & -1\frac{1}{2}\\ -\frac{1}{2} & \frac{1}{2} & 0\\ 0 & -\frac{1}{2} & \frac{1}{2}\end{matrix}}_{A^{-1}}\right]$$

$$\textcircled{b_1} = \underline{b_1} + \underline{(-3)\,b_3}$$

$$\begin{array}{rccc|cccl} b_1: & 1 & 0 & 3 & 2 & -1\frac{1}{2} & 0 & \\ -3b_3: & 0 & 0 & -3 & 0 & 1\frac{1}{2} & -1\frac{1}{2} & + \\ \hline \textcircled{b_1}: & 1 & 0 & 0 & 2 & 0 & -1\frac{1}{2} & \end{array}$$

$$A^{-1} = \begin{bmatrix} 2 & 0 & -1\frac{1}{2}\\ -\frac{1}{2} & \frac{1}{2} & 0\\ 0 & -\frac{1}{2} & \frac{1}{2}\end{bmatrix}$$



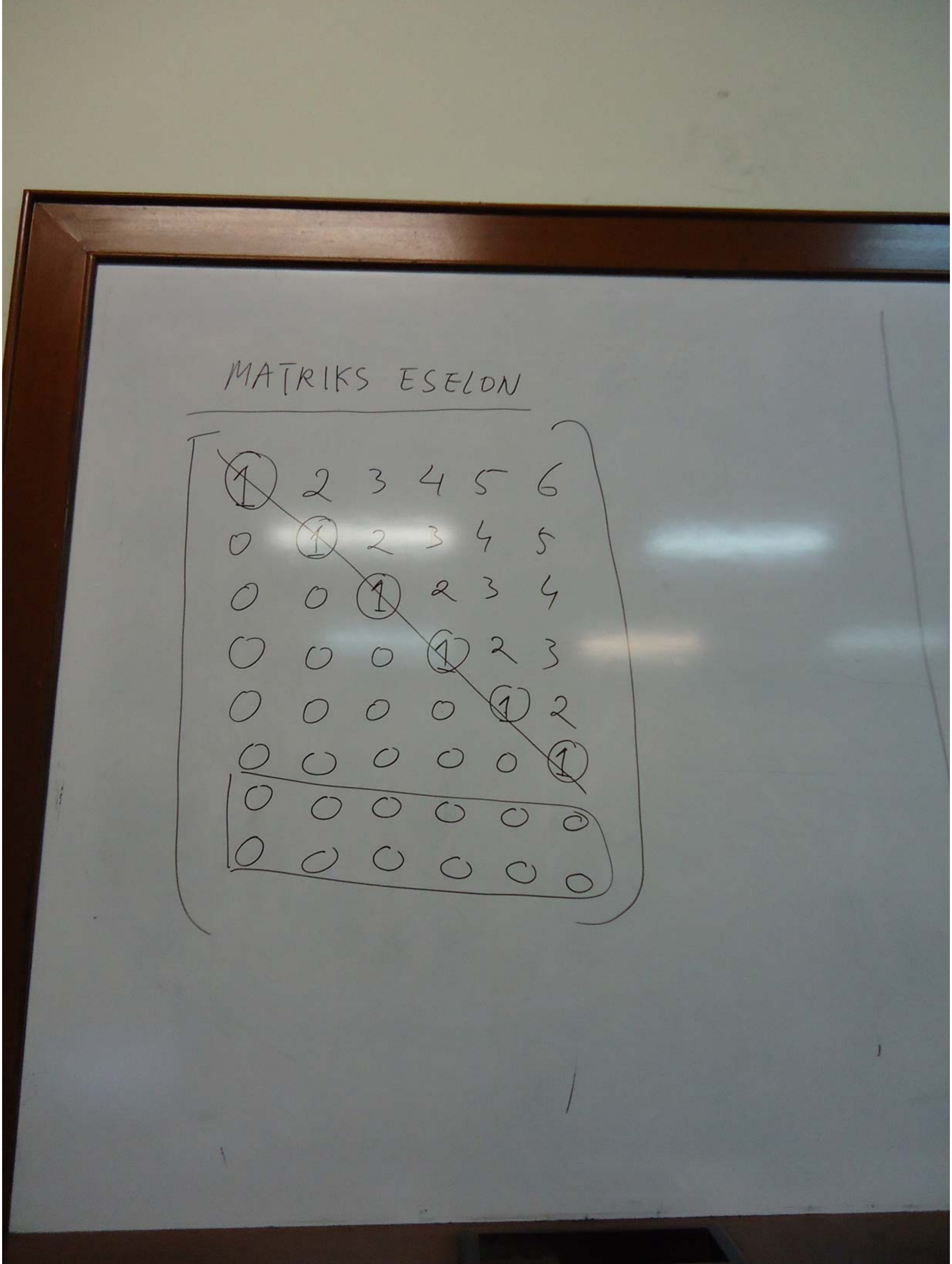
```
> A := array([[-1,0,-2],[0,1,1],[1,0,1]
  );
```

