

ORDINARY DIFFERENTIAL EQUATION (ODE) = PERSAMAAN DIFFERENSIAL BIASA (PDB)

BUNGA BANK = $p_0 \times Modal$

$$\downarrow$$
$$\boxed{\frac{dM}{dt} = p_0 \times M}$$

PENURUNAN JUALATI PENDUDUK (P)
TERHADAP WAKTU:

$$\boxed{\frac{dP}{dt} = (K) P}$$

$X = \int \text{RAK}$

$$\boxed{V = \frac{dX}{dt}}$$

$$, a = \frac{dV}{dt}$$

$$a = \frac{d}{dt} \left(\frac{dX}{dt} \right)$$

$$\boxed{a = \frac{d^2 X}{dt^2}}$$

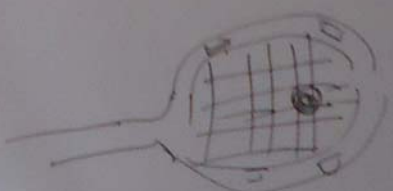
$$\left(\frac{d^3}{dx^3} \right)$$

PD TINGKAT-5
PD ORDE-5

PANGKAT-3

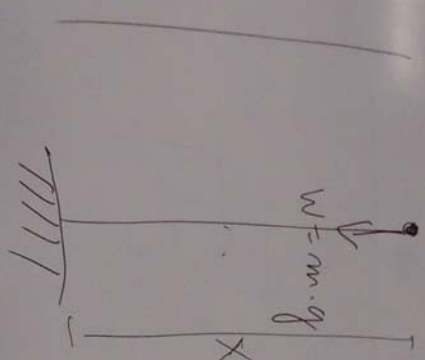
DEKAT-3
(DEGREE-3)

IMW



$$I = m \cdot v$$

$$I = m \cdot \frac{dx}{dt}$$



$$F = w - e \cdot v^2$$

$$m \left(\frac{dx}{dt} \right)^2 = m \cdot g - e \cdot \left(\frac{dx}{dt} \right)^2$$

PD ORDE-2

(STOCK LAW)

$$\textcircled{1} \frac{dy}{dx} = 3x^2 - 6x + 5$$

$$dy = (3x^2 - 6x + 5) dx$$

$$dy - (3x^2 - 6x + 5) dx = 0$$

$$\int dy - \int (3x^2 - 6x + 5) dx = \int 0$$

$$y - \left(\frac{3}{3}x^3 - \frac{6}{2}x^2 + 5x \right) = C$$

$$y - (x^3 - 3x^2 + 5x) = C$$

$$y = x^3 - 3x^2 + 5x + C$$

Solusi umum PD

Sol um PD

SEORANG PEDAGANG MENABUNGKAN UANGNYA DI BANK
DENGAN BUNGA 10% PER TAHUN.

KAPAN TABUNGAN (MODAL) MENJADI LIMA KALI LIPAT?

Solusi:

BUNGA PER TAHUN = 10% * M.

$$\frac{dM}{dt} = \frac{10}{100} \cdot M$$

$$\frac{dM}{M} = \frac{1}{10} \cdot dt$$

$$\frac{dM}{M} - \frac{1}{10} dt = 0$$

$$\int \frac{dM}{M} - \frac{1}{10} \int dt = \int 0$$

$$\ln M - \frac{1}{10} t = C$$

$$\ln M - \ln e^{\frac{1}{10}t} = \ln C$$

$$\ln \frac{M}{e^{\frac{1}{10}t}} = \ln C$$

$$\frac{M}{e^{\frac{1}{10}t}} = C$$

Tabungannya menjadi 5X lipat setelah 10ln5 TAHUN

YA DI BAWAH

$$5 = \ln e^5$$

LIPAT?

$$= \int_0$$

$$= C$$

$$= \ln C$$

$$M(t) = C \cdot e^{\frac{1}{10}t}$$

$$t=0 \text{ th} \rightarrow M(0) = C \cdot e^{\frac{1}{10}(0)}$$

$$M(0) = C$$

$$M(t) = M(0) \cdot e^{\frac{1}{10}t}$$

$$5 M(0) = M(0) \cdot e^{\frac{1}{10}t}$$

$$5 = e$$

$$\ln 5 = \ln e^{\left(\frac{1}{10}t\right)}$$

$$\ln 5 = \frac{1}{10}t$$

$$t = 10 \ln 5$$

② MISAL KAN JUMLAH PENDUDUK SEKARANG = $P(0)$
 JUMLAH PENDUDUK SETELAH 5 TAHUN MENJADI
DUA KALI LIPAT.
 SETELAH BERAPA TAHUN JUMLAH PENDUDUK
 MENJADI 10 KALI LIPAT ?