$$A := \begin{bmatrix} -1 & 0 & -2 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \qquad (10)$$

```
> inverse(A);
```

$$\begin{bmatrix} 1 & 0 & 2 \\ 1 & 1 & 1 \\ -1 & 0 & -1 \end{bmatrix}$$

```
>
```



MATRIKS ESELON

$$\left[\begin{array}{cccccc}
\textcircled{1} & 2 & 3 & 4 & 5 & 6 \\
0 & \textcircled{1} & 2 & 3 & 4 & 5 \\
0 & 0 & \textcircled{1} & 2 & 3 & 4 \\
0 & 0 & 0 & \textcircled{1} & 2 & 3 \\
0 & 0 & 0 & 0 & \textcircled{1} & 2 \\
0 & 0 & 0 & 0 & 0 & \textcircled{1} \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{array}\right)$$

MATRIKS ESELON TEREDUKSI

$$\begin{bmatrix} \textcircled{1} & 0 & 0 & 0 & 0 & 0 \\ 0 & \textcircled{1} & 0 & 0 & 0 & 0 \\ 0 & 0 & \textcircled{1} & 0 & 0 & 0 \\ 0 & 0 & 0 & \textcircled{1} & 0 & 0 \\ 0 & 0 & 0 & 0 & \textcircled{1} & 0 \\ 0 & 0 & 0 & 0 & 0 & \textcircled{1} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$I_3=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\xrightarrow[b_{13}]{b_{31}}E=\begin{bmatrix}0&0&1\\0&1&0\\1&0&0\end{bmatrix}\xrightarrow[b_{13}]{b_{31}}I_3=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$I_3=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\xrightarrow{b_2(5)}E=\begin{bmatrix}1&0&0\\0&\boxed{5}&0\\0&0&1\end{bmatrix}\xrightarrow{b_2(\frac{1}{5})}I_3=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$I_3=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\xrightarrow{b_{23}(4)}E=\begin{bmatrix}1&0&0\\0&1&\boxed{4}\\0&0&1\end{bmatrix}\xrightarrow{b_{23}(-4)}I_3=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$\boxed{b_2}=b_2+(4)b_3$$

$$\begin{array}{rccc l}b_2: & 0 & 1 & 0 & \\ 4b_3: & 0 & 0 & 4 & + \\ \hline \boxed{b_2}: & 0 & 1 & 4 & \end{array}$$

$$\boxed{b_2}=b_2+(-4)b_3$$

$$\begin{array}{rccc l}b_2: & 0 & 1 & 4 & \\ -4b_3: & 0 & 0 & -4 & + \\ \hline \boxed{b_2}: & 0 & 1 & 0 & \end{array}$$

$$I \xrightarrow{b_{ij}} E$$

$$\boxed{E.A = A \xrightarrow{b_{ij}}}$$

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \xrightarrow[b_{21}]{b_{12}} E = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$E.A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 0+3 & 0+4 \\ 1+0 & 2+0 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix},$$

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \xrightarrow[b_{21}]{b_{12}} \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$$

OBE (OPERASI BARIS ELEMENTER)

1) $\xrightarrow{b_{ij}}$ $\xrightarrow[b_{21}]{b_{12}}$

2) $\xrightarrow{b_i(p)}$ $\xrightarrow{b_2\left(\frac{1}{5}\right)}$ $\boxed{1}$

3) $\xrightarrow{b_{ij}(p)}$ $\xrightarrow{b_{21}(-3)}$

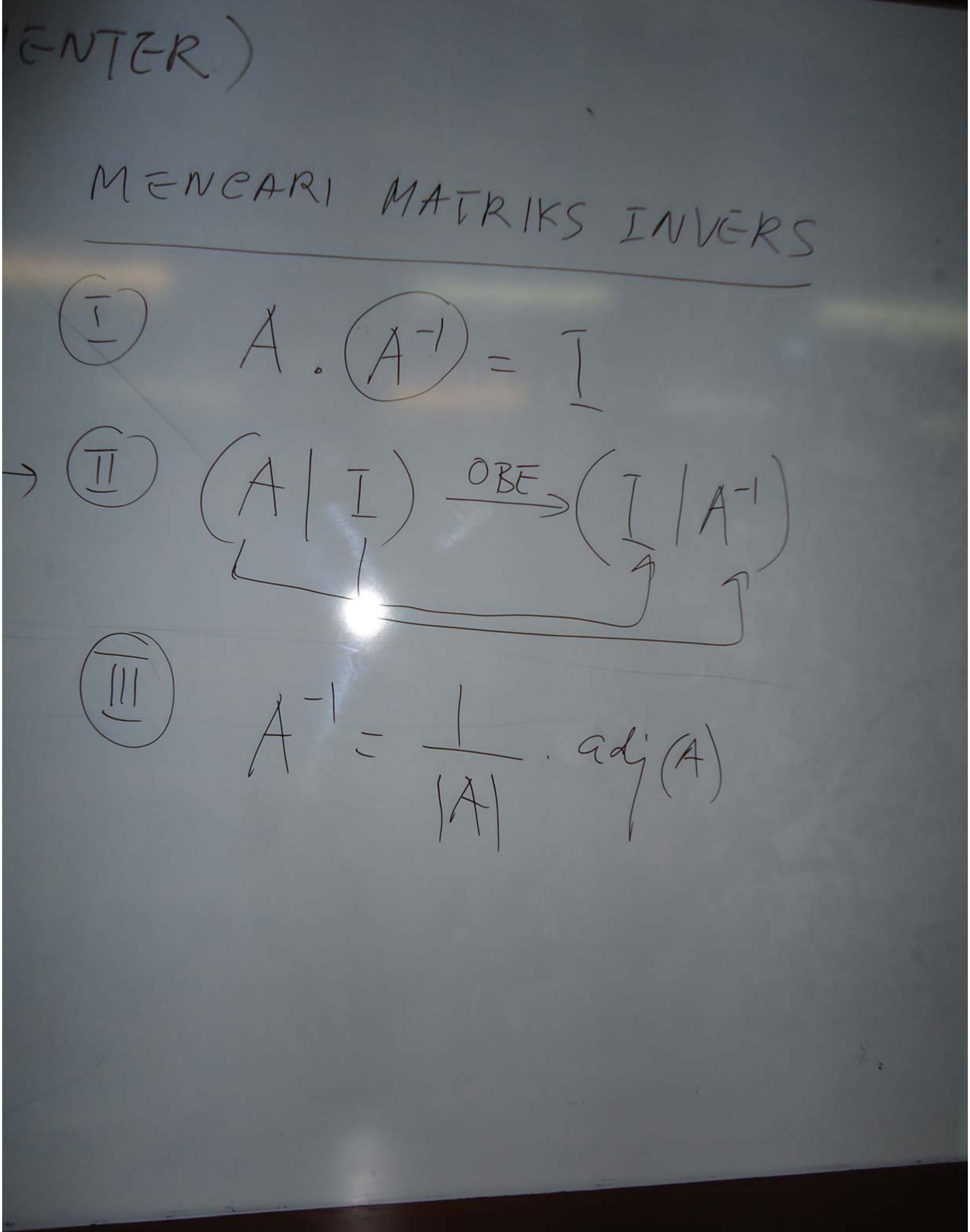
$b_2 = b_2 + (-3)b_1$ $\boxed{0}$

MEN

Ⅰ

→ Ⅱ

Ⅲ



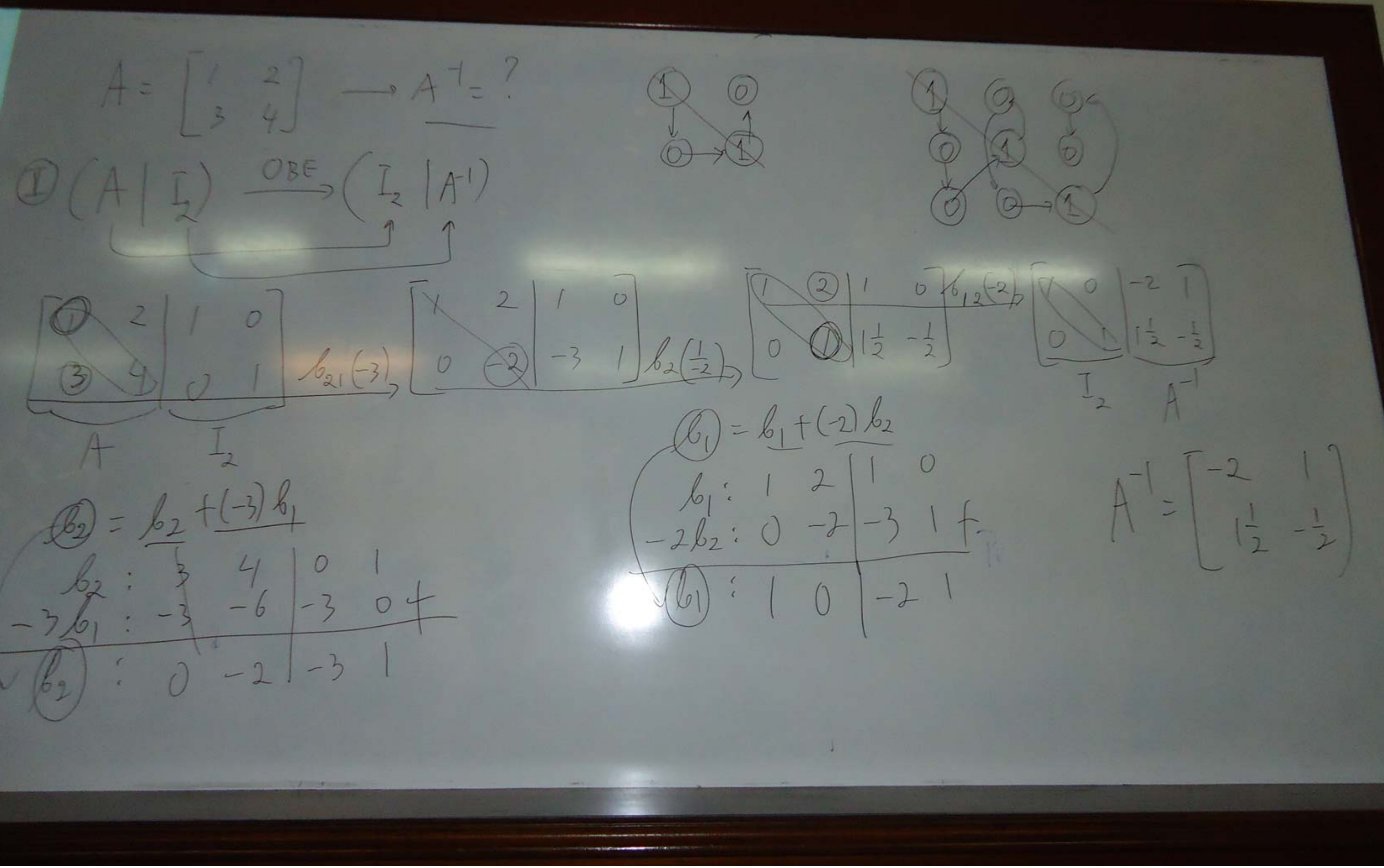
ENTER)

MENCARI MATRIKS INVERS

I $\quad A \cdot A^{-1} = I$

→ II $\quad (A \mid I) \xrightarrow{OBE} (I \mid A^{-1})$

III $\quad A^{-1} = \frac{1}{|A|} \cdot \mathrm{adj}(A)$



$A = \begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix} \longrightarrow \underline{A^{-1} = ?}$

① $(A \mid I_2) \xrightarrow{OBE} (I_2 \mid A^{-1})$

$\left[\underbrace{\begin{matrix}1 & 2\\ 3 & 4\end{matrix}}_{A}\middle|\underbrace{\begin{matrix}1 & 0\\ 0 & 1\end{matrix}}_{I_2}\right] \xrightarrow{b_{21}(-3)} \left[\begin{array}{cc|cc}1 & 2 & 1 & 0\\ 0 & -2 & -3 & 1\end{array}\right] \xrightarrow{b_2\left(\frac{1}{-2}\right)} \left[\begin{array}{cc|cc}1 & 2 & 1 & 0\\ 0 & 1 & 1\frac{1}{2} & -\frac{1}{2}\end{array}\right] \xrightarrow{b_{12}(-2)} \left[\underbrace{\begin{matrix}1 & 0\\ 0 & 1\end{matrix}}_{I_2}\middle|\underbrace{\begin{matrix}-2 & 1\\ 1\frac{1}{2} & -\frac{1}{2}\end{matrix}}_{A^{-1}}\right]$