SOLUSI :

LAJU PERTAMBAHAN PENDUDUK :

$$\boxed{\frac{dP}{dt} = k \cdot P}$$

$$\frac{dP}{P} = k \cdot dt$$

$$\boxed{\frac{dP}{P} - k \cdot dt = 0}$$

$$\int \frac{dP}{P} - k \cdot \int dt = \int 0$$

$$\ln P - k \cdot t = C$$

$$\ln P - \ln e^{kt} = \ln C$$

$$\ln \frac{P}{e^{kt}} = \ln C$$

$$\frac{P}{e^{kt}} = C$$

$$P(t) = e \cdot e^{kt}$$

$$t=0 \text{ th} \rightarrow P(0) = e \cdot e^{k(0)}$$

$$P(0) = e$$

$$\rightarrow P(t) = P(0) \cdot e^{kt}$$

$$t=5 \rightarrow 2 P(0) = P(0) \cdot e^{k(5)}$$

$$2 = e^{5k}$$

$$\ln 2 = \ln e^{5k}$$

$$\ln 2 = 5k$$

$$k = \frac{\ln 2}{5}$$

$$P(t) = P(0) \cdot e^{\left(\frac{\ln 2}{5}\right)t}$$

$$10 \cdot P(0) = P(0) \cdot e^{\left(\frac{\ln 2}{5}\right)t}$$

$$10 = e^{\left(\frac{\ln 2}{5}\right)t}$$

$$\ln 10 = \ln e^{\left(\frac{\ln 2}{5}\right)t}$$

$$\ln 10 = \frac{\ln 2}{5} \cdot t$$

$$5 \ln 10 = (\ln 2) t$$

$$t = \frac{5 \ln 10}{\ln 2}$$

JUMLAH PENDUDUKNYA MENJADI 10 X LIPAT
 SETELAH $\frac{5 \ln 10}{\ln 2}$ TAHUN

Soal-soal:

1. $\frac{dy}{dx} = \frac{1+y}{2+x}$

$$\rightarrow y = c(2+x) - 1$$

2. $y(1-x) dx + x^2(1-y) dy = 0$

$$\rightarrow y = c.x.e^y e^{\frac{1}{x}}$$

3. $\frac{dy}{dx} = \frac{y^2 + xy^2}{x^2y - x^2}$

$$\rightarrow y = \frac{cx}{\frac{1}{1+y}}$$

4. $(1+2x^2) y y' = 2x(1+y^2)$

$$\rightarrow y^2 + 1 = c(1+2x^2)$$

5. $\cos^2 x \frac{dy}{dx} = y + 3$

$$\rightarrow \ln(y+3) = \tan x +$$

$$1) \boxed{\frac{dy}{dx} = \frac{1+y}{2+x}}$$

$$\frac{dy}{1+y} = \frac{dx}{2+x}$$

$$\boxed{\frac{dy}{1+y} - \frac{dx}{2+x} = 0}$$

$$\int \frac{dy}{1+y} - \int \frac{dx}{2+x} = \int 0$$

$$\ln(1+y) - \ln(2+x) = \ln c$$

$$\ln \frac{(1+y)}{(2+x)} = \ln c$$

$$\frac{1+y}{2+x} = c$$

$$(1+y) = c(2+x)$$

$$y = c(2+x) - 1$$

②

$$y(1-x)dx + x^2(1-y)dy = 0 \quad -\ln e^{\frac{1}{x}} - \ln$$

$$y(1-x)dx = -x^2(1-y)dy$$

$$\frac{(1-x)dx}{x^2} = -\frac{(1-y)dy}{y}$$

$$\left(\frac{1}{x^2} - \frac{x}{x^2}\right)dx = -\left(\frac{1}{y} - \frac{y}{y}\right)dy$$

$$\left(\frac{1}{x^2} - \frac{1}{x}\right)dx = -\left(\frac{1}{y} - 1\right)dy$$

$$\frac{dx}{x^2} - \frac{dx}{x} + \frac{dy}{y} - dy = 0$$

$$\int x^{-2} dx - \int \frac{dx}{x} + \int \frac{dy}{y} - \int dy = \int 0$$

$$\frac{1}{-2+1} x^{-2+1} - \ln x + \ln y - y = C$$

$$\underline{-\frac{1}{x} - \ln x + \ln y - y = C}$$

$$1) dy = 0 \quad -\ln e^{\frac{1}{x}} - \ln x + \ln y - \ln e^y = \ln c$$

$$\ln \frac{y}{e^{\frac{1}{x}} \cdot x \cdot e^y} = \ln c$$

$$\frac{y}{x \cdot e^{\frac{1}{x} + y}} = c$$

$$y = c \cdot x \cdot e^{\frac{1}{x} + y}$$

SOL. UM. PD

1.3 Typ

Persamaan
semuanya

Contoh:

1. $F(x)$

$F(x)$

$F(x)$

P.D. HOMOGEN

$$M(x,y) dx + N(x,y) dy = 0$$

$$\underbrace{(3x+5y)}_{M(x,y)} dx + \underbrace{(5x+3y)}_{N(x,y)} dy = 0$$

$$M(x,y) = 3x + 5y$$

$$N(x,y) = 5x + 3y$$

$$M(tx,ty) = 3(tx) + 5(ty)$$

$$N(tx,ty) = 5(tx) + 3(ty)$$

$$= 3(t)x + 5(t)y$$

$$= 5tx + 3ty$$

$$= t(3x+5y)$$

$$= t(5x+3y)$$

$$= t \cdot M(x,y)$$

$$= t \cdot N(x,y)$$

P.D. HOMOGEN DERAJAT (1)