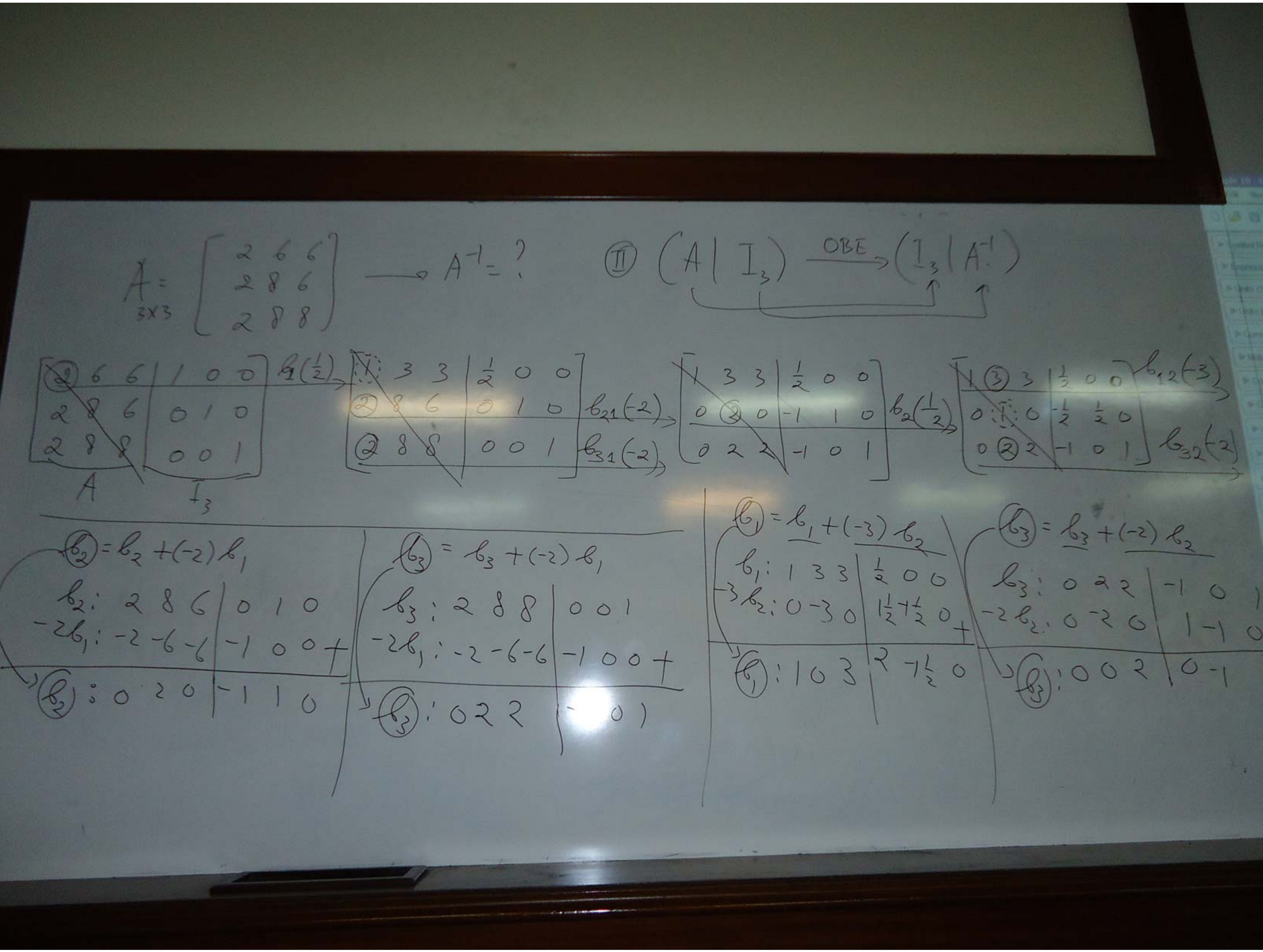
$b_2 = \underline{b_2} + \underline{(-3)\,b_1}$

$$\begin{array}{rcrr|rrl}
b_2 & : & 3 & 4 & 0 & 1 & \\
-3b_1 & : & -3 & -6 & -3 & 0 & + \\
\hline
b_2 & : & 0 & -2 & -3 & 1 &
\end{array}$$

$b_1 = \underline{b_1} + \underline{(-2)\,b_2}$

$$\begin{array}{rcrr|rrl}
b_1 & : & 1 & 2 & 1 & 0 & \\
-2b_2 & : & 0 & -2 & -3 & 1 & + \\
\hline
b_1 & : & 1 & 0 & -2 & 1 &
\end{array}$$

$A^{-1} = \begin{bmatrix}-2 & 1\\ 1\frac{1}{2} & -\frac{1}{2}\end{bmatrix}$



$A_{3\times3} = \begin{bmatrix}2&6&6\\2&8&6\\2&8&8\end{bmatrix} \longrightarrow A^{-1} = ?$

Ⅱ $(A \mid I_3) \xrightarrow{OBE} (I_3 \mid A^{-1})$

$$\left[\begin{array}{ccc|ccc}2&6&6&1&0&0\\2&8&6&0&1&0\\2&8&8&0&0&1\end{array}\right] \xrightarrow{b_1(\frac{1}{2})} \left[\begin{array}{ccc|ccc}1&3&3&\frac{1}{2}&0&0\\2&8&6&0&1&0\\2&8&8&0&0&1\end{array}\right] \xrightarrow[b_{31}(-2)]{b_{21}(-2)} \left[\begin{array}{ccc|ccc}1&3&3&\frac{1}{2}&0&0\\0&2&0&-1&1&0\\0&2&2&-1&0&1\end{array}\right] \xrightarrow{b_2(\frac{1}{2})} \left[\begin{array}{ccc|ccc}1&3&3&\frac{1}{2}&0&0\\0&1&0&-\frac{1}{2}&\frac{1}{2}&0\\0&2&2&-1&0&1\end{array}\right] \xrightarrow[b_{32}(-2)]{b_{12}(-3)}$$

A I_3

$b_2 = b_2 + (-2)b_1$

b_2: 2 8 6 | 0 1 0

$-2b_1$: -2 -6 -6 | -1 0 0 +

b_2: 0 2 0 | -1 1 0

$b_3 = b_3 + (-2)b_1$

b_3: 2 8 8 | 0 0 1

$-2b_1$: -2 -6 -6 | -1 0 0 +

b_3: 0 2 2 | -[illegible] 0 1

$b_1 = b_1 + (-3)b_2$

b_1: 1 3 3 | $\frac{1}{2}$ 0 0

$-3b_2$: 0 -3 0 | $1\frac{1}{2}$ $-1\frac{1}{2}$ 0 +

b_1: 1 0 3 | 2 $-1\frac{1}{2}$ 0

$b_3 = b_3 + (-2)b_2$

b_3: 0 2 2 | -1 0 1

$-2b_2$: 0 -2 0 | 1 -1 0

b_3: 0 0 2 | 0 -1

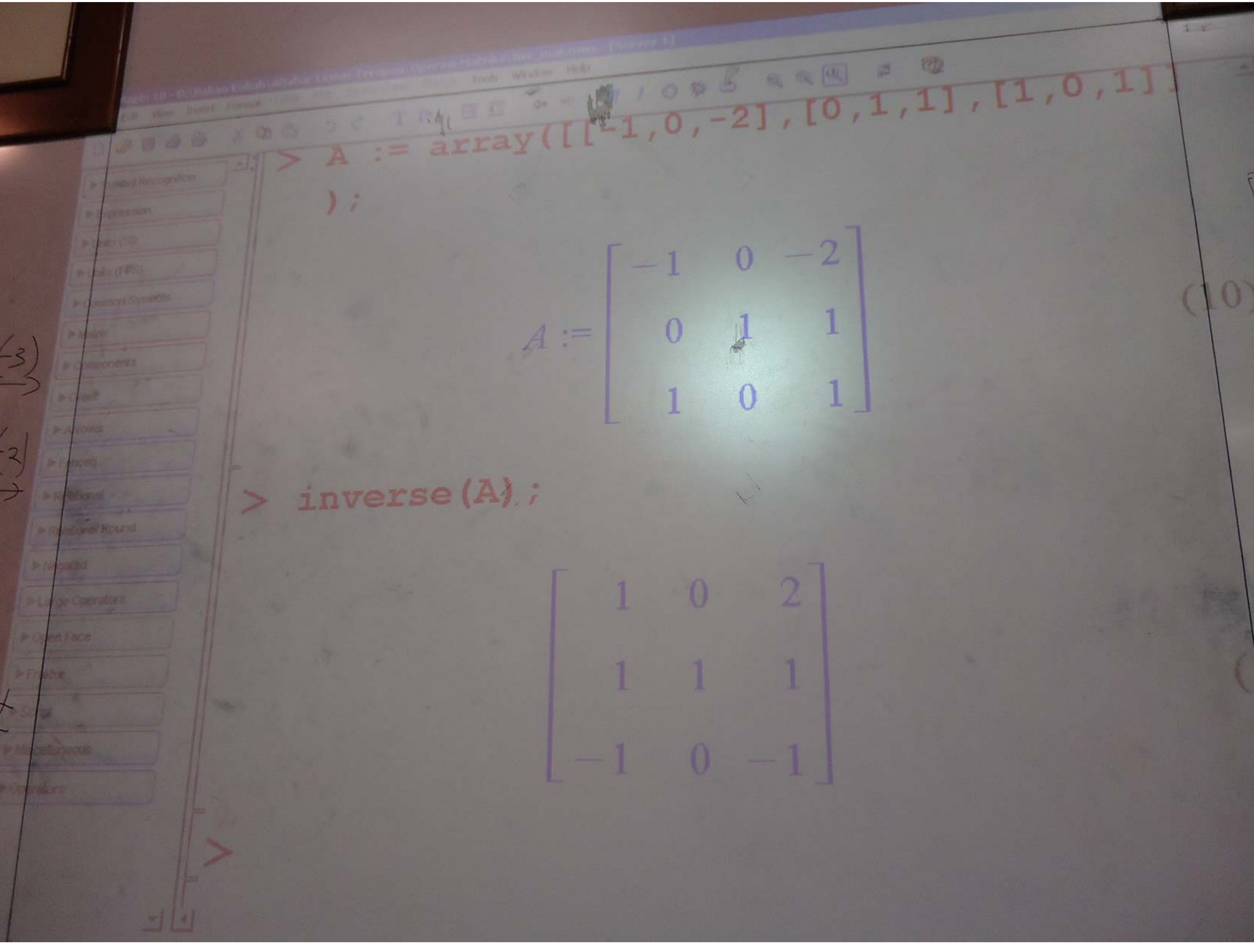
$$\left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 2 & -1\frac{1}{2} & 0 \\ 0 & 1 & 0 & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & \textcircled{2} & 0 & -1 & 1 \end{array}\right] \xrightarrow{b_3\left(\frac{1}{2}\right)} \left[\begin{array}{ccc|ccc} 1 & 0 & \textcircled{3} & 2 & -1\frac{1}{2} & 0 \\ 0 & 1 & 0 & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 & 0 & -\frac{1}{2} & \frac{1}{2} \end{array}\right] \xrightarrow{b_{13}(-3)} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 0 & -1\frac{1}{2} \\ 0 & 1 & 0 & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 & 0 & -\frac{1}{2} & \frac{1}{2} \end{array}\right]$$