MISALKAN:

$$z = \frac{y}{x}$$

$$\rightarrow y = x \cdot z$$

$$\frac{dy}{dx} = (x)' z + x \cdot (z)'$$

$$\frac{dy}{dx} = \frac{dx}{dx} \cdot z + x \cdot \frac{dz}{dx}$$

$$dy = z \cdot dx + x \cdot dz$$

$$dy = x \cdot dz + z \cdot dx$$

$$(3x + 5z) dx + (5x + 3z) dy = 0$$

$$(3x + 5xz) dx + (5x + 3xz)(x dz + z dx) = 0$$

$$(3x + 5xz) dx + (5x^2 + 3x^2z) dz + (5xz + 3xz^2) dx = 0$$

$$(3xz^2 + 10xz + 3x) dx = -(5x^2 + 3x^2z) dz$$

$$x(3z^2 + 10z + 3) dx = -x^2(5 + 3z) dz$$

$$\frac{dx}{x} = - \frac{(3z+5) dz}{3z^2+10z+3}$$

$$d(3z^2+10z+3)$$

$$\frac{dx}{x} + \frac{(3z+5) dz}{3z^2+10z+3} = 0$$

$$\int \frac{dx}{x} + \int \frac{(3z+5) dz}{3z^2+10z+3} = \int 0$$

$$\ln x + \frac{1}{2} \int \frac{\overset{(2)}{(3z+5)} dz}{d(3z^2+10z+3)} = C$$

$$\ln x + \left(\frac{1}{2}\right) \ln(3z^2+10z+3) = C$$

$$\textcircled{2} \ln x^2 + \ln(3z^2+10z+3) = \textcircled{2C} \quad \textcircled{\times 2}$$

$$\ln x^2 + \ln(3z^2+10z+3) = \ln C$$

$$\ln x^2(3z^2+10z+3) = \ln C$$

$$\begin{aligned}(3z^2 + 10z + 3) &= (6z + 10)dz \\ &= \textcircled{2} \underline{(3z + 5)dz}\end{aligned}$$

$$z =$$

$$x^2 (3z^2 + 10z + 3) = C$$

$$x^2 \left(3 \cdot \frac{y^2}{x^2} + 10 \cdot \frac{y}{x} + 3 \right) = C$$

$$3y^2 + 10xy + 3x^2 = C$$

$$3x^2 + 10xy + 3y^2 = C$$

Sol. UM. PD

$$\textcircled{1} \quad \frac{dy}{dx} = \frac{3y^3 - x^3}{3xy^2}$$

$$(3y^3 - x^3)dx = 3xy^2dy$$

$$\underbrace{(3y^3 - x^3)}_{M(x,y)} dx - \underbrace{3xy^2}_{N(x,y)} dy = 0$$

$$M(x,y) = 3y^3 - x^3 \leftarrow$$

$$M(tx, ty) = 3(ty)^3 - (tx)^3$$

$$= 3 \cancel{t^3} y^3 - \cancel{t^3} x^3$$

$$= \cancel{t^3} (3y^3 - x^3)$$

$$= \cancel{t^3} M(x,y)$$

$$N(x,y) = -3xy^2 \leftarrow$$

$$N(tx, ty) = -3(tx)(ty)^2$$

$$= -3(tx)(t^2y^2)$$

$$= -3 \cancel{t^3} xy^2$$

$$= \cancel{t^3} (-3xy^2)$$

$$= \cancel{t^3} N(x,y)$$

P.D. HOMOGEN DERAJAT - 3

MISALKAN :

$$\boxed{z = \frac{y}{x}} \rightarrow \boxed{y = x \cdot z} \rightarrow y^3 = (xz)^3 = x^3 z^3$$

$$\boxed{dy = x \cdot dz + z \cdot dx}$$

$$(3y^3 - x^3)dx = 3xy^2 dy$$

$$(3x^3 z^3 - x^3)dx = 3x(x^2 z^2)(x dz + z dx)$$

$$(3x^3 z^3 - x^3)dx = 3x^3 z^2 (x dz + z dx)$$

$$(3x^3 z^3 - x^3)dx = 3x^4 z^2 dz + 3x^3 z^3 dx$$

$$-x^3 dx = 3x^4 z^2 dz$$

$$-dx = 3x z^2 dz$$

$$-\frac{dx}{x} = 3z^2 dz$$

$$0 = 3z^2 dz + \frac{dx}{x}$$

$$\int 3z^2 dz + \int \frac{dx}{x} = \int 0$$

$$\frac{3}{3} z^3 + \ln x = C$$

$$\boxed{z^3 + \ln x = c}$$

$$\ln e^{z^3} + \ln x = \ln c$$

$$\ln x \cdot e^{z^3} = \ln c$$

$$x \cdot e^{z^3} = c$$

$$\boxed{x \cdot e^{\frac{y^3}{x^3}} = c}$$

$$\frac{y^3}{x^3} + \ln x = c$$

$$\frac{y^3}{x^3} = c - \ln x$$

$$\underline{y^3 = x^3 (c - \ln x)}$$