$$\underbrace{\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}}_{I_3} \quad \underbrace{\begin{matrix} 2 & 0 & -1\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} & \frac{1}{2} \end{matrix}}_{A^{-1}}$$

$$\textcircled{b_1} = \underline{b_1} + \underline{(-3)\, b_3}$$

$$\begin{array}{rccc|ccc} b_1: & 1 & 0 & 3 & 2 & -1\frac{1}{2} & 0 \\ -3b_3: & 0 & 0 & -3 & 0 & 1\frac{1}{2} & -1\frac{1}{2} \; + \\ \hline \textcircled{b_1}: & 1 & 0 & 0 & 2 & 0 & -1\frac{1}{2} \end{array}$$

$$A^{-1} = \begin{bmatrix} 2 & 0 & -1\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$$



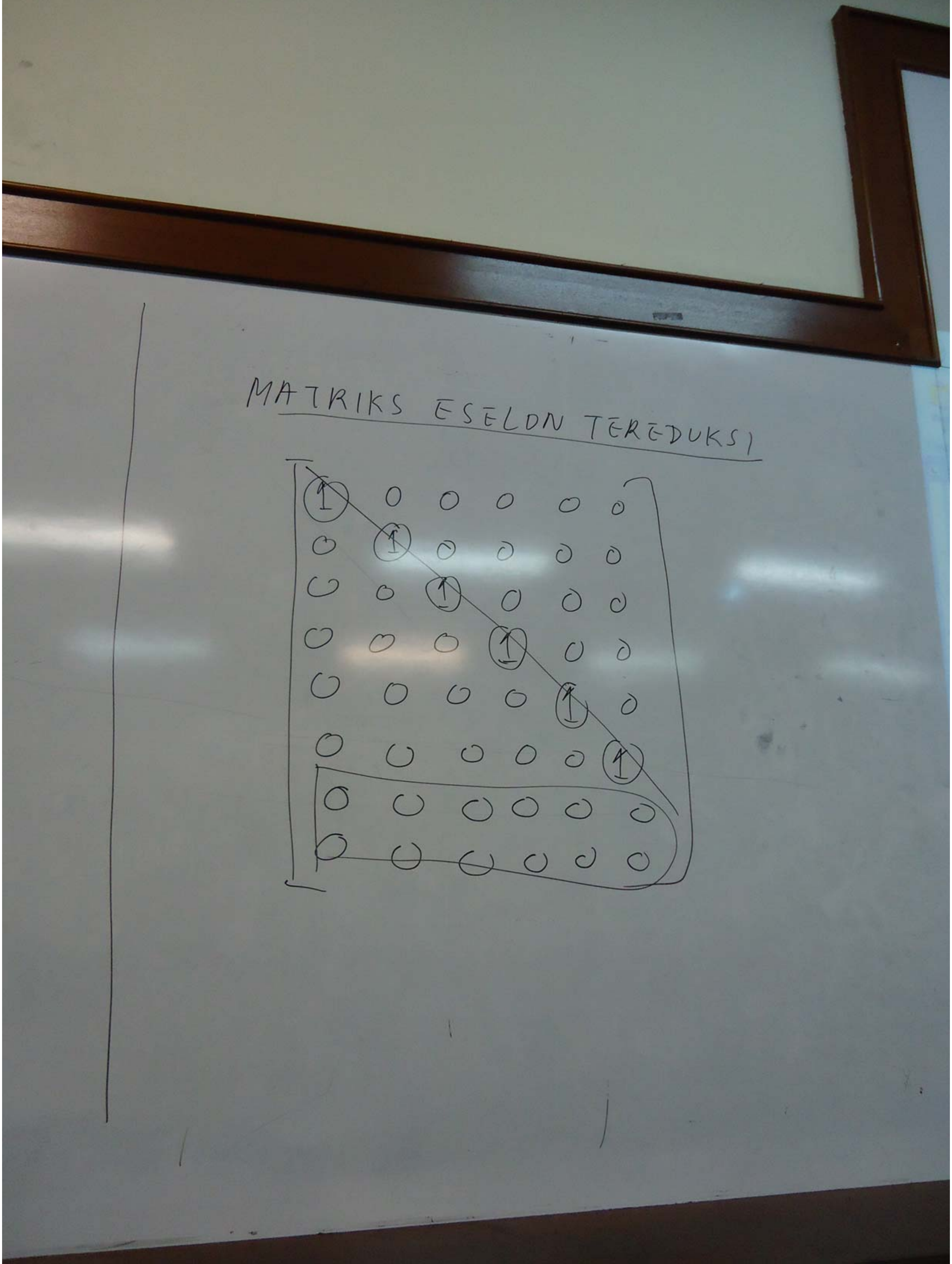
```
> A := array([[-1,0,-2],[0,1,1],[1,0,1]]
);
```

$$A := \begin{bmatrix} -1 & 0 & -2 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \tag{10}$$

```
> inverse(A);
```

$$\begin{bmatrix} 1 & 0 & 2 \\ 1 & 1 & 1 \\ -1 & 0 & -1 \end{bmatrix}$$

```
>
```



MATRIKS ESELON TEREDUKSI

$$\begin{bmatrix} \textcircled{1} & 0 & 0 & 0 & 0 & 0 \\ 0 & \textcircled{1} & 0 & 0 & 0 & 0 \\ 0 & 0 & \textcircled{1} & 0 & 0 & 0 \\ 0 & 0 & 0 & \textcircled{1} & 0 & 0 \\ 0 & 0 & 0 & 0 & \textcircled{1} & 0 \\ 0 & 0 & 0 & 0 & 0 & \textcircled{1} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$I_3 = \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix} \xrightarrow[b_{13}]{b_{31}} E = \begin{bmatrix}0&0&1\\0&1&0\\1&0&0\end{bmatrix} \xrightarrow[b_{13}]{b_{31}} I_3 = \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$I_3 = \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix} \xrightarrow{b_2(5)} E = \begin{bmatrix}1&0&0\\0&5&0\\0&0&1\end{bmatrix} \xrightarrow{b_2(\frac{1}{5})} I_3 = \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$I_3 = \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix} \xrightarrow{b_{23}(4)} E = \begin{bmatrix}1&0&0\\0&1&4\\0&0&1\end{bmatrix} \xrightarrow{b_{23}(-4)} I_3 = \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$b_2 = b_2 + (4)b_3$$

$$\begin{array}{rccc} b_2: & 0 & 1 & 0 \\ 4b_3: & 0 & 0 & 4 \quad + \\ \hline b_2: & 0 & 1 & 4 \end{array}$$

$$b_2 = b_2 + (-4)b_3$$

$$\begin{array}{rccc} b_2: & 0 & 1 & 4 \\ -4b_3: & 0 & 0 & -4 \quad + \\ \hline b_2: & 0 & 1 & 0 \end{array}$$

$$I \xrightarrow{b_{ij}} E$$

$$\boxed{E . A = A \xrightarrow{b_{ij}}}$$

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \xrightarrow[b_{21}]{b_{12}} E = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

$$E . A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 0+3 & 0+4 \\ 1+0 & 2+0 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}.$$

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \xrightarrow[b_{21}]{b_{12}} \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$$